Ultrasonics 91 (2019) 231–236

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Simultaneous ultrasonic parameter estimation of a multi-layered material by the PSO-based least squares algorithm using the reflection spectrum

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ARTICLE INFO

Keywords:
Multi-layered material
Least squares inversion
Simultaneous measurement
Particle swarm optimization
Ultrasonic parameters
Reflection spectrum

ABSTRACT

Advanced multi-layered materials with superior performance are required for many applications. The non-destructive characterization of multi-layer properties is a hot spot of current research. The least squares inversion method using the reflection spectrum has been developed and widely used to estimate the properties of thin single layers simultaneously. However this method has the problems of a loss in speed and simplicity, and a local optimal solution, especially in the cases of a multi-layered structure because of the increasing estimated parameters and the uncertainty influence from the parameters. Particle swarm optimization (PSO) is a robust global search algorithm similar to 'bird' foraging, which can be used to improve the performance of the least squares inversion algorithm. This paper has proposed a PSO-based least squares estimation using the ultrasonic reflection spectrum to make simultaneous measurement. The simulation and experiment, carried out on the aluminum-TC4 bi-layered material, tested and proved the capability of the new algorithm. The real measured parameters and the estimated parameters were obtained. The results have been compared to analyze the errors of the estimated parameters.

1. Introduction

Multi-layered composite materials are an advanced form of materials which are made of two or more constituents with significantly different properties from individual materials, such as polymers, metals, ceramics, fibre or particle reinforced materials [1]. Many applications require the use of multi-layered materials because of their superior performance. Many researchers have studied the non-destructive characterization of the properties of multi-layered materials [2–6].

Some authors have applied the inversion algorithm to estimate the properties of layers based on the data from ultrasonic transmission tests [2,7]. They used the least squares inversion method to characterize the layered materials based on an idealized model of the physical problem and the data obtained from ultrasonic wave transmission measurements. When the contact transducers are replaced by immersion transducers, this methodology can be used for thin multi-layered materials because they usually have a large residual stress. Some researchers have studied the simplified computations of the reflection coefficients of multi-layered structures [4]. This has simplified the computation of the reflection spectrum when characterizing the multi-layered structures. However, it is not a method that can derive the properties from the experimental reflection spectrum. Other authors have simultaneously estimated the thicknesses and the ultrasonic longitudinal wave velocities of layers in multi-layered structures by exploiting the phase information in the complete absence of an interface echo based on one dimensional wave propagation [6]. However, this method is not expected to yield solutions when the interface is within 1 mm of the front wall or the back wall. Therefore this method also has limitations. A parametric model has also been proven to be able to reconstruct the signal waveform composed of multiple overlapping echoes from multi-layered structures based on the estimated parameter vector [3]. This model-based estimation method can avoid the complexity of overlapping echoes from multi-layered structures, although it has an obvious loss in simplicity, speed and numerical stability when the number of parameters increases. Others have simultaneously evaluated the interfacial normal stiffness and the longitudinal wave velocity of the layers in multi-layered structures based on the local extremal frequencies of the reflection spectrum [5]. They could only evaluate multi-layered structures with an imperfect inter-layer interface, where the properties of each layer were similar. Thus this method is not useful for multi-layered materials with different constituents.

The least squares inversion method, which has been widely used, has been proven efficient in estimating the properties of thin layers [8–10]. This method cannot ensure the global optimal value when the

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https://doi.org/10.1016/j.ultras.2018.08.003
Received 12 December 2017; Received in revised form 24 July 2018; Accepted 6 August 2018
Available online 11 August 2018

Nomenclature

c_1	self-cognition acceleration coefficient
c_2	social-cognition acceleration coefficient
e	termination criteria
$gbest^t$	most optimistic position of the swarm at its t^{th} iteration
L	maximum number of iterations
m	number of layers
N	number of particles
$pbest_i^t$	most optimistic position of the particle i at its t^{th} iteration
r_1	random functions in the range[0, 1]
r_2	random functions in the range[0, 1]
v_i^t	speed of the particle i at its t^{th} iteration
w	inertia weights coefficient
x_i^t	position of the particle i at its t^{th} iteration

number of estimated parameters increases, which is the limitation of the algorithm itself. The interaction of parameters in two layers also make it impossible to find a convergence domain for each parameter. Thus, this algorithm is not suitable for direct use for multi-layered materials. The properties of thin layers can also be directly derived from the reflection spectrum [11,12]. However, the computation of the reflection coefficients of multi-layered structures is so complex that it is impossible to directly calculate their properties.

This paper has combined the particle swarm optimizer (PSO) and the least squares inversion method to estimate the ultrasonic parameters of multi-layered materials using the reflection spectrum. The parameters to be estimated were the transition time τ, the acoustic impedance Z, and the attenuation α of each layer. PSO can quickly converge to a reasonable output with a very accurate initial guess [13]. The least squares function was computed according to the theoretical reflection spectrum from the estimated parameters and the experimental reflection spectrum. The simulation and experiment of bi-layered materials have been done to test the methodology based on the PSO-based least squares algorithm using the reflection spectrum. The individual material of each layer has been measured to obtain the real properties to compare with the estimated results of the new algorithm.

2. Theoretical model

The acoustic impedance of n-layered structures in multi-layered material is computed by the recursive method as [14]:

$$Z_\lambda^{(n)} = Z_n \frac{Z_\lambda^{(n-1)} - Z_n \frac{e^{2ik_{zn}d_n}-1}{e^{2ik_{zn}d_n}+1}}{Z_n - Z_\lambda^{(n-1)} \frac{e^{2ik_{zn}d_n}-1}{e^{2ik_{zn}d_n}+1}}, \tag{1}$$

where $n \geqslant 2$, $Z_\lambda^{(n)}$ is the total acoustic impedance of the n-layered structure, $Z_n = \rho_n c_{Ln}$ is the acoustic impedance of the nth layer, $k_{zn} = \frac{\omega}{c_{Ln}}(1 + \alpha_n i)$ is the wave number in the z direction of the nth layer, whose imaginary part denotes the frequency dependent attenuation, d_n is the thickness of the nth layer, ρ_n is the density of the nth layer, c_{Ln} is the longitudinal wave velocity of the nth layer, α_n is the attenuation of the nth layer.

Four properties of each layer: d_n, ρ_n, c_{Ln} and α_n are included in the formula. These properties are coupled into three independent ultrasonic parameters of each layer: τ_n, Z_n, and α_n. The transition time τ_n of each layer is defined as:

$$\tau_n = \frac{d_n}{c_{Ln}}. \tag{2}$$

The simplified expression of Eq. (1) is

$$Z_\lambda^{(n)} = Z_n \frac{Z_\lambda^{(n-1)} - Z_n \frac{e^{2i\tau_n\omega(1+\alpha_n i)}-1}{e^{2i\tau_n\omega(1+\alpha_n i)}+1}}{Z_n - Z_\lambda^{(n-1)} \frac{e^{2i\tau_n\omega(1+\alpha_n i)}-1}{e^{2i\tau_n\omega(1+\alpha_n i)}+1}}, \tag{3}$$

where there are three independent properties Z_n, τ_n and α_n in each layer. Then the reflection coefficient corresponding to the acoustic impedance is:

$$R_t(\omega) = \frac{Z_\lambda^n - Z_{n+1}}{Z_\lambda^n + Z_{n+1}}, \tag{4}$$

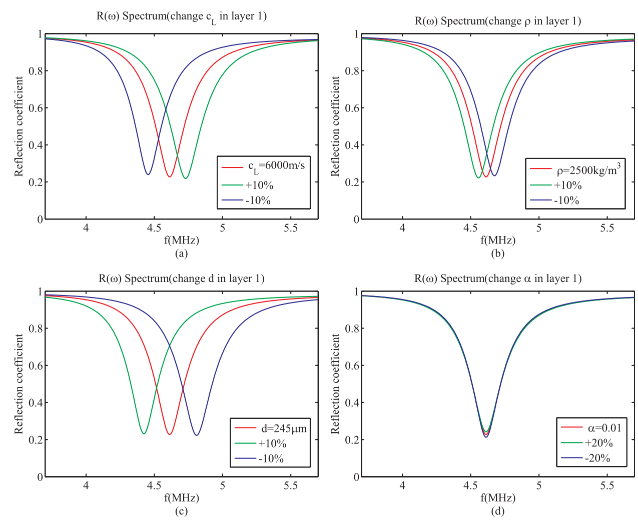

Fig. 1. The theoretical reflection spectrum of the aluminum-TC4 bi-layered structure with the changeable properties in layer 1 including the (a) longitudinal wave velocity (b) density (c) thickness and (d) attenuation.

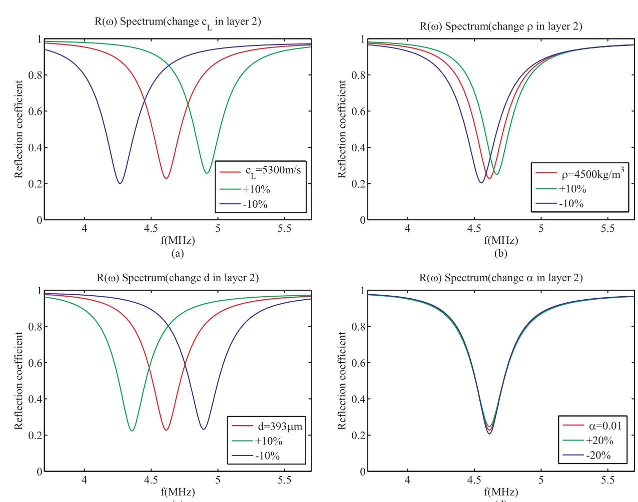

Fig. 2. The theoretical reflection spectrum of the aluminum-TC4 bi-layered structure with the changeable properties in layer 2 including the (a) longitudinal wave velocity (b) density (c) thickness and (d) attenuation.

where Z_{n+1} denotes the acoustic impedance of the infinite half-plane above the surface here.The effects of wave velocity, density, thickness and attenuation on the amplitude of the normal reflection spectrum from a bi-layered structure have been shown in Figs. 1 and 2. The numerical analysis used the properties in Table 1. The numerical results showed the complex effect of the layers' properties on the spectrum.

Table 1
The initial estimated properties of the bi-layered material.

Material	ρ (kg/m^3)	c_L (m/s)	d (μm)	α	Z (10^6Pa s/m)	τ (μs)
Al	2500	5300	245	0.01000	23.85	0.074
TC_4	4500	6000	393	0.01000	15.00	0.041

The thickness is the mean value of the side-view height, which is shown by mean ± 3σ.

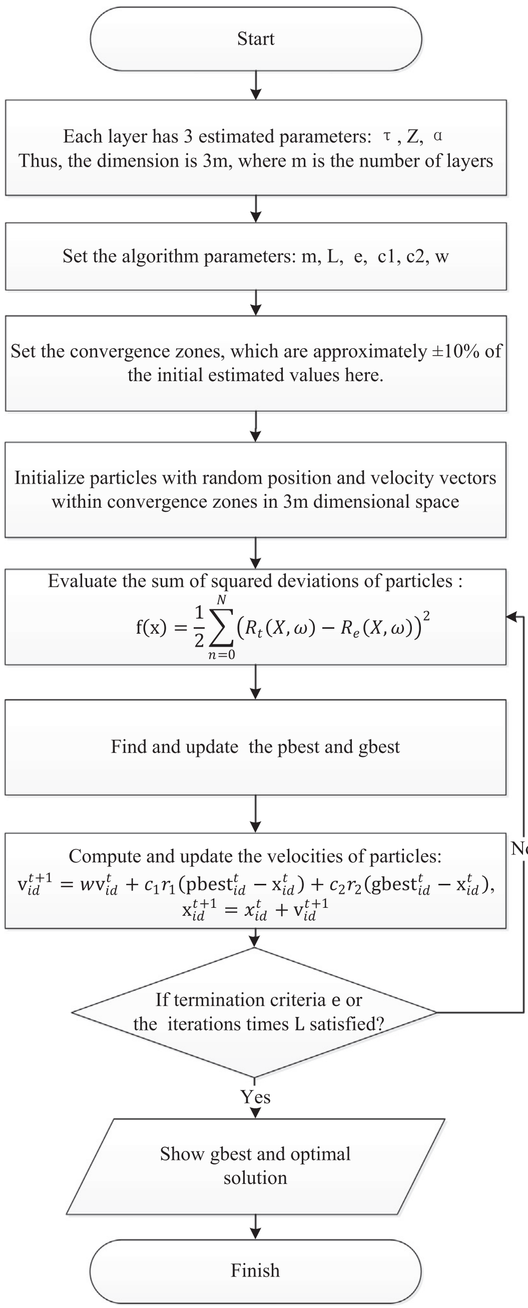

Fig. 3. Flow chart of the proposed PSO-based least squares algorithm for estimating the properties of multi-layered materials using the reflection spectrum.

3. Algorithm

3.1. Least squares inversion method

The properties are determined by minimizing the sum of the squared deviation between the theoretical and experimental reflection spectrum using the least squares inversion method. The object function is:

$$f(X) = \frac{1}{2}\sum_{n=0}^{N}(R_t(X,\omega) - R_e(X,\omega))^2,\ X \in \mathbb{R}^{3m}, \tag{5}$$

where R_t is the theoretical reflection coefficient, R_e is the experimental reflection coefficient, X are the parameters in a $3m$ dimensional space which consist of $3m$ variables, m is the number of layers, n are the data points at various frequencies, N is the number of the data points.

In previous cases of thin layers, an accurate initial estimation of the parameters is necessary through the use of pre-experiments. In fact, the effect of single layer properties on the reflection spectrum is explicit; e.g. the position of the valley of the curve is only influenced by the transition time. Thus, it is easy to use pre-experiments to find the narrow convergence. For example, if the experimental reflection spectrum has been obtained, the approximate values of τ and Z can be directly judged [15]. The traditional least squares method does not allow any local minima in the convergence. However, pre-experiments are not useful for multi-layered structures. It can be seen in Figs. 1 and 2 that it is difficult to distinguish the effect from each properties of the multi-layered material. The Gauss-Newton method used to solve the inversion problem requires the Jacobian (gradients) and the Hessian, which need the partial derivative of the objective function [3]. However, if the partial derivative of Eq. (4) is computed, there will be enormous computational complexity, which is mainly caused by Eq. (3).

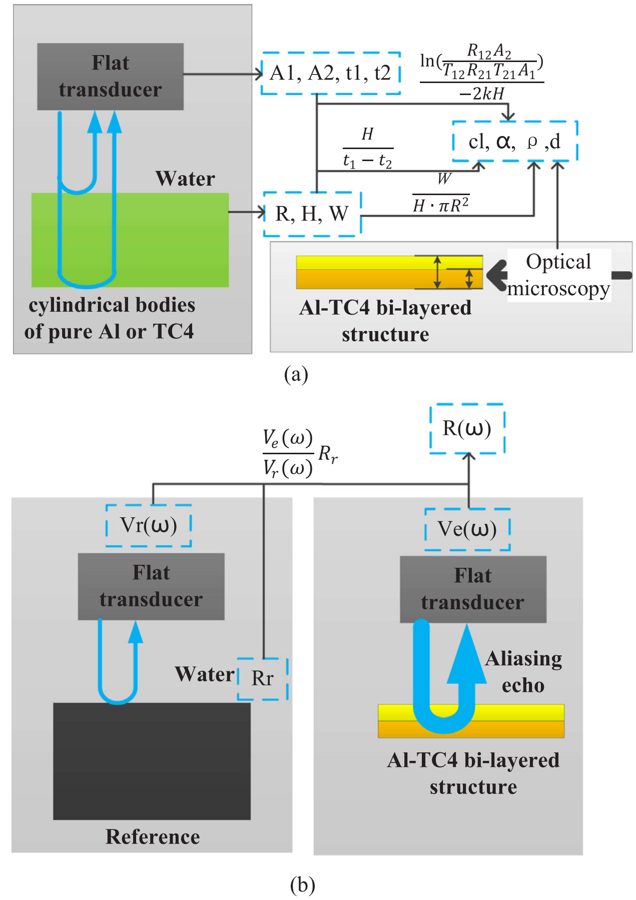

Fig. 4. The measurement principle of (a) the real properties and (b) the experimental reflection spectrum.

3.2. PSO-based algorithm

The particle swarm optimization resembles a school of flying birds.

Table 2
The real measured properties of pure aluminum and TC_4.

Material	ρ (kg/m^3)	c_L (m/s)	d (μm)	α	Z (10^6Pa s/m)	τ (μs)
Al	2731	6545	245 ± 54	0.00500	17.87	0.037 ± 0.008
TC_4	4379	5972	392 ± 75	0.00520	26.15	0.066 ± 0.012

TC_4 is a titanium alloy, whose composition is $Ti{-}6Al{-}4V$.
The thickness is the mean value of the side-view height, which is shown by mean ± 3σ.

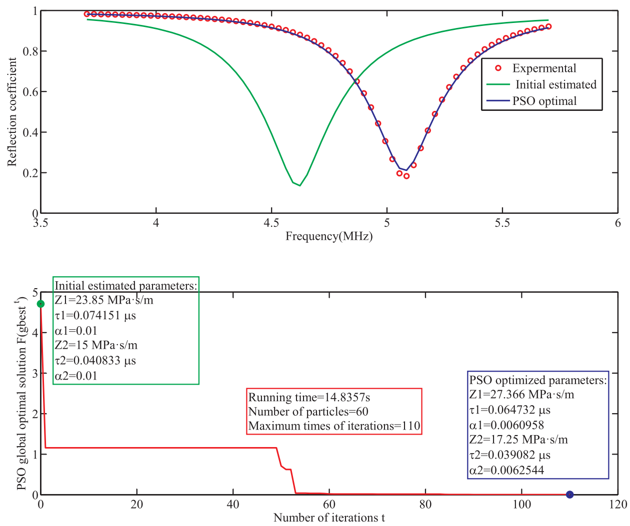

Fig. 5. (a) The reflection spectrum of the real measured parameters, initial estimated parameters, PSO optimized parameters and (b) the track of the global optimal solution in the simulation.

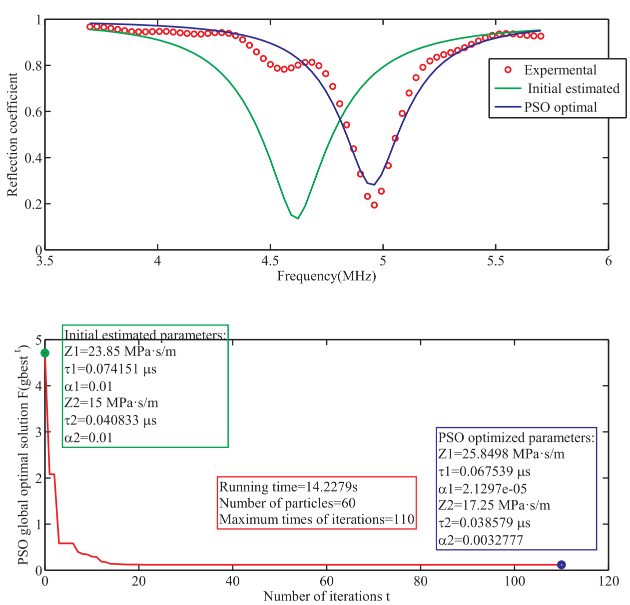

Fig. 6. (a) The reflection spectrum of the real measured parameters, initial estimated parameters, PSO optimized parameters and (b) the track of the global optimal solution in the experiment.

Each 'bird' is a particle representing a potential solution in the $3m$ dimensional space to the problem. They 'fly' around in the $3m$ dimensional search space with adjustable velocities, which are constantly updated according to their own flying experience and that of the whole swarm[16,17]. Thus all particles search the space for the solutions along their flight inertia while automatically moving closer to the historical global optimal solution. The velocity and position are manipulated by:

$$v_{id}^{t+1} = wv_{id}^{t} + c_1r_1(pbest_{id}^{t}-x_{id}^{t}) + c_2r_2(gbest_{d}^{t}-x_{id}^{t}), \tag{6}$$

$$x_{id}^{t+1} = x_{id}^{t} + v_{id}^{t+1}. \tag{7}$$

The flow chart of the PSO-based least squares algorithm has been shown in Fig. 3. The initial estimated parameters are also needed. Fortunately for implementation issues, the approximate material properties of each layer in a composite material are probably previously known. The height of each layer is initialed according to the side-view height using optical microscope. The potential solutions, called particles, are searching the whole convergence averagely. Even though there are some local minima in the convergence, as long as the population of the particles is large enough, the global optima can also be achieved. The selection of the search zones and the initial velocities are also feasible without ensuring that there is no local minimum.

4. Materials

The specimen was an aluminum-TC4 bi-layered material. TC4 is a titanium alloy, whose composition is Ti-6Al-4V. Aluminum was on the top. The cylindrical specimens of pure aluminum and TC4 were measured to obtain the real properties of each material. The real densities were measured by:

$$\rho = \frac{W}{H\cdot\pi R^2}, \tag{8}$$

where W is the weight of the cylindrical body, H is the height of the cylindrical body, R is the radius of the bottom. The effect of the pressure difference on the density was ignored in air and water, because the water layer was shallow during the experiments. The transition time and reflective echoes from the surface and the bottom were obtained to measure the longitudinal wave velocity and attenuation. The samples were suspended in water. The formulas are:

$$c_L = \frac{H}{t_1-t_2}, \tag{9}$$

$$\alpha = \frac{\ln\left(\frac{R_{12}A_2}{T_{12}R_{21}T_{21}A_1}\right)}{-2kH}, \tag{10}$$

where t_1 and t_2 are the arrival times of the reflective echoes from the surface and the bottom, A_1 and A_2 are the amplitudes of the reflective echoes from the surface and the bottom, R_{12} is the acoustic reflection coefficient of the liquid-solid interface, R_{21} is the acoustic reflection coefficient of the solid-liquid interface, T_{12} is the acoustic transmission coefficient of the liquid-solid interface, T_{21} is the acoustic transmission coefficient of the solid-liquid interface, k is the wave number. The mean thickness of each layer in the bi-layered structure was measured by an optical microscope in side-view. Fig. 4(a) has shown the measurement principle of the real properties. The real measured properties have been shown in Table 2.

5. Simulation

The computed reflection spectrum based on the real measured properties was the simulation of the experimental reflection spectrum

Table 3
The estimated parameters of the bi-layered material based on the PSO-based least squares algorithm in the simulation and experiment.

Layer	Simulation			Experiment		
	Z (10^6Pa s/m)	τ (μs)	α	Z (10^6Pa s/m)	τ (μs)	α
1	27.36 (4.64%)	0.065 (−1.63%)	0.00610 (21.92%)	25.85 (−1.15%)	0.068 (2.63%)	0.00002 (−99.57%)
2	17.25 (−3.49%)	0.039 (4.40%)	0.00625 (56.36%)	17.25 (−3.49%)	0.038 (3.06%)	0.00328 (−18.05%)

The errors are in the parentheses.

according to Eqs. (3) and (4). Each layer has 3 estimated parameters. Thus, there are totally 6 estimated parameters with 2 layers. The dimension of the search zone is 6. The initial estimated parameters have been shown in Table 1. The convergence zones are set according to the initial values, which are approximately ± 10% of the initial estimated values here. The algorithm was realized by Matlab software in a 64-bit system. The CPU was a 3.10 GHz Intel i3-2100. The frequency band of the reflection spectrum in the simulation was from 3.7 MHz to 5.7 MHz which was the bandwidth of the transducer in the experiment. The PSO-based least squares algorithm quickly adjusted the estimated parameters to the real measured parameters. The track of the algorithm has been shown in Fig. 5.

6. Experiment

The measurement principle of the experimental reflection spectrum has been shown in Fig. 4(b). The reference material was a thick cylinder of stainless steel. Its transverse plane was smooth enough so that its reflection coefficient was independent of frequency. Therefore, the transverse plane whose reflection coefficient kept a constant was used as calibration to observe the reference wave. The experimental reflection coefficient of the specimen is:

$$R_e(\omega) = \frac{V_e(\omega)}{V_r(\omega)} R_r, \tag{11}$$

where V_e is the complex spectrum of the wave reflected by the bi-layered structure, V_r is the complex spectrum of the reference wave, R_r is the reflection coefficient of the reference material.

Similarly, the PSO-based least squares algorithm was applied to quickly adjust the estimated parameters to make the theoretical reflection spectrum consistent with the experimental reflection spectrum. The initial estimated parameters have also been shown in Table 1. The track of the algorithm has been shown in Fig. 6.

7. Results

The estimated properties of the bi-layered material based on the PSO-based least squares algorithm in the simulation and experiment have been shown in Table 3. The errors were computed by comparing the results with the real measured values. It can be seen in Figs. 5(a) and 6(a) that the experimental and the simulated reflection spectrum were not exactly the same. This was the allowable error mainly caused by the errors of the instrument and measurement technique. The important algorithm parameters such as the running time, number of particles and number of iterations have been shown in Figs. 5(b) and 6(b). The algorithm performance could be evaluated by these parameters.

8. Discussion

The differences between the initial estimated parameters and the real parameters were set to more than 10% of the initial estimated parameters to completely confirm the capability of the proposed method. The initial estimated attenuation had errors of much more than 10%. The final estimated parameters results shown in Table 3 have obviously fewer errors than the initial errors except for the attenuation. The results clearly depicted coherence between the simulation and experiment.

The errors of the attenuation were very large in both the simulation and experiment, especially in the latter case. It can be seen in Figs. 1 and 2 that the attenuation had little effect on the reflection spectrum. The attenuation was very small and was greatly influenced by the porosity in the metallic materials, which was always the raw material of the multi-layered composite materials [18]. Thus the noise and the numerical fluctuations would seriously influence the inversion of the attenuation.

Sometimes the solution of the proposed method also had the problem of local optima. The chances of being trapped in local optima can be reduced by increasing the number of particles and the number of iterations while also increasing the running time linearly. The initial estimated parameters had initial errors of more than 10% in these experiments. The initial estimated parameters and the search zones can be optimized on the basis of all the previous knowledge about the raw materials. Therefore the chances of the local optimal solution can be greatly reduced in the actual cases. Pre-experiments should be carried out to find the convergence zone in the minimum least squares method. A perfect convergence zone without any local minimum is unrealistic for the multi-layered material. The PSO algorithm has made it feasible by allowing some local minima in the convergence zone. The population of the particles should be large enough to ensure the large probability of global optima. Short computing time and fast convergence are the inherent advantages of the PSO method.

9. Conclusions

The least squares inversion method using the reflection spectrum has been widely used to achieve simultaneous measurement of the properties of a thin single layer. However the current least squares inversion algorithm cannot avoid the problem of convergence and local optima for multi-layered materials. This is not only because of the increasing number of the estimated parameters, but also the unpredictable effect of each parameter on the reflection coefficient spectrum, which make it unrealistic to find the convergence. The direct calculation of the properties from the reflection spectrum is unrealistic due to the complexity of Eqs. (3) and (4). Many measurement techniques for the measurement of the properties of multi-layered structures have been also proposed. However most of these methods are not suitable for thin multi-layered structures. Some of them have limits in computing speed and stability. In this paper, a PSO-based least squares algorithm has been proposed for simultaneously estimating the ultrasonic parameters of each layer in a multi-layered structure. This method has followed the least squares inversion algorithm using the reflection spectrum, which is widely used in thin single layer measurement. In order to solve specific implementation issues in multi-layered structures, PSO has been used to improve the performance of the algorithm for parameter estimation. An aluminum-TC4 bi-layered material specimen has been used to obtain the experimental reflection spectrum. The real properties were obtained by measuring cylindrical specimens of pure aluminum and TC4. The capability of the new algorithm has been confirmed by the simulation and the experiment.

Acknowledgements

This work is supported by National Natural Science Foundation of China (Grant No. 51425504, 51575488, 51521001, 51605429), Science Fund for Creative Research Groups of National Natural Science Foundation of China 51521064, Zhejiang Provincial Natural Science Foundation of China LZ13E050001, Project of Application Research on Public welfare Technology 2014C31086.

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