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Symmetry analysis of the floor ornaments of the San Marco cathedral in Venice

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Abstract

The marble floor ornaments of San Marco Cathedral in Venice are analysed according to their symmetries and motifs. We find that all the possible crystallographic plane symmetry groups are represented with the exception of threefold symmetric $p3$, $p3m1$, $p31m$, and sixfold symmetric $p6mm$. The occurrence of the fourfold-symmetry group $p4mm$ prevails with a rich collection of motifs. The wealth of this symmetry may have resulted from the aim to cover the surface with crosses in order to keep the evil forces away from the sacred place.

Keywords: Architecture, Art, Applied mathematics

1. Introduction

In this Report, I use the term San Marco Cathedral (SMC) for *Il Basilica di San Marco a Venezia* and analyse the symmetry of its tessellated marble floor, for which several expressions are being used, e.g., *sectili marciani*, *pavimentum sectile*, *opus sectile*, *opus tessellatum*. My appreciation for the marble floor of SMC began after studying the achievements of Mary Griep [1] and culminated in the observation of a digital versatile disk (DVD) containing a one-to-one orthophoto of the floor [2] in full resolution, which was the stimulus for the present report.

Beginning with the displacement of the bones of the alleged evangelist Saint Marc from Alexandria to Venice in the IX. century, SMC was founded and became the

major attraction in Venice as a Byzantine church until the present day. Its splendid interior and exterior appearance has improved in parallel and simultaneously with the expansion of the Venetian naval power in the Eastern Mediterranean, especially with the so-called Crusades [3]. The presence of the Lagoon, on the other hand, acts as an effective marine shelter, and Venice is one of the least destroyed civil centres in Europe by invading enemies. Thus, the beauty of the city and the SMC dominate, not least through the preservation and restoring efforts of the municipal authorities [4].

In addition to the magnificent architecture of the Byzantine church, two of its elements are of great cultural importance: The mosaics of the walls and domes and the marble floor. The mosaics depict events from the Old and New Testaments and have been the subject of extensive studies [5]. The marble floor, mostly consisting of beautifully tessellated ornaments, has aroused artistic enthusiasm [1, 6]. Due to the vulnerability of the floors to the external detrimental effects, like flood or human foot steps, there were serious destructions and also serious efforts of renovation [4, 7].

There exists a comprehensive account on SMC along with other churches of the Venetian Lagoon area [8]. The creation of ortophotos of the floor ornaments of SMC, however, has required an enormous effort during several years of technical work, and the collection of the photos represents an extremely valuable archive material [9].

At this point, I will only deal with the tessellated floor ornaments. The encyclopaedic work of Owen Jones [10] classifies the ornaments according to ethnic or cultural groups in which they were created. According to this work, the floor ornaments of SMC are considered *Byzantine*. Thus, the classical school uses the decorative elements, motifs, the harmony of colours and shapes as the key elements for the classification. This has the advantage of preserving the beautiful details of the artwork.

Besides these beauties, an earlier report [11] contains an indication of the presence of 17 symmetry groups hidden in the floor patterns of SMC. In the following Section, I will first discuss these symmetry concepts applicable to planar periodic patterns, report then on the procedures for extracting images of floor patterns from a DVD [2] and classify the captured images according to their symmetry. The next Section presents the results based on symmetry groups. I will continue to report on the detailed structure of the patterns by analysing the fundamental regions, their motifs. The last Section contains the Discussions.

2. Methods

Decorations and ornaments, in particular regularly tessellated patterns, are a collection of intelligent signs created by men of a cultural group. They are expressions and basic tools of human intelligence to process and pass it on to others.

This information is characteristic of the group of craftsman, artisans or architects who express their state of mind, developed and accumulated over time. Therefore, the ornaments are characteristic of the ethnic or cultural group, and one wants to identify this group and to learn details about it by studying the ornaments and symmetries contained therein [12, 13].

The routine studies of the ornaments are established in archaeology and art history. They are mainly based on the detailed analysis of motifs that make up the ornaments and decorations and emphasise the harmony of colours and figures [1, 6, 10]. The seminal work on this field is the doctoral thesis of Edith A. Müller at the University Zurich, who introduced another approach to the study of ornaments [14]. Her method consists of determining the *space groups* of two-dimensional repeating patterns using Group Theory. This method of classification of the patterns presents a unique advantage that it is rigorous, easily documented, and not affected by personal impression [15].

Müller's novel technique was based on a theory developed by a group of mathematicians who proved that a two-dimensional regular pattern, like a crystal surface, can be described with just 17 different symmetry groups [16, 17]. During her dissertation Müller had the ideal mentoring by Andreas Speiser in the Department of Mathematics [18]. Another mathematician in Zurich, George Pólya [19], had been the source of inspiration for the scientists of the field. According to this classification scheme, which revolutionised the field of pattern analysis, the ornamental patterns are divided into discrete groups, as is the case in crystallography. In her dissertation, Müller has found that all but four of the symmetry groups are represented in the Alhambra Palace in Granada, Spain. Müller's dissertation has encouraged several researchers to apply her technique and further investigate the number of space groups represented in the Alhambra and to find out whether the missing members are present in other Mozarabic artefacts or similarly absent.

The pioneering work of Müller in the Alhambra has paved the way for the study of Islamic geometric patterns in Western Asia, the Middle East, including Asia Minor, and Northern Africa as well as in the Far East. To the best of my knowledge, there has been no systematic investigation of the tessellations created during the Byzantine period using the technique of Müller. This report is intended to be a first study and is partly presented at the 16th Architectural Biennale in Venice, 2018, in the Turkish Pavilion [20].

2.1. The mathematical background

The symmetry properties of an ornament give us the connection with the theory of mathematical groups. A practical way to understand the concept of symmetry

can be visualised by applying possible point-group operations on a motif. These operations are *rotations* through an axis perpendicular to the plane (*cf.* Figure 1a) and *mirror reflections* in the plane (*cf.* Figure 1b) and do not move the motif in the two-dimensional space. The application of these operations creates 10 different two-dimensional *crystallographic point groups* [17]. In the next step, each of these groups is embedded into the 5 possible plane lattices, the *Bravais lattices*, to cover the two-dimensional space. In this operation the object is moved continuously by *translation vectors* in two different directions. Figure 1c shows *translation* t in one direction. Five *Bravais lattices* thus constructed consist of oblique, primitive or centred rectangle, square, and hexagonal lattices. The combination of the point-group symmetry with the plane lattice is responsible for a further operation, the *glide reflection* (*cf.* Figure 1d), and this combination results in 17 translational planar groups [17]. They can usually be found on fabrics, wall and floor coverings or wallpapers and are therefore referred to as *wallpaper group*. The designation of each symmetry follows a four-digit recipe, as acknowledged by the International Union of Crystallography [21].

In a periodic pattern of unknown symmetry, one first determines by inspection the *unit cell* that repeats itself to cover the entire surface without leaving blank spaces or producing overlaps. It is the *translation vectors* that span the unit cell and carry it in two dimensions. The lattice thus constructed can be *primitive* or *centred*. This finding determines the first letter of group designation. It is a p for the *primitive* or c for the *centred* lattice. In the next step, we search for rotations in the pattern that map it to itself. The angular part n of 2π is called the order of rotation (*cf.* Figure 1a), which can be 1, 2, 3, 4, and 6. Accordingly, the pattern is called *e.g.* threefold symmetric for $n = 3$. The order $n = 5$ is crystallographically impossible, because fivefold symmetry is not compatible with the *translational symmetry*, and $n > 6$ would result in overlaps. The order of rotation is the second digit in the group designation. The

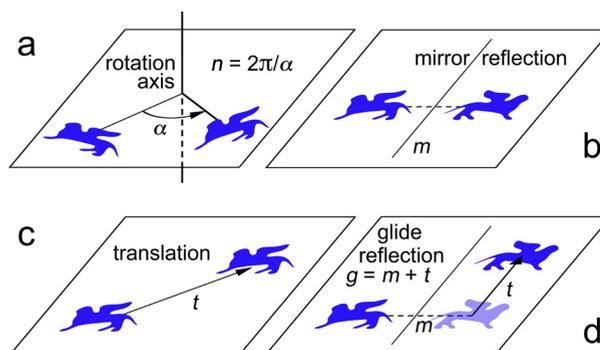


Figure 1. Four symmetry operations that move an object onto itself in a pattern. a. Rotation of the order n around an axis normal to the plane, b. Mirror reflection m across a line in the plane, c. Translation by t , a linear displacement of the shape. d. Glide reflection g across and along a line. a and b are point-group operations that do not move the object away from its location.

third digit is either a *mirror reflection* m or a *glide reflection* g that is present in the pattern perpendicular to the *translation vectors*. The last digit is also an m or a g along another major axis. If there is no *mirror* or *glide*, one writes 1. Grouped in rotational symmetries, one differentiates the following elements: $p1$, $p1m1$, $p1g1$, $c1m1$, $p211$, $p2mm$, $p2mg$, $p2gg$, $c2mm$, $p4$, $p4mm$, $p4gm$, $p3$, $p3m1$, $p31m$, $p6$, $p6mm$. The groups $p1$ and $p2$ are embedded in an oblique lattice. Other rotations of order $n = 2$ call for a rectangular lattice, $n = 4$ for square, and $n = 3$ and $n = 6$ for hexagonal lattices. The *motif* is the smallest, not further divisible area of the *unit cell* upon which the appropriate symmetry operations are applied in order to construct the *unit cell*.

2.2. Extraction of images of floor ornaments

A DVD, containing more than 650 000 jpg-formatted files, presents the *opus sectile* in a one-to-one precision of unprecedented resolution [2]. These files are flash-assembled into a single image of the floor that can be viewed on any magnification and moved on the computer screen. I have observed the floor tiles on a 22"-large screen and recorded the images of the desired ornaments. I have noted the exact position on a relatively large plan of SCM obtained from Ref. [8]. I took care of only recording areas with more than two periodicities. In several locations there are patterns with varying periodicity and motif size like in a rosette. They have not been included. The errors of the craftsmen, temporal degradations, and material faults were also not considered. Consistent colours have been properly accredited. I obtained 730 discrete domains with a particular symmetry. I estimate my error in this procedure to be around 1%.

A visual inspection showed that there are several places with the same symmetry. I was often able to determine the symmetry properties of a region by mere visual inspection using the steps described above in Section 2.1. Sometimes I had to consult to a transparent copy of the image to safely determine the isometries that map the pattern to itself, i.e., rotations, mirror and glide reflections. Once the isometries were determined, I used the ‘school recipe’ [17] to obtain the wallpaper group of the ornament. I also estimate my error in this process to be about 1%.

3. Results

3.1. Symmetry groups

Table 1 summarises the results. There are a few striking points worth mentioning. The symmetries $p1g1$, $p2gg$, and $p4$ occur suspiciously rare, while the symmetries $p3$, $p3m1$, $p31m$, and $p6mm$ do not occur at all. The group $p4mm$, on the other hand, accounts for more than 78% of all occurrences, and the related $p4gm$ almost another

Table 1. Occurrence of each symmetry group of the floor ornaments. Rotational orders are grouped together.

Symmetry group	Occurrence
<i>p1</i>	35
<i>p1m1</i>	19
<i>p1g1</i>	2
<i>c1m1</i>	8
<i>p211</i>	4
<i>p2mm</i>	27
<i>p2mg</i>	8
<i>p2gg</i>	1
<i>c2mm</i>	6
<i>p3</i>	0
<i>p3m1</i>	0
<i>p31m</i>	0
<i>p4</i>	2
<i>p4mm</i>	573
<i>p4gm</i>	36
<i>p6</i>	9
<i>p6mm</i>	0

5%. One reason for the dominance of these fourfold-symmetric patterns is the fact that they are used in several auxiliary areas.

Figure 2 presents colour drawings of selected ornaments, mentioned in Table 1. Only one example is shown for each symmetry group. The images in the figure are arranged in the same manner as they are mentioned in Table 1. There are 4 groups of rotation with order $n = 1$, five groups of $n = 2$, three groups of $n = 4$, and one group of $n = 6$. The group $n = 3$ in the floor ornaments is not represented at all. The symmetry operations are indicated within a *unit cell*, which is spanned by two *translation vectors* (blue arrows). *Translation* is present in all groups. *Mirror* (red) and *glide* (green) reflections are drawn whenever they do not coincide with *translation vectors*. Rotations of order 2 (diamonds), 3 (triangles), 4 (squares), and 6 (hexagons) are also represented.

The predominance of a single group implies a dull appearance of the floor of SMC and raises a question about the artisans who produced it. It even suggests that the floor coverage was created within a short period of time or acquired from a source specialised in such a work.

This speculation is erroneous and diminishes the artistic value of the *marble carpet*. The dominance of *p4mm* does not necessarily mean that there are simple square shapes that occupy more than 80% of the total area of the floor. It can only be understood that the majority of the motifs are arranged in fourfold orientation, without any statement about the motifs themselves. It is only the property of the mathematical procedure that overestimates the overall symmetry and disregards the details of the motifs.

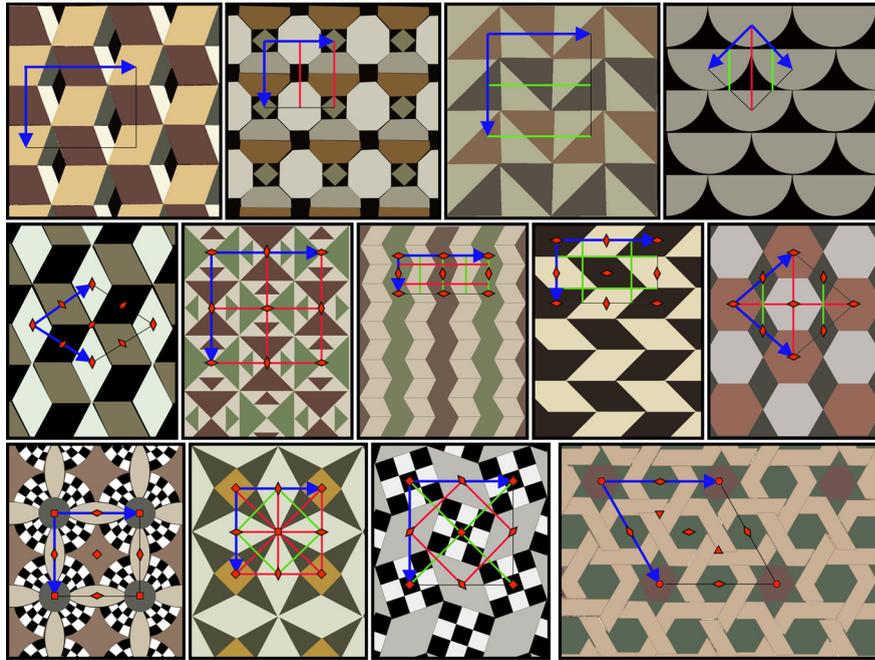


Figure 2. A collection of floor ornaments found in the San Marco Cathedral together with the appropriate symmetry properties superposed on the images (see text for details). From left to right, top row: $p1$, $p1m1$, $p1g1$, $c1m1$; middle row: $p2$, $p2mm$, $p2mg$, $p2gg$, $c2mm$; bottom row: $p4$, $p4mm$, $p4gm$, and to the right $p6$. The drawings are composed by inspection of images that are adopted from a DVD provided by the *Procuratoria di San Marco, 30124 Venice*; their orientation and magnification are modified in order to accommodate the appropriate *unit cell* of similar size in all groups.

Indeed, each motif can have any shape designed by the artisans; this is the *local* property of an ornament with characteristic details. Hence, $p4mm$ implies a square shape for the *unit cell* that constitutes the entire ornament, and that is its *global* appearance.

In order to shed light onto the relationship between motif and ornament, Figure 3 shows a part of the southern façade of the Ducal Palace at St. Marc's Square in Venice, in registry with the symmetry elements of the wallpaper group $p4gm$. It consists of fourfold- and twofold-rotation axes, which are represented as tiny green squares and violet diamonds, respectively. Red lines denote axes of *mirror reflection*, green lines are the axes of the *glide reflection*. We observe the large square made up by tiny green squares, indicated by fine black lines connecting four green fourfold-symmetry axes. This is equivalent, but not identical, to the square made up by tiny violet diamonds. We can consider both as a *unit cell* of the entire pattern, i.e., the whole façade can be covered by *translation* of the *unit cell* in two dimensions without voids and overlaps. Consider the 'black' square, which consists of eight equivalent areas, one of which is shaded blue on the figure. This is the motif, the smallest independent element of the structure. By applying symmetry operations characteristic of $p4gm$ to the motif, the unit cell is constructed. Thus, the *local* motif

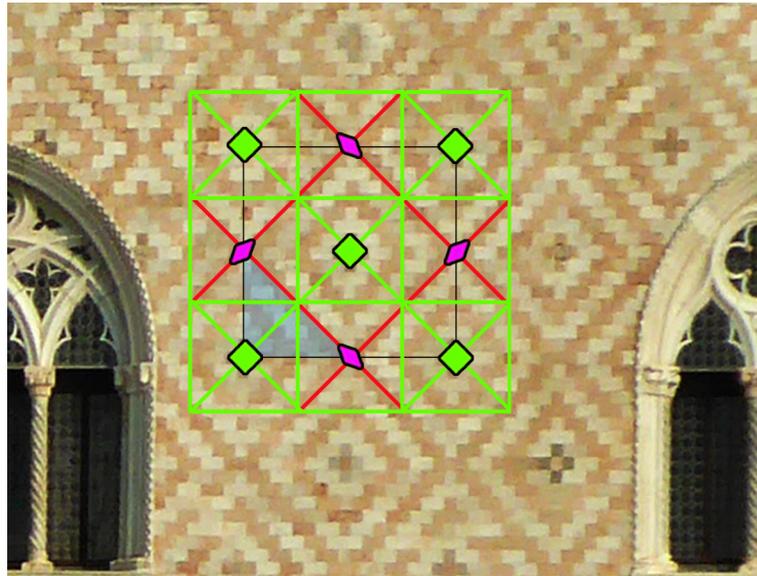


Figure 3. The southern façade of the Ducal Palace in Venice with superimposed symmetry elements of the group $p4gm$, drawn in registry. One unit cell of the pattern is indicated by fine black lines. The motif of the symmetry group is highlighted in blue.

is the seed of the *global* main structure. We can therefore say that the main features of the wall covering are contained in the motif. Hence, the motif deserves to be referred to as the *fundamental region* of a pattern.

Figure 4 illustrates the motif and the unit cell of the pattern on the Ducal Palace to demonstrate the relationship between the motif and the unit cell. To generate the unit

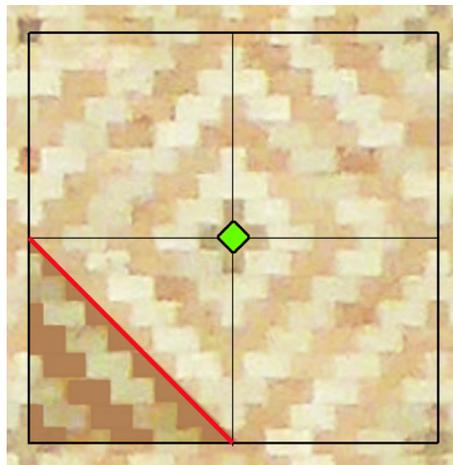


Figure 4. The motif in vivid colours and the resulting unit cell after applying the symmetry operations of the group $p4gm$ in fading colours. The small green square is a fourfold-symmetry axis and the red diagonal line is a mirror-symmetry axis. The large square within the black lines indicates the unit cell, and the motif corresponds to the blue-shaded area in the previous figure. One fourfold-symmetry axis is shown as a small green square in the centre of the unit cell. The pattern comes from the southern façade of the Ducal Palace.

cell, the motif in vivid colours is first reflected across the red diagonal in the figure, which represents one of the mirror axes of the symmetry group, and subsequently the resulting square is rotated three times around the fourfold-symmetry axis each by $2\pi/4$ to complete the large square within the black border lines. The thin black lines are drawn as an aid to the eye.

Now we proceed the reverse way, starting off with the motif and generating the entire ornament. Figure 5 shows an example of the $p4mm$ symmetry group with black and white inlays on the upper left side. The lines represent mirror reflections. The motif is reflected across either the vertical or the diagonal mirror line, followed by three times a rotation by $2\pi/4$ around the centre, required by the $p4mm$ symmetry. This process leads to the unit cell shown below. The entire structure, the square lattice, is generated by subsequent translation of the unit cell in two mutually perpendicular directions.

Quite frequently, the motif changes, but the overall symmetry can remain the same. In the following, I will therefore concentrate on the individual motifs of the 573 floor ornaments, which all belong to the $p4mm$ symmetry group. They contain colourful examples of artistic work of the artisans instead of limiting the classification of the floor ornaments by determining the *global* symmetry groups.

3.2. Motifs

The motif of an ornament is the smallest element that rotates, mirror or glide reflects, and finally translates though the entire two-dimensional space to complete

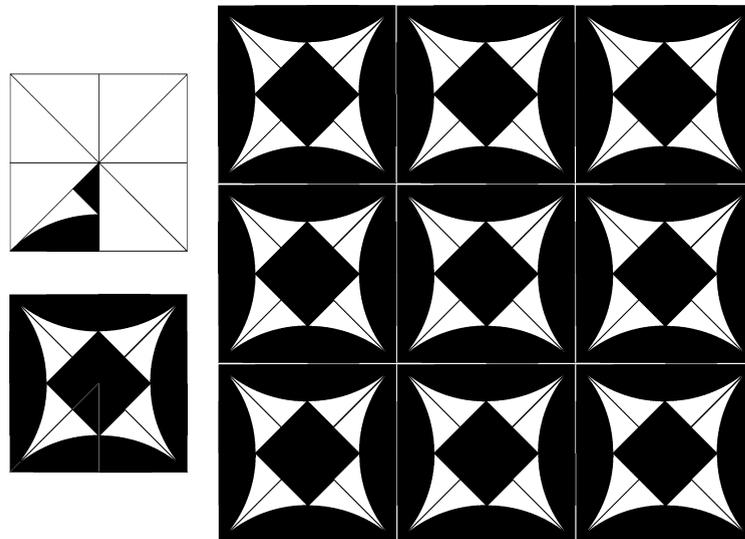


Figure 5. The motif is shown in the upper part on the left-hand side of the figure, the unit cell below it. The translation of the unit cell in two directions creates the entire ornamental pattern, as can be seen on the right-hand side. This ornament is located north of the *Atrium*, north of the *West Dome*, and west of the *North Dome*.

the ornament. One example is shown in Figure 4 for the symmetry group $p4gm$, another in Figure 5 for the $p4mm$. The number of motifs in these two examples is 8; in each 17 ornament group it is 1, 2, 2, 2, 2, 4, 4, 4, 4, 4, 8, 8, 3, 6, 6, 6, and 12, respective the groups mentioned in Section 2.1 and listed in Table 1. In surface crystallography, these numbers correspond to the number of atoms in the unit cell in one-atomic systems. The motifs of SMC are aesthetically designed and skilfully decorated with marbles and other precious stones from the Eastern Mediterranean to complete the pattern [22].

We now focus on the motifs belonging to the $p4mm$ symmetry group observed on the tessellated floor of SMC. I have found 573 different regions displaying this symmetry. The analysis of these 573 groups reveals that there are 40 different motifs that make up these ornaments. They are shown in Figure 6. I have arranged these motifs according to the distinctive shapes that are the main components: Some show circular segments (4, upper row), some thick flat arrows (10, lower ten motifs), others rhombi (6, third group of rows below), and the last group has squares (20, last four rows). The numbers in round brackets correspond to the number of occurrences with a total of 40 different motifs. Here the motif is placed in the unit cell, each corner of which is a fourfold-rotation axis, as well as the centre which is a different type of fourfold-rotation axis. The midpoints of the edges are all twofold-rotation axes. All the lines represent a mirror symmetry axis.

The beauty of these motifs is unprecedented. The circular segment in the 4 motifs in the upper group and the adjacent stone can be the most difficult shape to create. One of these motifs was examined in more detail in a recent report [20]. The next group of 10 motifs contains components whose corners are multiples of 45° . Only in the next group of diamond shapes other angles are found. The last group of 20 motifs has a majority of square-shaped fields that occupy at least half of the total area. The last motif has ten different inlays, while the first motif has only one.

4. Discussion

There exists a description of the floor pavement, relating the pattern together with the mosaics to Judeo-Christian legends [23]. Beyond this description, Figure 7 illustrates the rigorous spatial distribution of the symmetry groups of ornaments on the floor of the SMC with the intension of possibly obtaining some hints about the reason for the uneven distribution. This figure is an abstraction, since the floor plan of SMC is shown in scale, but only the ornaments are visible; they are coloured according to their rotational symmetry. There are few sixfold-symmetric regions ($p6$ in brown) at the *West Vault* (WV) and under the *Central Dome* (CD) in the middle of the figure. The letters in parentheses are the abbreviations in Figure 7. The groups $p1$ and $p1m1$ make up about 85% of all symmetries with $n = 1$. I have grouped them in red

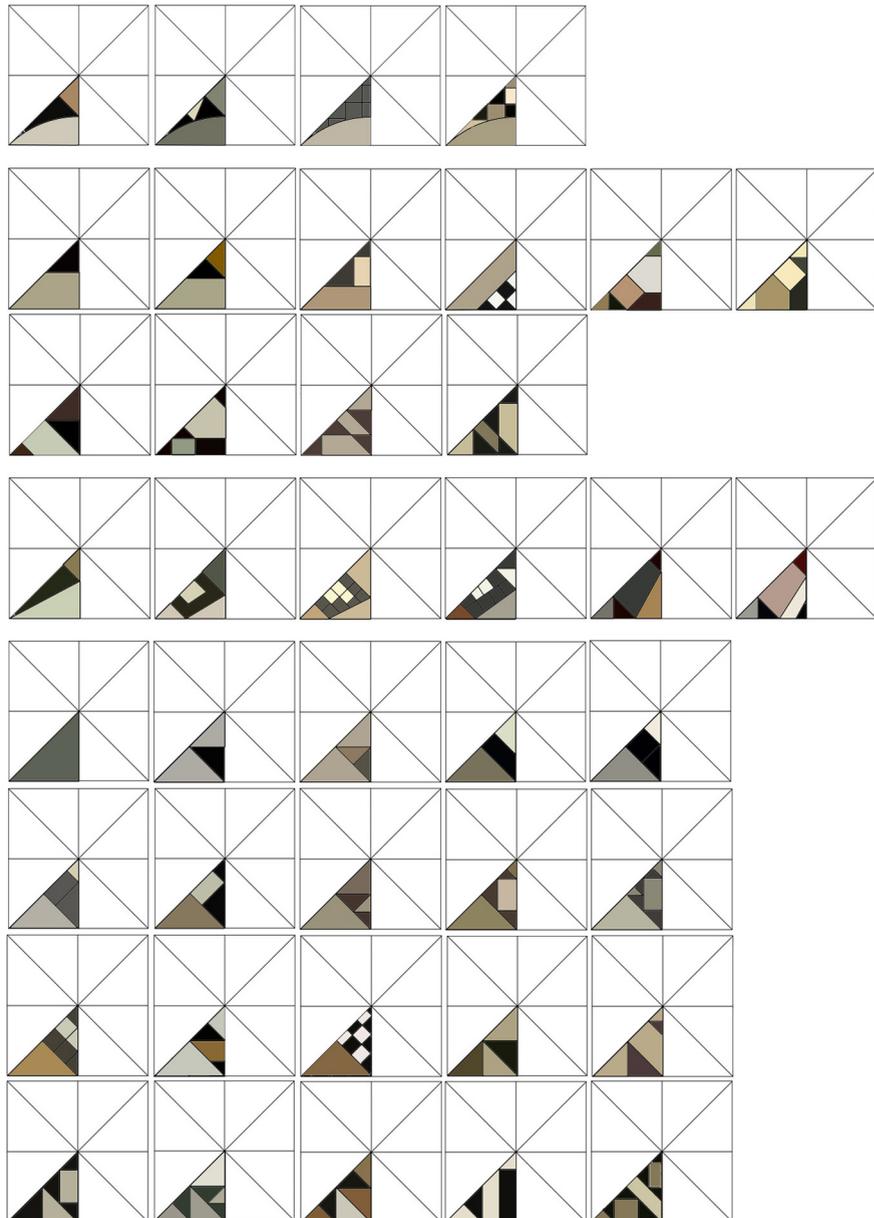


Figure 6. The motifs of the entire floor ornaments belonging to the $p4mm$ group, presented in four groups, embedded in the unit cell. Observe the great diversity of fundamental regions, which all generate the same symmetry group after the application of the appropriate operation.

colour. These ornaments are mainly below the *South Dome* (SD). There are some other red areas distributed almost randomly throughout the entire floor, except for the *Main Entrance* (ME). Green areas are ornaments with rotational symmetries of order $n = 2$. It is interesting that the floor of the *Chapel of St. Zeno* (CZ) is uniformly paved with such ornaments. Some twofold symmetric ornaments are found near the *Capella di San Pietro* (SP). Ornaments with symmetries $n = 4$ predominantly consist of $p4mm$; they cover almost the entire floor, but in particular the *Main Apse* (MA),

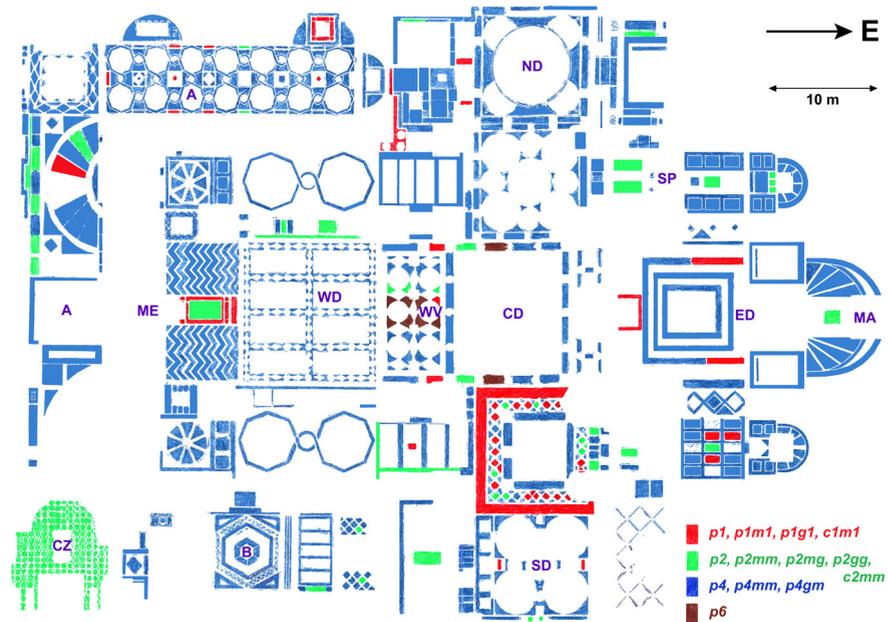


Figure 7. The floor of the San Marco Cathedral high-lighted with colours indicating the rotational symmetry of the local pavement. No threefold-symmetric ornaments could be found. Selected locations are indicated with letters (see text for abbreviations).

the *East Dome* (ED), the *North Dome* (ND), the *West Dome* (WD), the *Atrium* (A), and the *Baptistry* (B).

The location of each ornament, depicted in Figure 2, is found in Figure 7. The example for $p1$ is found west of the *North Dome*, $p1m1$ around north of the *South Dome*, $p1g1$ below the *South Dome*, and $c1m1$ north of the *Atrium*. The examples for $n = 2$ are located as follows: $p211$ south of the *Central Dome*, $p2mm$ and $p2mg$ north of the *East Dome* and near the *Capella di San Pietro*, $p2gg$ south of the *East Dome*, and $c2mm$ covers the entire floor of *Chapel of St. Zeno*. The example shown for $p4$ is located west of *Baptistry*, $p4mm$ is found north of the *Atrium*, and $p4gm$ north of the *West Dome*. $p6$ is seen at the south and north of the *Central Dome* and at the *West Vault*.

In early Christianity, crosses of different kinds and forms all are used to symbolise the spirit of Jesus Christ; it was him who kept Satan and similar evil forces away. To prevent Satan from entering a room, crosses were placed at the windows and entrances, preferably on the floor. The early churches in the Middle East frequently contain an abundance of crosses in the floor pavements. With the evolution of Christianity, this habit became increasingly exaggerated and degenerated into a pagan practice of superstition. Finally, the Byzantine Emperor Theodosius II issued an edict in 427 prohibiting the depiction of crosses on floors. This also preserved respect for the sacred site [24].

In the following centuries, Theodosius' edict was not followed seriously, and the Quinisext Council, which met in 692 in Constantinople under Justinian II, reissued the earlier canons prohibiting crosses and equivalent motifs to be placed on floors.

A clever solution to avoid the conspicuous exposure of crosses on floors is to use them in a concealed way, by adopting their global symmetry properties. The equal-armed Greek and the Maltese crosses both have fourfold symmetry. I suspect that a floor ornament with fourfold symmetry can represent a cross and would therefore serve the original ecclesiastical purpose of keeping the forces of evil away from the sacred place.

The majority of the *opus sectile* ornaments of SMC has fourfold symmetry. I am tempted to claim that these parts of floor were created with the purpose of repelling the evil forces from the church. Even if the individual craftsman did not create the floor ornaments directly with this idea in mind, the habits of these professional workers could have been influenced by this idea during centuries. Thus, the dominance of the fourfold-symmetrical ornaments is nearly to be expected. We have seen in Figure 6 forty different kinds of motifs which are used to generate the fourfold-symmetric ornaments. Observing these motifs alone gives no hint about a cross or anything similar. This speaks for the intelligence of the artisans who have so cleverly concealed the cross, if my conjecture about the fourfold symmetry is true.

The lack of threefold-symmetry groups is not easily conceivable. As a reaction, one is tempted to associate the threefold symmetry with the Holy Trinity and is deeply surprised that there is not a single ornament in this symmetry. In conclusion, I have found that the floor ornaments of SMC show a distinctly uneven distribution of 17 planar symmetry groups that provides a unique character and allows an easy comparison with other church buildings. Unfortunately, such studies expect the attention of art historians or Byzantine experts and alike. With this preliminary work such efforts are to be promoted.

Declarations

Author contribution statement

Mehmet Erbudak: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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The authors declare no conflict of interest.

Additional information

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