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Skin tissue responses to transient heating with memory-dependent derivative

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ABSTRACT

In this work, the new concept of "memory dependent derivative" in the Pennes' bio-heat transfer process of skin tissues is employed to investigate the one-dimensional problem of a skin tissue under sinusoidal heat flux conditions. Laplace transform technique is utilized to solve the problem. We investigate, numerically, the bio-heat transfer equation with memory-dependent derivative to find the effect on the tissue temperature of the kernel function and the time-delay parameter which are characteristic of memory dependent derivative heat transfer. Correlations are made with the results obtained in the case of the absence of memory-dependent derivative parameters. The effects of the time-delay on the temperature distribution in skin tissue for different forms of kernel functions are examined.

1. Introduction

Understanding the temperature distribution resulting from Bio-heat transfer and heat-actuated pressure in blood-perfused tissues is vital in many medical therapies and physiological examinations. It is essential for the improvement of biomedical equipment used for thermotherapy and design of heating or cooling garments (Arkin et al., 1994).

Current clinical medications and pharmaceuticals, such as, cryosurgery, cryopreservation, growth hyperthermia, and thermal malady diagnostics require the comprehension of temperature conduct in living tissues. Moreover, skin burns caused by presenting human body to fire or to being in contact with hot substances are probably the most regularly experienced perils in day by day life and in industry. Along these lines, contemplating bio-heat transfer in human bodies has been a hotly debated issue and is valuable for adequately outlining clinical thermal treatment hardware, for precisely assessing skin consume, and for setting up thermal securities for different purposes (Zhao et al., 2005).

During recent years, the comprehension of thermal and mechanical properties of human tissues has advanced by gigantic steps by the usage of central designing standards in the investigation of many heat and mass transport applications. During the previous two decades, there has been an undeniably serious enthusiasm for the study of bio-heat exchange phenomenon, with specific accentuation on remedial and analytic applications. Depending on innovative computational methods, the improvement of complex scientific models has significantly upgraded our capacity to break down different kinds of bio-heat transfer process. The coordinated efforts among physiologists, clinicians, and specialists in the bio-heat exchange field have brought about changes in counteractive action, treatment, safeguarding, and insurance procedures for organic frameworks. This includes utilization of heat or cold medications to devastate tumors and to enhance patients' states after cerebrum damage, and to the insurance of safety of people from extraordinary ecological conditions.

The main contribution in this field was made by Pennes (1948), who published a study on the temperature distribution in the human body and constructed a scientific model depicting heat stream inside natural tissue. It was a fundamental paper and the created scientific model (Bio-heat equation) has turned out to be broadly utilized for depicting heat conduct in natural tissue. Pennes added to the fundamental thermal condition a medium reaction convective term intended to portray the alleviating impact of blood stream. This term portrays heat transport by means other than dissemination.

The Pennes' bio-heat transfer equation uses the classical Fourier law (PCF) to evaluate the thermal conduct. Unfortunately, Fourier's law predicts a limitlessly quick spread of thermal signal, clearly in contrast with physical reality. It identifies with heat flux ($\boldsymbol{q}$) in the following way

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$$\boldsymbol{q}(x,t) = -\ k\ \nabla T(x,t) \tag{1}$$

where $T(x, t)$ is the temperature at point x and time t and k is the thermal conductivity.

The modeling of heat-related phenomena such as bio-heat transfer and heat-induced stress is useful for the development of biological and biomedical technologies, such as thermotherapy and design of heating or cooling garments (Arkin et al., 1994). Wissler (1998) demonstrated the legitimacy of Pennes' model when contrasted with typical heat appropriation in living tissue. Pennes' model precisely portrays the decay of the temperature from the center of the body to the surface (Cundin et al., 2009). Dehghan and Sabouri (2012) introduced a numerical solution for Pennes' bio-heat transfer equation by using a spectral element method to study the thermal behavior in living tissue. Das and Mishra (2014) studied the equivalence of Pennes' bioheat equation and Wulff continuum model for one dimensional planar tissue.

Cattaneo and Vernotte theory (1958a, 1958b) allows for the existence of thermal waves, which propagate at finite speeds and considering the delay between the temperature gradient and the energy flux. Under this theory, Fourier's law is replaced with

$$\boldsymbol{q}(x,t+\tau) = -\ k\ \nabla T(x,t) \tag{2}$$

where $\tau = \lambda/\kappa^2$ denotes relaxation time of the tissue, λ thermal diffusivity and κ velocity of wave propagation in the medium. The relaxation time represents the time lag required to establish steady heat conduction in a volume element once a temperature gradient has been imposed across it. Here it is defined as the characteristic time needed for accumulating the thermal energy required for progressive transfer to the nearest element within the non-homogeneous inner structures (Kaminski, 1990). This characteristic time can be caused by the non-equilibrium between fluids and solids, as well as by the effect of cell membrane which acts as the energy storage and conversion element in biological systems. This theory fails in some cases since it still establishes an instantaneous response between the temperature gradient and the energy transport.

Equation (2) ordinarily leads to a hyperbolic heat transfer equation. Starting from Maxwell's idea (Truesdell and Muncaster, 1980) and from the papers (Glass and Vick, 1985 and Dreyer and Struchtrup, 1993), an extensive amount of literature (Ignaczak and Ostoja-starzewski, 2009; Chandrasekharaiah, 1998; El-Attar et al., 2019; Lata 2019a,b; Abbas and Youssef, 2012; Ezzat, 2004; Ezzat and Abd-Elaal, 1997; Ezzat and El-Karamany, 2003) have contributed to the elimination of the paradox of instantaneous propagation of thermal disturbances.

Among the few works devoted to the applications of Cattaneo theory to Pennes' bio-heat transfer equation, we can refer to the survey of Liu (2000), who studied the mechanism of the wave like behaviors of heat transfer in living tissues through introducing a new concept of multi-mode energy coupling. Numerical calculations were performed to study the temperature transition in a three layers stratified skin with different thermal parameters. The thermal wave model of bio-heat transfer was introduced to investigate the physical mechanics and behaviors of heat transfer in living tissues by Yang (1993), Ozisik and Tzou (1994) and Liu et al. (1999).

In recent studies, it has been demonstrated that fractional calculus assumed an extremely pivotal job in clarifying the idea of genuine issues. This theory has been confirmed as a more conservative and proper methodology for taking care of these issues in contrast with the traditional methodology. Povstenko (2005) has inferred a quasi-static uncoupled thermoelastic model dependent on the heat conduction equation with fractional order time derivatives. He utilized the Caputo fractional derivative (1967) and got the stress components to the fundamental solution of a Cauchy problem for the fractional order heat conduction equation for both the one-dimensional and the two-dimensional cases.

Ezzat (2011, 2012) presented a fractional order theory of continuum mechanics in which heat conduction law was proposed as

$$\boldsymbol{q}\ +\ \frac{\tau^{\beta}}{\Gamma(1+\beta)}\ \frac{\partial^{\beta}\boldsymbol{q}}{\partial t^{\beta}} = -\ k\ \nabla T\left(x,t\right) \tag{3}$$

where β is the fractional order parameter. Lately, Sherief et al. (2010) set up another model of fractional heat conduction. One can refer to Abbas (2015), Sherief and Raslan (2016), Sherief and Abd El-Latief (2013, 2014), Aldawody et al. (2019), Hendy et al. (2019a) and Lata (2019a,b) for an overview of utilizations of fractional calculus.

A new mathematical model for Pennes' bio-heat equation using the methodology of fractional calculus was constructed by Ezzat et al. (2014a). Ghanmi and Abbas (2019) introduced the bioheat equation under fractional derivatives to study the thermal damage within the skin tissue during thermal therapy. Zhu et al. (2002) estimated the deposition of light energy in tissue and the rate process model for the thermal injuries by using the theory of diffusion. Gupta et al. (2010, 2013) introduced a homotopy perturbation method and the finite-decomposition method. Esneault and Dillenseger and Esneault (2010) used the finite difference method to study the improvement of temperature over time in hypothermia. Diethelm (2010) analyzed a fractional differential equation utilizing the methodology of Caputo-type (1967).

The memory-dependent derivative is defined in an integral form of a common derivative with a kernel function on a slipping interval. So this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). Its definition is more intuitive for understanding the physical meaning and the corresponding memory dependent differential equation has a force that is more expressive. Caputo (1967) gave the accompanying definition to fractional order derivative of function $f(t)$ of order α as:

$$D_a^{\beta}f(t) = \int_a^t K_{\beta}(t-\xi)\, f^{(\varepsilon)}(\xi)\ \mathrm{d}\xi, \quad K_{\beta}(t-\xi) = \frac{(t-\xi)^{\varepsilon-\beta-1}}{\Gamma(\varepsilon-\beta)}; \quad \varepsilon-1<\beta<\varepsilon$$

and

$$D_{\varepsilon}^{\beta}f(t) = \frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}f(t), \quad \beta=\varepsilon, \quad \beta>0, \quad t>\varepsilon$$

where $K_{\beta}(t-\xi)$ is the kernel function and $f^{(\varepsilon)}$ denotes the common ε-order derivative, which has a specific physical meaning.

From the above definition, it is clear that the fractional derivative at time t is not characterized locally at time t but relies upon the aggregate values of the derivatives on the interval $[\epsilon, t]$. Consequently, this idea of fractional derivatives can be utilized to portray the variety of a system in which the current change rate relies upon the past state, which is known as the memory impact.

In any case, we realize that the memory impact of a genuine procedure essentially results from the values in a segment of time $[t-\xi, t]$, where $\xi>0$ can be referred to as the time-delay. Disregarding a few utilizations of fractional calculus, it has a few bad drawbacks. Because of this, the idea of fractional order derivative has been replaced by Wang and Li (2011) by what has been named as memory dependent derivative. This new derivative can be written mathematically as:

$$D_{\omega}^{\beta}f(t) = \frac{1}{\omega}\int_{t-\omega}^{t} K(t-\xi)\, f^{\beta}(\xi)\ \mathrm{d}\xi, \tag{4}$$

where ω is the time delay and $K(t-\xi)$ is the kernel function. From the viewpoint of applications, different processes need different kernels to reflect their memory effects, so the kernel form $K(t-\xi)$ can also be chosen freely which may be more practical. The kernel is a monotone function with $K=0$ for the past time $t-\xi$ and $K=1$ for the present time t. In case, $K(t-\xi)\equiv 1$, we have

$$D_\omega f(t) = \frac{1}{\omega}\int_{t-\omega}^{t} f^{'}(\xi)\,\mathrm{d}\xi = \frac{f(t)-f(t-\omega)}{\omega} \to f^{'}(t)$$

This means that the common derivative $\mathrm{d}/\mathrm{d}t$ is the limit of D_ω as $\omega \to 0$.

In fact, the memory effect of a real process occurs on a segment of time, i.e., on the delayed interval $[t-\xi, t]$ ($\xi(>0)$ indicates the time-delay). Enlightened by these, the novel concept of derivative was initiated as the "memory-dependent derivative" (MDD) to reflect the memory effect in a distinct manner. One may state that the definition of MDD is more intuitive in realizing the physical significance and accordingly, the corresponding memory-dependent differential equations are more effective in real-world problems. Yu et al. (2014) introduced memory-dependent derivative into the Lord and Shulman generalized thermoelasticity theory.

Lately, Ezzat et al. (2014b, 2015, 2016a,b) built another generalized thermo-viscoelasticity model using memory–dependent derivatives, to signify memory-dependence, as,

$$\boldsymbol{q} + \omega\, D_\omega\, \boldsymbol{q} = -\ k\,\nabla T. \tag{5}$$

Equation (5) has clear physical significance. One can refer to Ezzat and El-Bary and Tiwari and Mukhopadhyay, 2018 for a survey of applications of memory-dependent derivative calculus. Sur and Kanoria (2018) introduced a mathematical model of three-phase-lag generalized thermoelasticity with memory-dependent derivative. Shaw (2019) theoretically demonstrated two aspects of a generalized model with memory-dependent derivatives. The characteristics of transient effects in an isotropic, thermoelastic medium were analyzed in terms of memory-dependent thermoelasticity theory. Li et al. (2019) solved a problem of transient responses of a hollow cylinder under thermal and chemical shock based on generalized diffusion-thermoelasticity with memory-dependent derivative. Hendy et al. (2019b) developed a model of generalized thermoelasticity with memory-dependent derivative heat conduction law for a thermoelectric half-space.

For a sinusoidal heating on the skin tissue surface, Liu and Xu (1999) systematically studied the Pennes' bio-heat transfer equation. Deng and Liu (2004) in like manner presented exact associations between skin thermal model information and human pathophysiology for possible diagnostics. Shih et al. (2007) considered the effect of the temperature response of a semi-infinite organic tissue caused by a sinusoidal heat transition at the skin surface.

The present work is an endeavor to apply the modified model of Pennes' bio-heat transfer equation with memory-dependent derivative (PMDD) to one-dimensional problems to examine the temperature distribution in a skin with immediate surface heating. Laplace transform procedure is used to get the solution in a closed form. As indicated by the numerical outcomes and its diagrams, a decision about the new model has been built. Correlations are made with the outcomes obtained with the non-attendance of the MDD parameter. The effects of the time-delay on the temperature for different forms of Kernel functions are discussed.

2. Determination memory-dependent derivative Pennes' bio-heat equation (PMDD)

Heat transfer in biological systemf is usually described by the Pennes' bio-heat equation based on the classical Fourier law (1) and the heat flux regarding the energy equation is given by Pennes (1948).

$$\rho\ C\frac{\partial T(\boldsymbol{x},t)}{\partial t} = -\ \ \nabla\cdot\ \boldsymbol{q}(\boldsymbol{x},t) + \ \ W_b C_b[T_a - T(\boldsymbol{x},t)] + \ Q_m \tag{6}$$

where C is the specific heat of tissue, ρ is the tissue's density, W_b is volumetric blood perfusion rate, C_b is the specific heat of blood, T_a is the temperature of the blood in the arteries, Q_m is the energy that is generated within the tissue as a result of the metabolic activity. Here, the term $W_b C_b\,[T_a - T(\boldsymbol{x},t)]$ demonstrates the heat transfer between blood and the tissue (Lin and Chou, 2005). Since (6) is mainly used to describe the collective thermal effect for tissues not adjacent to large vessels, the present method based on this model also has the same limitation and the estimated perfusion, thus, cannot account for the directional convective mechanism of heat transfer due to blood flow (Liu et al., 2002).

Considering Q_m as a constant and subtracting the steady state temperature field from (6), one has

$$\rho\ C\frac{\partial\theta(\boldsymbol{x},t)}{\partial t} + \ \ W_b C_b\theta(\boldsymbol{x},t) \ = -\ \ \nabla\cdot\ \ \boldsymbol{q}(\boldsymbol{x},t) \tag{7}$$

where

$\theta = T(\boldsymbol{x},t) - T_a.$

Taking the memory-time derivative of both sides of Eq. (7), we get

$$\rho\ C\frac{\partial}{\partial t}D_\omega\ \theta(\boldsymbol{x},t)\ \ + W_b C_b\ D_\omega\theta(\boldsymbol{x},t) = -\ \nabla\ \cdot\ \ D_\omega\ \boldsymbol{q}(\boldsymbol{x},t) \tag{8}$$

Multiplying Eq. (8) by ω and adding to Eq. (7), we obtain

$$(1+\omega D_\omega)\left[\rho\ C\frac{\partial\theta(\boldsymbol{x},t)}{\partial t}\ + \ W_b C_b\ \theta(\boldsymbol{x},t)\right] = -\ \nabla\ \cdot\ (1\ +\ \omega D_\omega)\boldsymbol{q}(\boldsymbol{x},t) \tag{9}$$

Substituting from Eq. (5), we get

$$(1+\omega D_\omega)\left[\rho\ C\frac{\partial\theta(\boldsymbol{x},t)}{\partial t}\ + \ W_b C_b\ \theta(\boldsymbol{x},t)\right] = k\ \nabla^2\theta(\boldsymbol{x},t) \tag{10}$$

By using Eq. (4), Eq. (10) transforms to

$$k\nabla^2\theta\left(\boldsymbol{x},t\right) = \rho\,C\frac{\partial\theta(\boldsymbol{x},t)}{\partial t}\ + W_b C_b\,\theta\left(\boldsymbol{x},t\right) + \int_{t-\omega}^{t} K\left(t-\xi\right)\ \left(\rho\,C\frac{\partial^2\theta(\boldsymbol{x},\xi)}{\partial\xi^2}\ + \ W_b C_b\frac{\partial\,\theta(\boldsymbol{x},\xi)}{\partial\xi}\right) d\xi \tag{11}$$

Eq. (11) represents a hyperbolic model of Pennes' energy equation with memory-dependent derivative, taking into account the time delay ω in biological systems and the parabolic Pennes' bio-heat transfer equation (PCF) follows as the limit case when $\omega \to 0$, so that $|D_\omega f(\mathrm{x},t)| \le \left|\frac{\partial f(\mathrm{x},t)}{\partial t}\right| = \left|\lim_{\omega\to 0}\frac{f(\mathrm{x},t+\omega)-f(\mathrm{x},t)}{\omega}\right|$.

This model of heat transfer eliminates the paradox of infinite speed of propagation of heat in the human living tissue and is more intuitive for understanding the physical meaning.

3. The physical problem

The derived mathematical model consists of energy balance in the tissue that incorporates the effects of metabolism and blood perfusion. The biological heat transfer equation in expanded form (1D) is given as

$$k\frac{\partial^2\theta(x,t)}{\partial x^2} = \rho\,C\frac{\partial\theta(x,t)}{\partial t}\ + W_b C_b\,\theta\left(x,t\right) + \int_{t-\omega}^{t} K\left(t-\xi\right)\ \left(\rho\,C\frac{\partial^2\theta(x,\xi)}{\partial\xi^2}\ + \ W_b C_b\frac{\partial\,\theta(x,\xi)}{\partial\xi}\right) d\xi \tag{12}$$

where x is the spatial variable.

The framework under consideration consists of a segment of skin tissue. To guarantee that the inner temperature stays consistent and that the temperature disseminations in the skin tissue are symmetrical, the size of the area ought not to be larger than 3 cm. Given the most extreme temperature that a typical tissue can stand before corruption, the highest temperature permitted will be 42. It will be assumed that blood and tissue properties are not modified by time or position, and furthermore, that blood perfusion will stay consistent. The exchange forms inside the tissue will dictate the surface temperature. For analytical solution, assuming the heat flux on the skin surface is taking as an arbitrary function of time and approaches zero deep in tissue $x = L$, L is the distance between the skin surface and the body core, which is also realistic for a biological body (Liu et al., 2002), then the boundary conditions (BC) can be written as

$$-k\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = (1+\omega D_\omega)\ f(t) \tag{13a}$$

$$-k\left.\frac{\partial\theta}{\partial x}\right|_{x=L} = 0 \tag{13b}$$

where

$$D_\omega f(t) = \frac{1}{\omega}\int_{t-\omega}^{t} K(t-\xi)\ f'(\xi)\ \mathrm{d}\xi \tag{14}$$

and the initial conditions (IC) are

$$\theta|_{t=0} = 0,\quad \left.\frac{\partial\theta}{\partial t}\right|_{t=0} = 0,\quad t>0 \tag{15}$$

Let us introduce the following non-dimensional variables (Sharma and Sharma, 2014):

$$x^* = \frac{x}{L},\quad t^* = \frac{k}{\rho\, C L^2}\, t,\quad W_b^* = \frac{C_b L^2}{k} W_b,\quad \theta^* = \alpha_t\, \theta,$$

where α_t is coefficient of linear thermal expansion (Xu et al., 2008).

By using the above values, Eq. (12) reduces to (dropping the asterisks for convenience)

$$\frac{\partial^2\theta}{\partial x^2} = (1+\omega D_\omega)\left(W_b + \frac{\partial}{\partial t}\right)\theta \tag{16}$$

and the boundary and initial conditions (13) and (15) become

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = -\ (1+\omega\, D_\omega) f(t) \tag{17}$$

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=L} = 0 \tag{18}$$

$$\theta|_{t=0} = 0,\quad \left.\frac{\partial\theta}{\partial t}\right|_{t=0} = 0,\quad t>0 \tag{19}$$

Since the kernel function form $K(t-\xi)$ can be chosen freely, we shall take

$$K(t-\xi) = m + n(t-\xi) \tag{20}$$

where m and n are constants.

Now we take three different cases of the kernel function such that:

- Case 1: $K(t-\xi) = 1/2;\quad m = 1/2,\ n = 0$
- Case 2: $K(t-\xi) = 1/2 - (t-\xi)/\omega;\quad m = 1/2,\ n = -1/\omega$
- Case 3: $K(t-\xi) = 1 - (t-\xi);\quad m = 1,\ n = -1$

4. The exact solution in the Laplace-transform domain

Performing the Laplace transform defined by the relation

$$\bar{g}(s) = \int_0^\infty e^{-st} g(t)\ dt$$

of equation (14) for all three different kernels, we obtain

- Case 1: $K(t-\xi) = 1/2;\quad m = 1/2,\ n = 0$

$$L\{\omega D_\omega f(t)\} = \left(\frac{1-e^{-\omega s}}{2}\right)\bar{f}(s) \tag{21}$$

- Case 2: $K(t-\xi) = 1/2 - (t-\xi)/\omega;\quad m = 1/2,\ n = -1/\omega$

$$L\{\omega D_\omega f(t)\} = \left(\frac{(1-e^{-\omega s})}{2} - \frac{(1-e^{-\omega s})}{\omega s} + e^{-\omega s}\right)\bar{f}(s) \tag{22}$$

- Case 3: $K(t-\xi) = 1 - (t-\xi);\quad m = 1,\ n = -1$

$$L\{\omega D_\omega f(t)\} = \left((1-e^{-\omega s}) - \frac{(1-e^{-\omega s})}{s} + \omega\, e^{-\omega s}\right)\bar{f}(s) \tag{23}$$

where $\bar{f}(s)$ is the Laplace transform of $f(t)$.

In general, we can take Laplace transform of Eq. (14), we get

$$G(s) = L\{\omega D_\omega f(t)\} = \left[\frac{(ms+n)\ (1 - e^{-\omega s})}{s} - n\,\omega\, e^{-\omega s}\right]\bar{f}(s) \tag{24}$$

Taking Laplace transform of both sides Eqs. 16–18 and using the homogeneous initial conditions (19), we obtain

$$\frac{\mathrm{d}^2\bar{\theta}(x,\ s)}{\mathrm{d}x^2} - (1+G)(W_b + s)\bar{\theta}(x,\ s) = 0 \tag{25}$$

$$\left.\frac{\mathrm{d}\bar{\theta}(x,s)}{\mathrm{d}x}\right|_{x=0} = -\ (1+G)\,\bar{f}(s) \tag{26}$$

$$\left.\frac{\mathrm{d}\bar{\theta}(x,s)}{\mathrm{d}x}\right|_{x=L} = 0 \tag{27}$$

The exact solution of Eq. (25) with boundary conditions (26) and (27) in the Laplace transform domain is given by:

$$\bar{\theta}\left(x,s\right) = \left(1+G\right)\left(\frac{ch\ \varpi(x-L)}{\varpi\ sh\ \varpi L}\right)\bar{f}\left(s\right) \tag{28}$$

where $\varpi = \sqrt{(W_b+s)\ (1+G)}$.

5. Inversion of the Laplace transforms

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{g}(s)$ be the Laplace transform of a function $g(t)$. The inversion formula for Laplace transforms can be written as Honig and Hirdes (1984):

$$g(t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\ \bar{g}(d+iy)\ \mathrm{d}y,$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(s)$.

Expanding the function $h(t) = \exp(-dt)g(t)$ in a Fourier series in the interval $[0,\ 2\ell]$, we obtain the approximate formula Honig and Hirdes (1984):

$$g(t) \approx g_N(t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k,\quad \text{for } 0 \le t \le 2\ell, \tag{29}$$

where

$$c_k = \frac{e^{dt}}{\ell}\ \mathrm{Re}\ \left[e^{ik\pi t/\ell}\ g(d + ik\pi/\ell)\right]. \tag{30}$$

Two methods are used to reduce the total error. First, the 'Korrektur' method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur-method uses the following formula to evaluate the function $g(t)$

$$g(t) = g_{NK}(t) \ = g_N(t) \ - \ e^{-2d\ell} \, g_{N'}(2\ell + t). \tag{31}$$

where N' is an integer such that $N' > N$.

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (29). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$, be the sequence of partial sums of (29).We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \ \ \varepsilon_{1,m} = s_m, \quad m = 1, 2, 3, \ldots$$

and $\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + 1/(\varepsilon_{n,m+1} - \varepsilon_{n,m}), \quad n, m = 1, 2, 3, \ldots$.

It can be shown from Honig and Hirdes (1984) that the sequence $\varepsilon_{1,1}$, $\varepsilon_{3,1}$, …, $\varepsilon_{N,1}$, … converges to $g(t) - c_0/2$ faster than the sequence of partial sums.

6. Numerical results

To obtain a closed-form solution for dimensionless temperature distribution in space and time domain, we must apply Laplace inversion formula to the Eq. (28). This has been done numerically using a method based on Fourier series expansion technique mentioned above. We use the Fortran 90 programming language on a personal computer with an I7 processor. The amount of calculation (and hence the execution time) depends on several parameters within the program. First, a parameter "ng" is the number of significant digits defining the maximum allowed relative error as $(10)^{-\mathrm{ng}}$. We usually take ng = 5. Near points of discontinuity of the function, the program might fail to converge, and we have to decrease ng. Another parameter is the maximum number of terms in the Fourier series to be added within one saw-tooth of the ε-algorithm. This is taken as 10000. The last parameter is the number of saw-teeth of the ε-algorithm to be considered. This is taken as 50. Overall, the program evaluates the value of θ at 50 points in less than 2 min (Sherief and Hussein, 2018).

For numerical evaluations, estimations of thermal properties for skin tissue and different parameters have been picked as the density of tissue (1000 kg/m^3); both particular heat capacity of tissue and blood are 4000 J/kg/^{o}C, the thermal conductivity of tissue is 0.5 W/m/^{o}C (Liu and Xu, 1999) and the coefficient of linear thermal expansion $\alpha_t = 1.78 \ (10)^{-4} \ /^oC$ (Xu et al., 2008). The distance between the skin surface and body core was chosen $L = 0.01$ m (Torvi and Dale, 1994) and $W_b = 0.0005 \ \ ml/s/ml$ (an average blood perfusion value for the human skin) (Liu and Xu, 1999).

The calculations were carried out for an arbitrary function $f(t)$. Therefore, we take $f(t)$ to represent a sinusoidal heat flux on skin surface such that:

$$f(t) = \begin{cases} \sin\left(\dfrac{\pi t}{a}\right) & 0 \leq \ t \ \leq \ a \\ 0 & \text{otherwise} \end{cases} \qquad \text{or} \qquad \bar{f}(s) = \frac{\pi a(1 \ + \ e^{-as})}{a^2 s^2 \ + \ \pi^2}$$

The numerical system laid out above was utilized to get the temperature θ and the heat flux q distributions for various cases at $a = 1.0$. The outcomes are represented graphically in Figs. 1–7, for different values of x, and t. In the present area, we have tried to demonstrate the impact of the kernel function and of the time-delay for every physical field, for example, temperature. The time-delay has a significant effect on the temperature distribution in the living tissue. The time when the propagating waves of heat reach the steady state is dependent upon both the estimated time-delay and the choice of the kernel function.

6.1. Behavior of the temperature in skin tissue

The variation of the non-dimensional temperature field has appeared in two different ways-for different kernel functions when the time-delay parameter is fixed and furthermore for a unique kernel function but for various estimations of the time-delay parameter.

Fig. 1 shows the variations of the non-dimensional temperature θ versus the skin tissue depth x for one value of non-dimensional time t = 20 and for a kernel function form $K(t, \xi) = 1/2\text{-}(t\text{-} \xi)\ /\omega$. Comparisons are performed between the solution corresponding to using classical Pennes' bio-heat transfer equation (ω= 0.0, PCF) and to using the bio-heat transfer equation with memory-dependent derivative (PMDD,

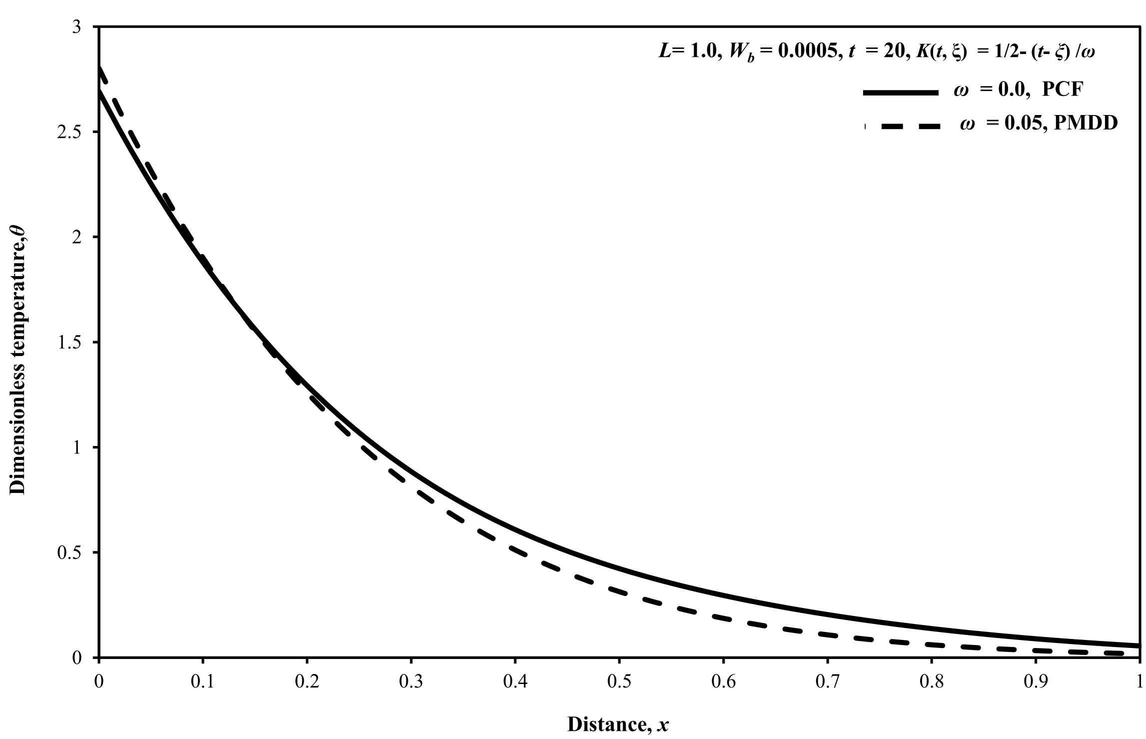

Fig. 1. The variation of temperature for different two theories.

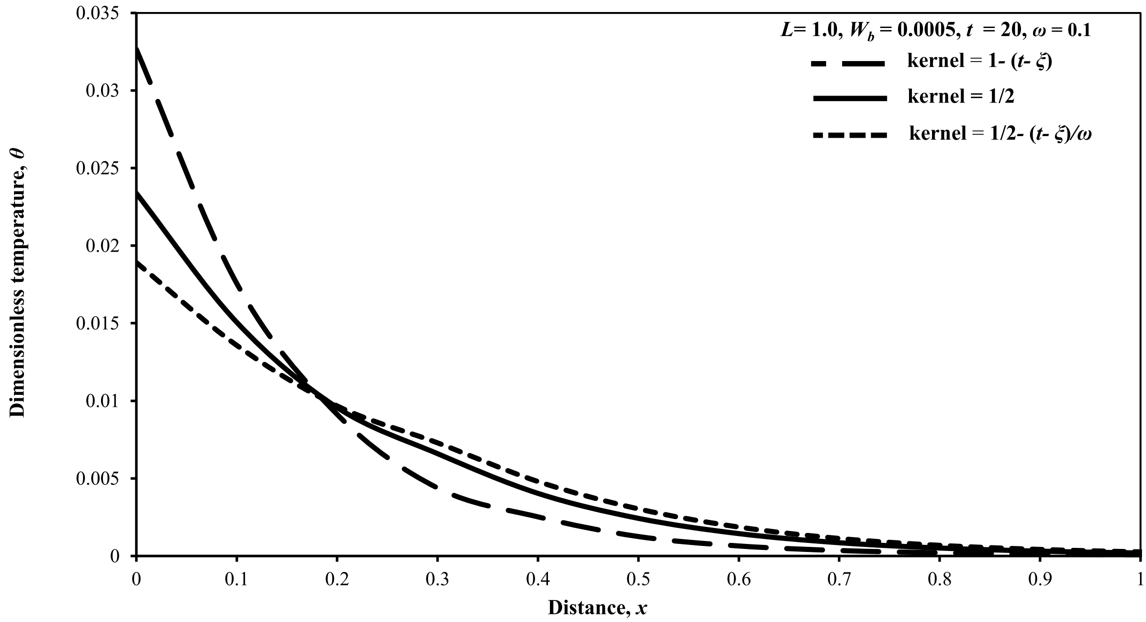

Fig. 2. Variation of temperature for different types of kernel function.

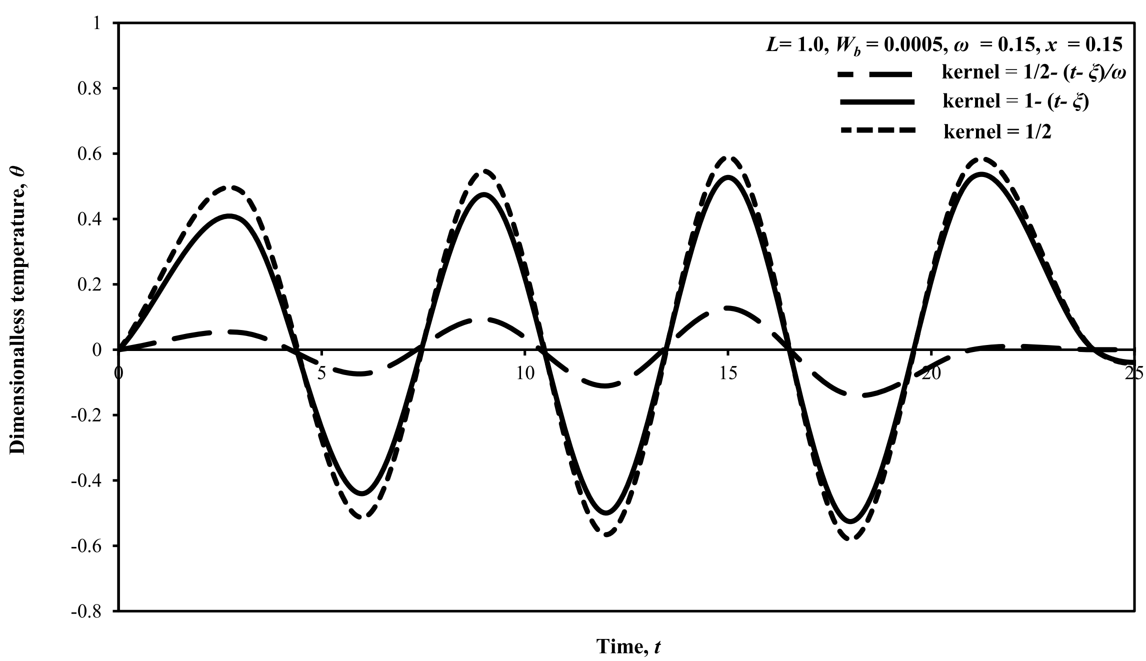

Fig. 3. Variation of temperature for three different types of kernel functions.

$\omega > 0$). In the first and more established theory the waves propagate with limitless rates, so the estimation of any of the functions isn't indistinguishably zero (however it might be small) for any large value of x (Ezzat et al., 2014a, 2016a). The locale of impact for the temperature is unmistakably demonstrated to be limited in all cases in which the time-delay parameter, $\omega > 0$, where the thermal waves travel with finite speeds. It is noted that the temperature field is continuous and has the maximum value at the initial points, i.e. when the distance $x = 0$(on the skin surface). Further, after some distance, the temperature field is observed to attain one local maximum value before it achieves the steady state. This means that for large values of the time-delay the particles of the skin tissue transport the heat to the other particles easily and this makes the decreasing rate of the temperature greater than the other one. This field has a decreasing trend as the depth in the skin tissue increases.

Fig. 2 presents the variation of the temperature versus the depth x skin tissue for wide range $0 \leq x \leq 1.0$ (triple-layered of skin interfaces) at value of time $t = 20$ and for different kernel functions but the time-delay parameter, ω is fixed and its value is 0.1. Computations are performed for three different kernel functions, namely $1 - (t - \xi)$; $1/2$; $1/2 - (t - \xi)/\omega$. We have noticed that for wide range of $0 \leq x \leq 0.2$ the temperature θ attains its maximum values of 0.032, 0.023 and 0.019, respectively for the types of kernel functions and then steadily decreases through the interval $0.2 < x < 1.0$ up to a constant value at $x = 1.0$.

Fig. 3 shows the temperature response for time range $0 \leq t < 25$ on the skin surface subjected to a sinusoidal heat flux at different types of kernel functions $1/2 - (t-\xi)/\omega$; $1 - (t-\xi)$ and $1/2$ and time-delay has a constant value, $\omega = 0.15$. In the present profiles, we see that the

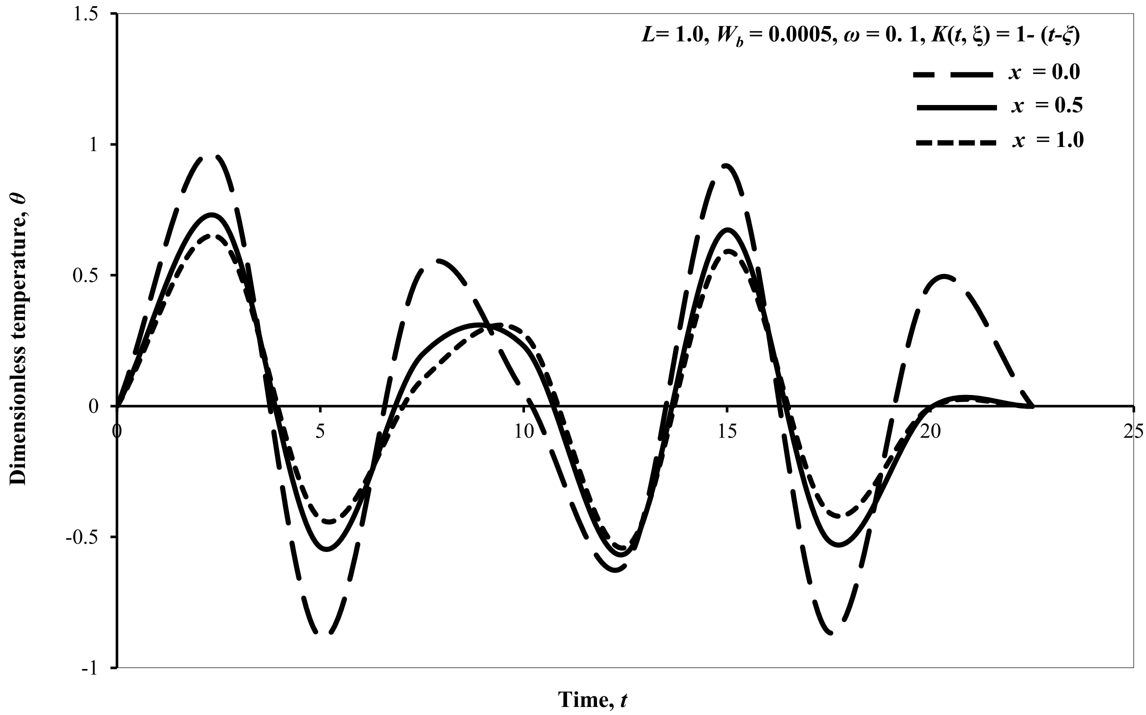

Fig. 4. The variation of temperature at three different locations.

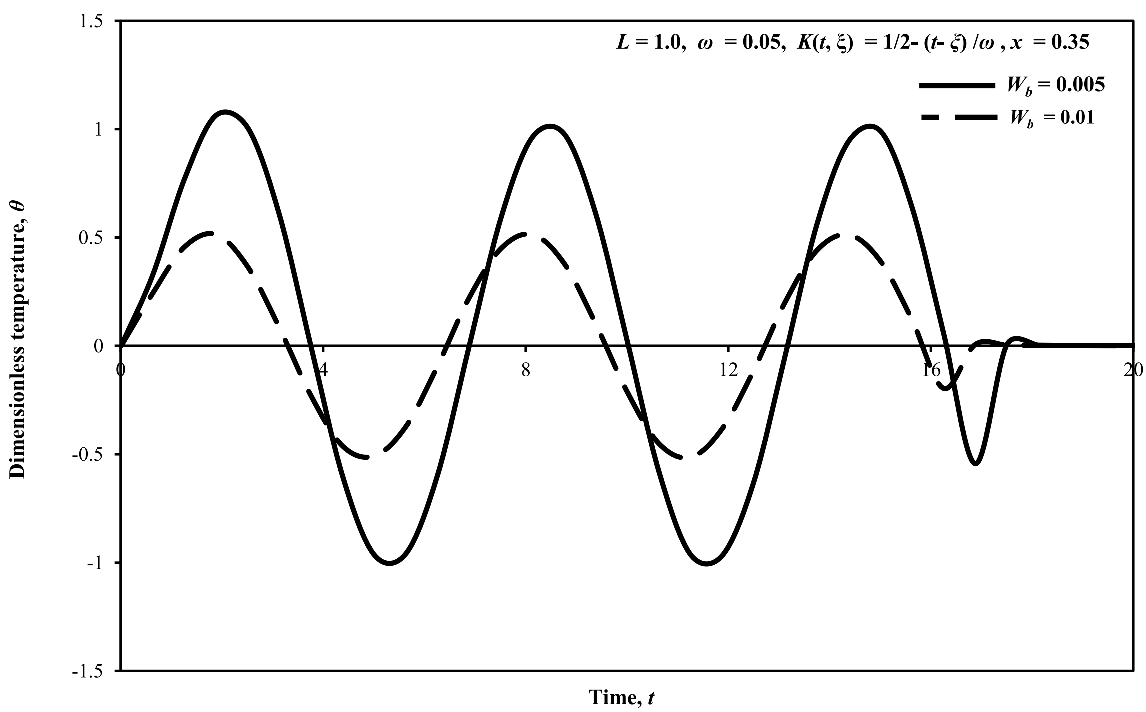

Fig. 5. Temperature responses at two perfusion levels at various blood perfusion rate.

influence of the kernel function is prominent. Moreover, we observe that the kernel functions are more effective at the vicinity of the local maximum at different layers of skin tissue. The local maximum is of its highest value for the linear kernel function $K(t,\ \xi) = 1/2\ \ - (t - \xi)/\ \omega$ and is of lowest value for the kernel function $K(t,\ \xi)\ =\ 1\ \ -\ \ (t\ -\ \xi)$. The temperature change is a ceaseless capacity, which implies that the particles transport the warmth to different particles effectively and this makes the diminishing rate of the temperature more noteworthy than the other case.

Fig. 4 shows the temperature oscillations inside the skin tissue inside the heated area. The amplitude of the temperature decays exponentially with the distance from the skin surface. We notice that a fast decay occurs and the temperature decays to zero within a short distance. Thus, the solution of the considered function of temperature vanishes identically outside a bounded region of a constant surface heating at distance from it equal to x depending only on the choice of t and is the location of the wave front.

Fig. 5 demonstrates the effect of the perfusion rate of blood on the

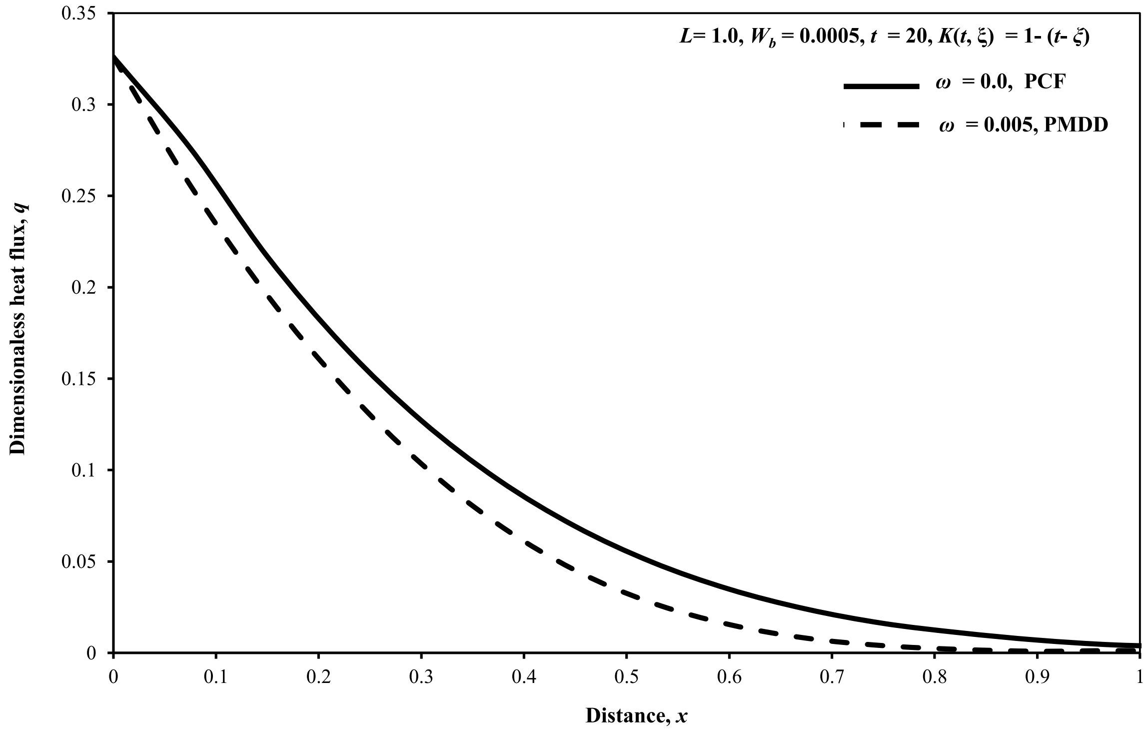

Fig. 6. The variation of heart flux for different two theories.

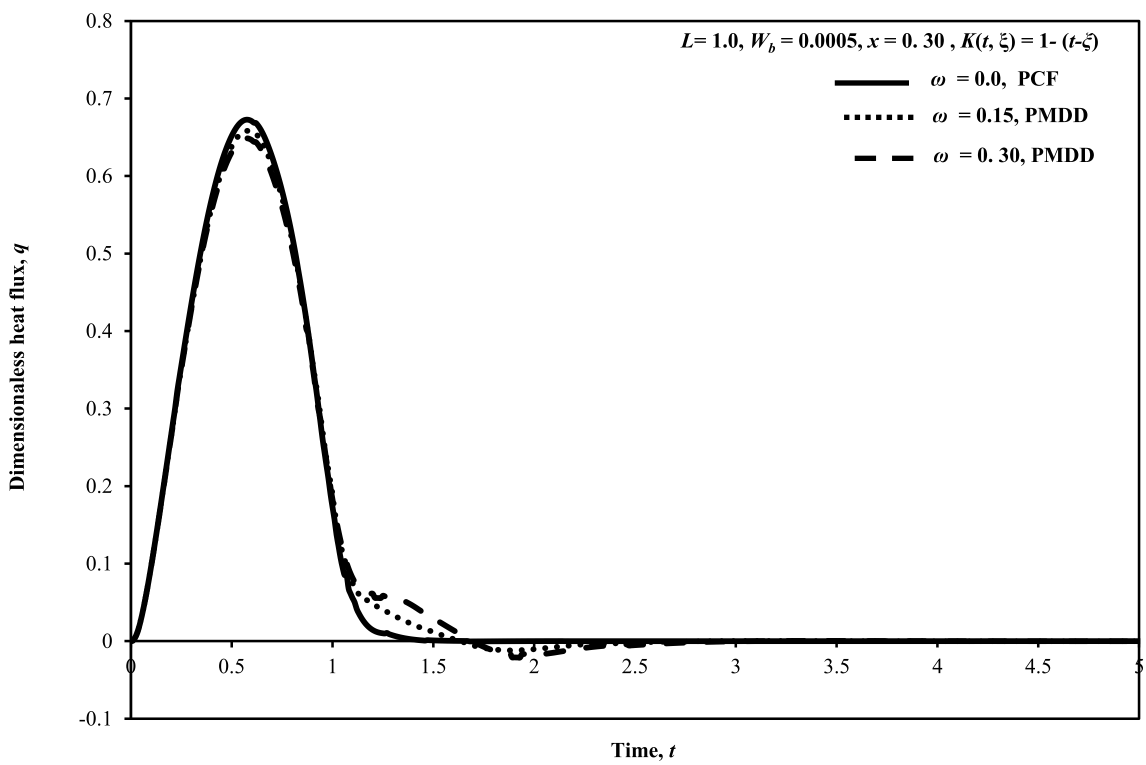

Fig. 7. The variation of heat flux different two theories.

temperature responses in the living skin tissue for two perfusion levels $W_b = 0.005$ and $W_b = 0.01$ at depth $x = 0.35$ from which the time required to achieve the steady state can be evaluated and it is under 18.75. Clearly, the higher blood perfusion is the shorter time. In such manner, the present strategy gives off an impression of being more appropriate for perfusion estimations in profoundly perfuse tissues.

6.2. Behavior of the heat flux in skin tissue

The variation of the temperature in the skin living tissue has appeared in two different ways. Firstly, we consider a unique kernel function for different values of the time-delay parameter. Secondly, for different kernel functions when the time-delay parameter is fixed.

Fig. 6 shows the variation of the heat flux versus the skin tissue depth x at two values of time-delay $\omega = 0$ and 0.05, for a fixed kernel function $K(t,\xi) = 1 - (t-\xi)$. The time-delay has a significant effect on the heat flux in the living tissue where the propagation waves are continuous functions, smooth and reach steady state depending on the value the time-delay. We also note that the increasing of the value of the time-delay ω causes a decrease in the heat flux field. The important phenomenon observed in these computations is that the heat flux with memory-dependent derivative vanishes identically outside a bounded region of a skin surface heating.

Fig. 7 demonstrates the heat flux oscillations inside the skin tissue amid the heating for the two theories. This demonstrates clearly the difference between the solution corresponding to the classical use of the Fourier heat equation (PCF model,$\omega = 0.0$) and to the use of the new generalized case (PMDD model, $\omega = 0.15,\ 0.30$ and $K(t,\xi) = 1 - (t - \xi)$). This result is very important since the new theory may preserves the advantage of the generalized theory, i.e. the response to the thermal effects does not reach infinity instantaneously but remains in the bounded region of space that expands with the passing of time.

7. Conclusion

- The memory-dependent derivative is defined in an integral form of a common derivative with a kernel function on a slipping interval. So, this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). Its definition is more intuitive for understanding the physical meaning and the corresponding memory dependent differential equation has aspects that are more representaive.
- A mathematical model of Pennes' bio-heat equation has been constructed in the context of a new consideration of heat conduction law with memory-dependent derivative.
- According to this new model, we have to construct a new classification for Pennes' bio-heat transfer according to their, time-delay ω where this parameter becomes a new indicator of its ability to conduct heat in skin tissue.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jtherbio.2019.102427.

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