



## Technical note

# An improved stiffness matrix model of the functional spinal unit for application to an improved understanding of pathological changes

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## ABSTRACT

The stiffness matrix is a useful way to describe the mechanical behaviour of the functional spinal unit, which is defined as the superior and inferior vertebrae, capsules and ligaments. This usefulness is extended by means of the concept of the “balance point”. The balance point is the load application point where the coupling coefficients of the stiffness matrix are minimized. Theoretical considerations are used to demonstrate that the stiffness matrix varies with load point location and thus a single stiffness matrix does not fully characterize the motion segment as well as to derive the stiffness matrix at any one specified point from the stiffness matrix at some other specified point. Special characteristics of the stiffness matrix obtained by loading through the “balance point” were shown. Some possible advantages derived from mechanical testing using the “balance point” concept are discussed. This study validates an improved stiffness matrix model that enhances the understanding of pathological changes by setting the gold standard of the behaviour of a normal functional spinal unit.

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## 1. Introduction

The mechanical behaviour of the Functional Spinal Unit (FSU) [1] represents an important aspect of both normal and pathological spinal function. The functional spinal unit (FSU) is defined as two adjacent vertebrae, the intervertebral disc and all collagenous tissues between them and it excludes the muscles.

The load-displacement behaviour of an FSU, as with all collagenous tissues, is nonlinear [2]. Within the total range of passive motion of any FSU, the typical load-displacement curve consists of two zones that exhibit very different biomechanical behaviours. Close to the neutral position of the FSU, this load-displacement behaviour is very compliant. This is the region known as the neutral zone [1]. During passive movement of the FSU, motion occurs with little resistance. Thus, a small load causes a relatively large displacement. The elastic zone follows from the end of the neutral zone to the point of maximum resistance (provided by the passive osteoligamentous tissues), thus limiting the range of motion. In the following work, we have treated the FSU with linear elastic behaviour, which is tantamount as studying at one strain rate.

Previous work [1,3–9] has shown that the lumbar FSU is viscoelastic, absorbs energy, moves with six degrees-of-freedom

(three translations and three rotations), exhibits coupled motion (motion in one direction affects motion in others), has limited fatigue tolerance and its mechanical behaviour is affected by its bony and ligamentous components.

In clinical practice, it is relatively easy to describe the shape of the spine [10] but it is more difficult to describe the three-dimensional motion of one segment relative to the adjacent segment [11,12]. Six degrees-of-freedom [11] positions of the *in-vivo* segment can be obtained by biplanar stereo radiography [13–17], CT or MRI. From *in-vivo* measurements, it is possible, but very difficult, to obtain data upon which to base a mechanical model. Measurements have been made, however, of intradiscal pressures in the classical studies of Nachemson [18] and electromyographic measurements [19] from which muscle forces have been estimated.

In clinical practice, it is possible to describe the motion [2,20] and, from the known loads, to calculate the stiffness matrix [4,21]. The stiffness matrix is defined as:

$$\underline{L} = [K]\underline{D}$$

where  $\underline{L}$  = general load vector,  $[K]$  = stiffness matrix and  $\underline{D}$  = general displacement vector.

This formulation assumes the existence of a series of linear relationships between the components of the applied load and the measured displacements. Previous workers have considered the intervertebral segment as a black box and the authors studied its global response to a certain kind of load or moment [7,8,22–26].

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Few groups have studied the six degrees-of-freedom (3 translations and 3 rotations) response to applied loads and torques of the minimally constrained vertebral body of the FSU [7,8,22,26]. These studies have defined the mechanical response by means of main and coupled stiffness coefficients. Using the main and coupled flexibility characteristics, it was assumed that a generalized stiffness matrix could be developed by inversion of the flexibility matrix, which would describe the behaviour of the segment. Determining this matrix would be of great benefit as it would permit comparison of segment properties as affected by loading, environmental conditions or disease history. Tencer and Ahmed [7] took into account the role of secondary variables in the measurement of the mechanical properties of the lumbar intervertebral joint.

The mechanical model of the segment is validated by its experimental behaviour. Two kinds of information are needed to define a mechanical model:

1. the mechanical behaviour of the material equivalent to the segment *in-vitro*, including the representation of the passive components of the segments, such as disc, ligaments, and bony elements; and
2. the mechanical behaviour of the segment in living situations to facilitate the understanding of the stabilizing action of the neuromuscular system.

For improved understanding of clinical practice, it is necessary to define an analytical model of the intervertebral segment (preferably both a static and dynamic model). In classical studies [2,8–17,26], the application of pure forces ( $f_x, f_y, f_z$ ) (i.e., pure loads are those unaccompanied by a moment) is desired. However, these loads are usually accompanied by coupled moments due to the fact that the application point of the load does not necessarily correspond to that point where, when a pure load vector is applied, minimal coupled rotations result. Through these coupled moments, the values reported of the coefficients of the stiffness matrix are modified, which may partly explain the different numerical values reported between authors [7,15,21,25]. A way to solve the problem of inconsistent response could be for all testing methods to use the same loading point. However, this would create difficulties since a consistent geometric location may not have consistent functional meaning. Both Rolander and Lin et al. [27,28] found points with consistent functional meaning and called them the “balanced position of the load” and an “operational centroidal axis” respectively. They were both defined as that point where, when a vertical load was applied, there was only a vertical deflection and no resultant rotation. The experimental methods used by these workers however, did not allow the motion segment to move in an unconstrained fashion.

One limitation of previous works is that there is no description of a reference based on mechanical function at the point at which the specimens are loaded. Except for the point of application of preload in the study by Panjabi et al. [26], and the description of the balance point in the work by Tencer and Ahmed [7], loads are applied to the motion segment over geometrically based reference locations. In particular, loads were applied either in line with the geometric centre of the upper vertebral body, or along an axis collinear with the line segment connecting the geometric centres of the centre of the upper and lower vertebral bodies of the motion segment. Tencer and Ahmed [7] did find a balance point, but did not mention its significance.

The significance of loading a specimen at its balance point is that it allows the determination of a stiffness matrix at a unique, functionally based reference. The geometric centre of the disc or vertebral body centrum may be a unique, geometrically based reference, but it is not a unique, functionally based reference. As long as the geometric centre is located at some distance away from the mechanical balance point, there will be some resultant rotation of

the specimen when a vertical load is applied there. A benefit of the balance point concept is that it will produce a stiffness matrix in which any coefficients approach zero.

As the balance point would be a unique descriptor of the mechanical properties of motion segment, it would change location as a result of changes in the mechanical properties of the segment. Therefore, its location might serve as a reliable outcome measure of the effects of property changes in the disc. It is imperative that the reliability of the balance point is sensitive to minimization desired in accordance with the coupled rotations or translations; load magnitude and direction; and load application rate due to the viscoelasticity that might change the effect of motion. It is therefore, necessary to specify all of these testing parameters, along with the location of the balance point found when describing the generalized stiffness matrix of the motion segment.

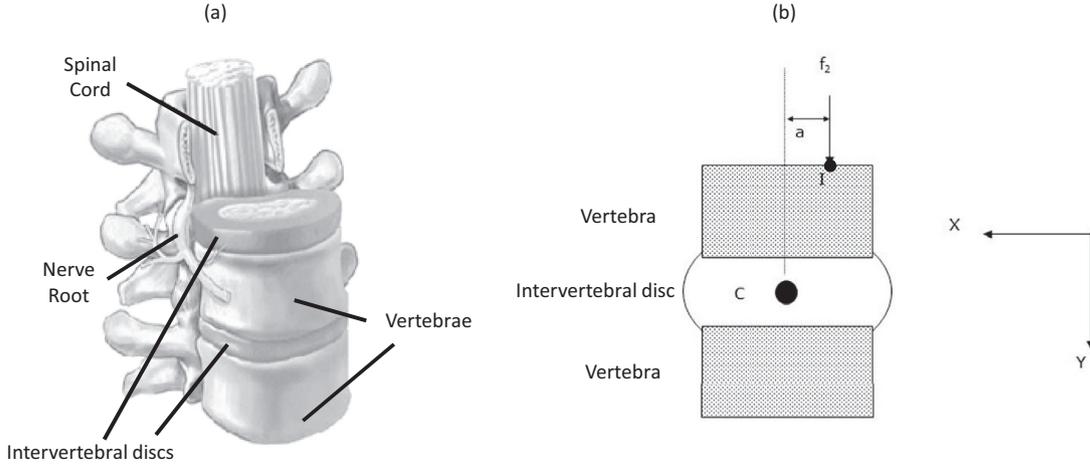
The concept of the balance point should have impact upon efforts to model the spine *in vivo*. It is assumed that the vertebral body centre is vertically in line with the pivot point of the motion segment. This pivot point is located in line with the geometric centre of the vertebral endplate. The pivot used should be located coincident with the experimentally found balance point. The differences between using a point in line with the geometric centre of the disc and the balance point as a pivot can be demonstrated with a simplified, mid-sagittal plane model of the motion segment. The model shows that there can be significant differences in loads in both the disc and the erector spinae muscles when the pivot point used is in line with the geometric centre as opposed to the experimentally determined balance point. Wilder et al. [29] performed an *in-vitro* experimental protocol to measure the flexibility matrix and demonstrated that a balance point could be identified for assessing the causal effect between mechanical changes and the load exposure history in the lumbar motion segment. A material testing apparatus equipped with motion sensors was developed to test experimentally the couple and main motion, viscoelastic, and compliance responses to axial compression loading applied at various point of a spinal motion segment. By collecting experimental data, the balance point was identified as the point where the lateral bending rotations and coupled flexion-extension was minimized. The average balance point location was posterior to the geometric centre. Its mean location ranged from a minimum of 5% to a maximum of 13.5% (of the endplate anterior-posterior diameter) posterior to the geometric centre. In one series of tests, one standard deviation away from the mean placed the balance point 24.5% posterior to the geometric centre [30].

Hence, the balance point would be especially important when trying to optimize task performance while minimizing a function of force on the spine or motion segment. The goals of the present study are twofold; (i) to demonstrate that a single stiffness matrix does not exist for the entire motion segment, but rather that the stiffness matrix coefficients vary from point to point within the specimen; and (ii) to derive the equations which allow the calculation from the value of the stiffness matrix at one point to the corresponding value at another point (which would allow comparison with the results given by other authors).

## 2. Methods and results

### 2.1. Theoretical consideration of stiffness matrix

Let  $\underline{L} = (L_1, L_2, L_3, L_4, L_5, L_6)$  be the general load vector applied at point C on the moving vertebra (Fig. 1).  $L_1, L_2, L_3$  are the components of force vector  $\underline{f} = (f_x, f_y, f_z)$  taken along the  $x, y,$  and  $z$  axes respectively of the  $R_c$  reference frame fixed to the moving vertebra.  $L_4, L_5, L_6$  are the components of moment vector  $\underline{M} = (M_x, M_y, M_z)$  taken about axes parallel to the  $x, y,$  and  $z$  axes of the  $R_c$  reference frame and passing through point C. Let  $\underline{D} = (D_1, D_2, D_3, D_4, D_5,$



**Fig. 1.** (a) A diagram showing the components of certain spinal region and (b) a schematic diagram in the sagittal plane showing the anterior view of a loading scheme of a motion segment with a vertical axial load  $f_2$ , applied at point I, at a distance  $a$ , from the geometric centre of the disc, C.

$D_6$ ) be the general displacement vector at the same point C on the moving vertebra were  $D_1, D_2, D_3$  are the components of the translation vector  $\underline{T} = (t_x, t_y, t_z)$  along the  $x, y$ , and  $z$  axes; and where  $D_4, D_5, D_6$  are the components of the rotation vector  $\underline{E} = (r_x, r_y, r_z)$  taken about axes parallel to the  $x, y$ , and  $z$  axes of the  $R_c$  reference frame and passing through point C.

We assume that at any time there exists a relation between each component,  $L_i$  of the generalized load  $\underline{L}$  and each component  $D_j$  of the generalized displacement  $D$ :

$$L_i = \sum_{j=1}^{j=6} K_{ij} D_j \quad \text{for } i = 1 \text{ to } 6 \quad (1)$$

Eq. (1) implies that each component  $L_i$  is linearly related to  $D_j$ , but there is no assumption about the terms  $K_{ij}$ , which may be either constant or dependent upon time, load magnitude, material properties, etc.

Eq. (1) may be written under the following matrix form:

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} \quad (2)$$

The  $K_{ij}$  terms with  $i$  from 1 to 6 are the direct stiffness coefficients. To help clarify the physical meaning of the  $K_{ij}$  we can write out the first relation of Eq. (2) with both indicial and physical notations:

$$L_1 = K_{11}D_1 + K_{12}D_2 + K_{13}D_3 + K_{14}D_4 + K_{15}D_5 + K_{16}D_6$$

$$f_x = K_{11}t_x + K_{12}t_y + K_{13}t_z + K_{14}r_x + K_{15}r_y + K_{16}r_z.$$

$K_{11}$  represents the coefficient between the component of the load along the  $x$  axis and the displacement in the same direction. It is the classical stiffness.

In addition to the direct displacement:  $t_x$ , the application of the load  $f_x$  creates coupled displacements (motions in a direction different from that of the applied force): coupled translations  $t_y$  and  $t_z$ , and coupled rotations  $r_x, r_y, r_z$ . The off-diagonal terms are the coupled stiffness coefficients.

## 2.2. Theoretical consideration of flexibility matrix

The stiffness matrix  $[K]$  is not usually directly determined in biomechanics, because it is easier to apply a load and to measure

the corresponding displacement than it is to do the converse [16]. So, the stiffness matrix  $[K]$  is obtained through the inversion of the flexibility matrix  $[F]$ , which is experimentally obtainable where  $[F]$  is defined so that:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\ F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\ F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36} \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} & F_{46} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} & F_{56} \\ F_{61} & F_{62} & F_{63} & F_{64} & F_{65} & F_{66} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \end{bmatrix} \quad (3)$$

The  $[F]$  matrix has been obtained by several groups [16,21,30] in the way of applying successive application of the 6 pure loads,  $L_i$  (3 components of pure load and 3 components of pure moment) that allows the calculation of the six rows of the flexibility matrix  $[F]$ .  $[K]$  is computed through inversion of the flexibility matrix to obtain:  $[F]^{-1}$ .

## 2.3. Critical analysis of available experimental results

Some authors assume sagittal plane symmetry and consider the stiffness matrix to be symmetrical about the diagonal, i.e.  $i=j$  [31–33]. Some assume sagittal plane symmetry of the motion segment, which implies that loads and moments applied in the sagittal plane do not result in displacements or rotations out of the sagittal plane [7,26,34]. Patwardhan et al. [34] assumed that the behaviour of the segment is elastic. This implies that the matrix  $[K]$  is symmetric according to the principles of conservation of energy. However, the components of the vertebral motion segments have non-linear behaviour even within the elastic region. Others [7,26] show that the application of a preload modifies the mechanical characteristics of the intervertebral material. Because of this non-linearity, no single value exists for the stiffness matrix  $[K]$ , to describe the state of the motion segment. But it is necessary to propose a method, which allows the comparison between the numerical results.

In order to apply a pure load to a material, it is necessary to specify the application point of the load vector. If rotations occur in response to an applied load, there is a mismatch between the load vector with its application point and the balance point (the point which when loaded exhibits minimal coupled rotation) of the material. When the load is applied away from the balance point, a moment is created which adds a rotational component to the material's response.

This is illustrated in an anterior view in Fig. 1. A longitudinal compressive force ( $f_2$ ) is applied to the upper vertebra of the motion segment, along a line parallel to the y-axis and passing through the geometric centre (point C) of the disc. Using more general terminology, the load is referenced to point C is  $\underline{L}_2 = (0, f_2, 0, 0, 0, 0)^T$ , and the resulting displacement is  $\underline{D}_2 = (D_{12}, D_{22}, D_{32}, D_{42}, D_{52}, D_{62})$ . Let point I be the intersection of the axis of lateral bend rotation with the coronal plane under consideration for the resulting non-zero displacement component  $D_{62}$ . Referenced to point I,  $\underline{L}'_2 = (0, f_2, 0, 0, 0, f_2a)^T$  (where “a” is the magnitude of the normal distance IC) (Fig. 1). Thus, the measured displacement  $\underline{D}_2$  is actually the sum of the displacements produced by both the force component  $L_{22} = f_2$ , and also the moment component  $L_{62} = f_2a$ . This introduces an error into the calculation of the flexibility matrix [F] in Eq. (3).

$$\begin{bmatrix} D_{12} \\ D_{22} \\ D_{32} \\ D_{42} \\ D_{52} \\ D_{62} \end{bmatrix}_{\text{Resulting}} = \begin{bmatrix} D_{12} \\ D_{22} \\ D_{32} \\ D_{42} \\ D_{52} \\ D_{62} \end{bmatrix}_{\text{Due to } f_2} + \begin{bmatrix} D_{12} \\ D_{22} \\ D_{32} \\ D_{42} \\ D_{52} \\ D_{62} \end{bmatrix}_{\text{Due to } f_2a} \quad (4)$$

$$\left\{ \begin{array}{l} D_{12} = F_{12}f_2 + F_{16}f_2a \\ D_{22} = F_{22}f_2 + F_{26}f_2a \\ D_{32} = F_{32}f_2 + F_{36}f_2a \\ D_{42} = F_{42}f_2 + F_{46}f_2a \\ D_{52} = F_{52}f_2 + F_{56}f_2a \\ D_{62} = F_{62}f_2 + F_{66}f_2a \end{array} \right\} \text{ in place of: } \left\{ \begin{array}{l} D_{12} = F'_{12}f_2 \\ D_{22} = F'_{22}f_2 \\ D_{32} = F'_{32}f_2 \\ D_{42} = F'_{42}f_2 \\ D_{52} = F'_{52}f_2 \\ D_{62} = F'_{62}f_2 \end{array} \right\} \quad (5)$$

In classical studies, the load is applied at the geometric centre point C of the upper mobile vertebral body centrum, or the point of intersection of the line connecting the centre of the motion segment vertebrae with the mid-transverse plane of the disc [8]. In several studies, there seems to be an assumption that the load point of minimal coupled rotation or balance point coincides with the centre C of the segment. However in general, this is not true [7,31–34].

#### 2.4. The position of this study

Several methods were developed to define a method allowing the comparison between the values of stiffness matrices corresponding to the same loading and experimentally calculated in different points as well as to propose an experimental protocol such that the loading point corresponds to the balance point (when a pure load vector is applied at the balance point with minimal coupled rotations result) allowing the possibility to obtain pure loading conditions, and then to study the parameters intervening on the magnitude of the terms of stiffness.

##### 2.4.1. Calculation of flexibility matrix when a load is applied at location other than at the balance point

Of interest is the relationship between the flexibility matrix [F] referenced to point I and the flexibility matrix [F'] referenced to some other point I', for a given load state. The generalized load vector at I is  $\underline{L}$  and at I' is  $\underline{L}'$ . Since the forces are constant ( $f' = f$ ), we have a reduced form:

$$L'_1 = L_1, L'_2 = L_2, L'_3 = L_3$$

The moments at I' and I are related by the equation:

$$M' = M + (I'I \times L)$$

where  $I'I \times L$  is the deviation of the moment from the measurement at some other point I' from the measurement at the referenced point I.

Thus, we can write:

$$L'_4 = L_4 - cL_2 + bL_3$$

$$L'_5 = L_5 + cL_1 - aL_3$$

$$L'_6 = L_6 - bL_1 + aL_2$$

where  $I'I = (ai + bj + ck)$  where a, b, c are the components of the vector  $I'I$ .

Using the matrix form, we can write:

$$\underline{L}' = [\Delta L]\underline{L} \quad (6)$$

$$\begin{bmatrix} L'_1 \\ L'_2 \\ L'_3 \\ L'_4 \\ L'_5 \\ L'_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -c & b & 1 & 0 & 0 \\ c & 0 & -a & 0 & 1 & 0 \\ -b & a & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \end{bmatrix}$$

The generalized displacement vector at I is D and at I' is D'. These are related by the equation:

$$T' = T + (I'I \times R)$$

where R is the rotation vector ( $D_4, D_5, D_6$ )<sup>T</sup>, T is the translation vector ( $D_1, D_2, D_3$ )<sup>T</sup>.

Thus, we can write:

$$D'_1 = D_1 - cD_5 + bD_6$$

$$D'_2 = D_2 + cD_4 - aD_6$$

$$D'_3 = D_3 - bD_4 + aD_5$$

and because  $R' = R$ , we have:  $D'_4 = D_4, D'_5 = D_5, D'_6 = D_6$ .

Using the matrix form, we can write:

$$D' = [\Delta D]D \quad (7)$$

or:

$$\begin{bmatrix} D'_1 \\ D'_2 \\ D'_3 \\ D'_4 \\ D'_5 \\ D'_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -c & b \\ 0 & 1 & 0 & c & 0 & -a \\ 0 & 0 & 1 & -b & a & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix}$$

For the stiffness matrices we have:

$$L = [K]D \quad (8)$$

$$L' = [K']D' \quad (9)$$

Substituting using Eqs. (6), (7) and (8), we can write:

$$[\Delta L]L = [K'][\Delta D]D$$

$$[\Delta L][K]D = [K'][\Delta D]D \quad (10)$$

$$[\Delta L][K][\Delta D]^{-1} = [K'] \quad (11)$$

Thus, we can calculate the stiffness matrix [K'] at any point I'. It can be seen that, in general, the matrix will vary from point to point, since  $[K'] = [K]$  only if  $[\Delta D] = [\Delta L] = [1]$  (the unity matrix), which is true only when  $a = b = c = 0$ , in other words, that  $I'I = (0i + 0j + 0k)$  and vanishes. This ability to calculate [K'] from [K] allows a comparison between the different results published in the literature, which have been obtained using comparable loading states.

#### 2.4.2. Calculation of the balance point location from the flexibility matrix found at any point

In this section, we will establish that the balance point location may be calculated and need not be determined experimentally.

Let  $L'$  and  $D'$  be the load and displacement vectors respectively in  $l'$  (which coincides with the balance point).

Let  $L$  and  $D$  be the corresponding vectors in the known point  $l$ . The balance point could be located theoretically by assuming that  $L'$  and  $D'$  be the load and displacement vectors respectively in  $l'$  where such point coincides with the balance point.

$$\text{We had } L' = [\Delta L]L \quad (12)$$

$$\text{and } D' = [\Delta D]D \quad (13)$$

Using the flexibility matrices:

$$D = [F]L \quad (14)$$

$$D' = [F']L' \quad (15)$$

$$D' = [\Delta D]D$$

$$D' = [\Delta D][F]L \quad (16)$$

$$[\Delta D][F]L = [F']L' = [F'][\Delta L]L \quad (17)$$

$$[\Delta D][F][\Delta L]^{-1} = [F'] \quad (18)$$

So it is possible to calculate the flexibility matrix  $[F']$  of balance point  $l'$  through  $[\Delta D]$ ,  $[F]$  and  $[\Delta L]^{-1}$ .

$$\text{where } [\Delta L]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & c & -b & 1 & 0 & 0 \\ -c & 0 & a & 0 & 1 & 0 \\ b & -a & 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

The resulting flexibility matrix  $[F']$  has the following form:

$$[\Delta D][F][\Delta L]^{-1} = [F'] = \begin{bmatrix} F'_{11} & F'_{12} & F'_{13} & F'_{14} & F'_{15} & F'_{16} \\ F'_{21} & F'_{22} & F'_{23} & F'_{24} & F'_{25} & F'_{26} \\ F'_{31} & F'_{32} & F'_{33} & F'_{34} & F'_{35} & F'_{36} \\ F'_{41} & F'_{42} & F'_{43} & F'_{44} & F'_{45} & F'_{46} \\ F'_{51} & F'_{52} & F'_{53} & F'_{54} & F'_{55} & F'_{56} \\ F'_{61} & F'_{62} & F'_{63} & F'_{64} & F'_{65} & F'_{66} \end{bmatrix} \quad (20)$$

Each coefficient of the flexibility matrix  $[F']$  in  $l'$  is a function of the flexibility  $[F]$  in  $l$  and of the components of the vector  $\underline{l}' = (ai + bj + ck)$ . Writing that the non-diagonal terms  $F'_{ij}$  must be minimum in  $l'$ , we can compute the values of  $a$ ,  $b$ ,  $c$ , satisfying these conditions. The function  $F_y$  is minimum in  $l'$ ; so we can write

$$(\partial F'_y / \partial a) = 0; \quad (\partial F'_y / \partial b) = 0; \quad (\partial F'_y / \partial c) = 0$$

The literal derivation gives linear equations in terms of  $a$ ,  $b$ ,  $c$  and any series of 3 non-diagonal values.  $F'_{12}$ ,  $F'_{13}$ ,  $F'_{23}$  for example are strictly sufficient for this calculation. The other values  $F'_{14}$ ,  $F'_{24}$ ,  $F'_{34}$  may be introduced for calculation of  $a$ ,  $b$  and  $c$ , considered to be a mean. For this we may use the least square method for any term  $F'_{ij}$ . The resolution of the numerical system may give the position of the balance point. On the contrary, if both the flexibility and stiffness matrices are known at the balance point, it is possible

to find the corresponding matrices in any point of 3-dimensional space.

To summarize, we introduce the use of the concept of balance point for practical reasons. Therefore, experiments and calculations are simpler and passage from knowledge of the stiffness matrix of the global segment to the physical modelling of each of its components is facilitated.

### 3. Discussion

In contrast to classical mechanical engineering, biological structures are more complex and do not easily reveal their function. Because they merely require conception and not the conscious thought of a human designer for their inception and fabrication, a kind of reverse engineering is needed to study them. In this case, the spine performs a function, but the details are not well known. Although the geometry of motion segment can be determined, it is impossible to use this to decipher completely the global stiffness characteristics, rotational characteristics, or mechanical model of its structure. Biomechanics researchers try to uncover global knowledge of the structure and function by means of the global stiffness matrix.

There are several limitations in this study. First, the improved stiffness matrix model did not include pathological specimens. Second, this study did not consider muscle activity.

The stiffness matrix is of obvious significance and allows the analytical description of the segment taking into account important mechanical data. However, it is not self-evident why we should study the balance point. There are several advantages to this concept; (i) the work has shown that the balance point serves as an example of load site dependency of the stiffness matrix; (ii) it provides an additional point at which the stiffness matrix is known. Thus, it is possible to provide additional knowledge of the motion segment behaviour, which in turn may be used for determination of properties of individual structural components; (iii) determination of the stiffness matrix;  $[K]$  is experimentally simpler using the balance point (BP) as the loading point, than using other loading points. The method of determining the stiffness matrix at the balance point has the advantage that it is possible to determine the flexibility matrix  $[F]_{BP}$  and the stiffness matrix  $[K]_{BP}$  at the balance point using only the application of a series of three forces:  $f_x$ ,  $f_y$ , and  $f_z$  applied in line with and at some known distance away from their respective balance points. The calculation of  $[K]$  is obvious from  $[F]$ .  $[K] = [F]^{-1}$ . Points  $BP_x$ ,  $BP_y$ , and  $BP_z$  may be chosen anywhere along the direction of the pure force which does not provide any rotation. If the three directions intersect, only one balance point will be defined. If the 3 directions do not intersect, a zone may be defined which contains the balance point; (iv) loading through BP as well as change of BP with the same applied load may be more sensitive for detecting changes produced by various interventions (e.g., discectomy, degeneration), than is loading through a geometric centre; (v) it is possible that the neuromuscular control system seems to minimize rotations during many loading circumstances, thus knowledge of the balance point may provide a useful tool to study such things as optimization of muscle forces and muscle activation strategies [31–34]; (vi) it is probable that the balance point changes under certain conditions and may affect the propensity to injury (e.g. disc degeneration, muscle fatigue, ligamentous tear) [31–34]; (vii) this study validates an improved stiffness matrix model that enhances the understanding of pathological changes by setting the gold standard of the behaviour of a normal functional spinal unit.

### Declaration of Competing Interest

None

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## Ethical approval

Not required.

## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.medengphy.2019.09.013](https://doi.org/10.1016/j.medengphy.2019.09.013).

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