



# Generation of negative group velocity Lamb waves by a moving laser source

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## ABSTRACT

Analytical existence conditions for negative group velocity (NGV) of an arbitrary mode Lamb waves in elastic plate was obtained. Experimentally, NGV waves were excited by means of a thermoelastic laser source moving across the sample plate with a controllable velocity. Such source is coupled with modes which phase velocity component parallel to the surface coincides with the source speed. Additional mode selection was performed by choosing a quasiperiodic shape of the laser spot, which specifies the range of the wavenumber  $k$ . This technique was successfully applied for generation of backward (NGV) waves. By tuning the speed of the source the propagation direction was switched from forward to reversal.

## 1. Introduction

Conventional optoacoustical methods for generation of Lamb waves utilize radiation of pulsed or modulated c.w. lasers absorbed in the sample plate, which gives rise to the thermoelastic stress in the material. Coupling of the laser radiation with the elastic perturbation of the solid can have different nature: thermal expansion, ablation or more complicated mechanisms like piezoelectric effect. Laser sources based on a pulsed laser can be optimized for certain Lamb mode generation by forming an appropriate spatial distribution of the laser radiation [1,2]. This determines the range of the wave number and therefore improves the efficiency of a selective generation of a desired acoustic mode, such as the wave with zero group velocity (ZGV). Alternative approach for mode selection is specifying the frequency range of the wave by means of modulation of a c.w. laser radiation [3,4]. Combination of both spatial and amplitude modulation theoretically allows for generation of a single wave in a multimodal structures such as an elastic plate (except for the points of modes intersection). In this work we have applied another method for mode selection: we specify the  $\omega/k$  ratio, or the phase velocity, along with the wavenumber  $k$ . A spot of a powerful c.w. laser was scanned across the sample surface at a constant velocity which can be precisely controlled. Such moving laser source is coupled mostly to the modes, which phase velocity (or its projection on the surface) coincides with the velocity of the laser source [5–7]. Additional optimization of the selective generation of the desired mode has been done by an appropriate shaping of the laser spot. For the source speed below the Rayleigh velocity only one Lamb mode can be generated, namely the lowest antisymmetric mode  $A_0$ , since all other modes lie

above the Rayleigh speed. Character of generation is affected by the frequency dispersion of the mode, which makes group and phase velocities differ and thus decrease the coupling effect between the source and the mode. In non-dispersive regime, such as the Rayleigh wave or the low-frequency limit of the  $S_0$  mode, the generated wave is a narrow broad-band pulse, whose width is determined by the spot size and amplitude grows linearly with propagation distance. Such growth is similar to the linear growth of the second harmonic in optics under the perfect phase synchronization. In this case the laser spot follows the acoustic pulse and continuously deposits energy through the linear thermoelastic mechanism. In the case of strong dispersion, the generated waves spread in time, acquiring narrow-band oscillatory shape.

## 2. Analysis of existence of NGV

Negative group velocity (NGV, and the special point with zero group velocity ZGV) can be observed in a wide range of material parameters and for different plate modes. It appears as a negative slope of the dispersion curve in the  $\omega - k$  domain, where the group velocity  $V_g = \partial\omega/\partial k < 0$ ,  $\omega$  and  $k$  are the angular frequency and the wave number respectively. Existence of this effect is completely determined by a single material parameter [8], for example the Poisson's ratio  $\nu$  or the ratio of longitudinal and transversal phase velocities of the material  $\kappa = V_l/V_T$ . These two parameters are related by  $\nu = \frac{1}{2} \frac{\kappa^2 - 2}{\kappa^2 - 1}$ . Physical limitation for the Poisson's ratio is  $-1 < \nu < 1/2$ , therefore  $\kappa$  must satisfy  $\kappa > 2/\sqrt{3}$ .

The necessary and sufficient condition for the NGV existence in a homogeneous isotropic plate is that the thickness resonance (at  $k = 0$ )

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of the mode corresponds to a local frequency maximum [9]:

$$\frac{\partial^2 \omega}{\partial k^2} \Big|_{k=0} < 0 \quad (1)$$

with  $\omega$  and  $k$  related by the Rayleigh-Lamb equation  $D_{s,a}(\omega, k) = 0$ , where subscripts  $a$  and  $s$  denote antisymmetric or symmetric wave respectively. For symmetric modes Eq. (1) decomposes into the following two inequalities for the material parameter  $\kappa$ :

$$(2n + 1)\pi\kappa^3 + 16 \tan((n + 1/2)\pi\kappa) < 0 \quad (2)$$

for the “odd” modes with  $\omega h = (n + 1/2)\pi V_L/2$  at  $k = 0, n = 0, 1, 2, \dots$ , and

$$n\pi\kappa - 8 \tan(n\pi/\kappa) < 0 \quad (3)$$

for the “even” modes with  $\omega h = n\pi V_T$  at  $k = 0, n = 1, 2, \dots$ , where  $h$  is the half of the plate thickness.

NGV effect is often explained as “mode repulsion” effect [10,11] between even and odd modes of the same type (both symmetric or antisymmetric) if they lie close enough at  $k = 0$ , or, in other words, if  $V_T$  approaches  $V_L/2$ .

Areas with NGV may also exist for antisymmetric modes, corresponding material conditions are given by:

$$n\pi\kappa^3 - 8 \tan(n\pi\kappa) < 0 \quad (4)$$

for “odd” modes with  $\omega h = n\pi V_L$  at  $k = 0, n = 1, 2, \dots$ , and

$$(2n + 1)\pi\kappa + 16/\tan((n + 1/2)\pi/\kappa) < 0 \quad (5)$$

for “even” modes with  $\omega h = (n + 1/2)\pi V_T$  at  $k = 0, n = 1, 2, \dots$

As can be seen from Eqs. (2) and (3), odd symmetric modes can exhibit NGV for all  $n$  starting from zero, whereas even modes require  $n \geq 1$ . “Odd” symmetric mode with  $n = 0$  and “even” mode with  $n = 1$  are  $S_1$  and  $S_2$  modes; repulsion between them is the cause for the lowest-mode NGV effect. Eq. (2) determines the range  $1.216 < \kappa < 2$  for  $n = 0$ , and Eq. (3) determines  $2 < \kappa < 3.390$  for  $n = 1$ . As  $\kappa$  passes the singular point  $\kappa = 2$ , even and odd modes exchange their relative positions, and the lowest branch exhibits NGV. Thus altogether NGV can be observed within the wide range  $1.216 < V_L/V_T < 3.390$  (or  $-0.544 < \nu < 0.45$ ). Besides this interval, there are other narrow intervals localized around  $\kappa = 4, 6, \dots$ , where the odd mode with  $n = 0$  gets closer to even modes with higher  $n$ . Interval for NGV effect in antisymmetric modes is somewhat narrower. The lowest NGV modes occur within the interval  $1.184 < V_L/V_T < 1.944$  with the transition point at  $\kappa = 3/2$  as a result of interaction (repulsion) between  $A_2$  and  $A_3$  modes, when  $V_L$  is close to  $3/2V_T$ . Ranges of the NGV existence given by Eqs. (2)–(5) corroborate the numerical studies [9,11]. Below we consider the NGV effect in the lowest symmetric mode, which in our experiment is the  $S_1$  mode.

Physically NGV Lamb waves have no principle peculiarities compared to normal modes. Effect of reversal energy flow is a result of essentially inhomogeneous character of Lamb waves. In those waves the amplitude and phase relations between different components vary with depth, and under certain conditions may change the direction of the local power flow. Local power flux through the elastic plate is defined by the acoustical Poynting vector  $p_i = -\langle \sigma_{ij} \dot{U}_j \rangle$  where  $\sigma_{ij}$  is the stress tensor,  $\dot{U}_j$  denotes the particle velocity components. Its component  $p_x$ , parallel to the wave propagation, varies with depth as shown in Fig. 1. Here the curves are normalized to the  $U_x$  displacement amplitude in the mid plane of the plate.

Total power flow, associated with group velocity  $V_g = \partial\omega/\partial k$  is the integral of  $p_x$  over the plate thickness. At the ZGV point this integral takes on zero value [12], whereas the character of the depth dependence of the energy flow remains qualitatively similar. Note that in the middle part of the plate the energy flows backwards even for “normal” wave.

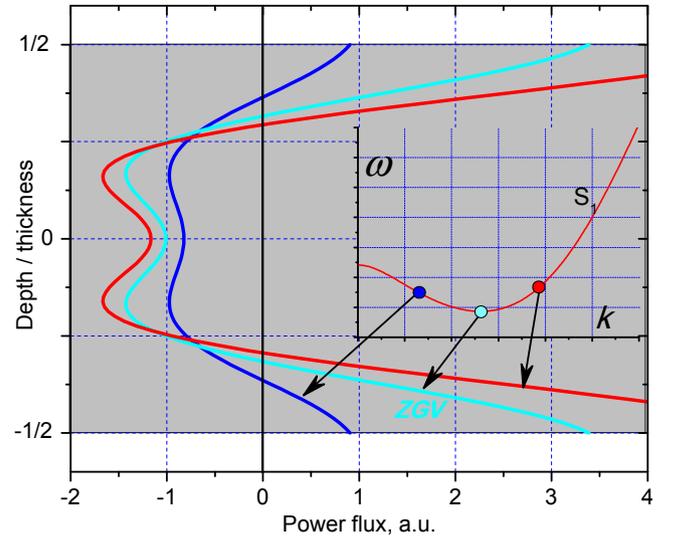


Fig. 1. Depth distribution of local acoustic power flux parallel to the wave propagation through the elastic plate (denoted with gray color).

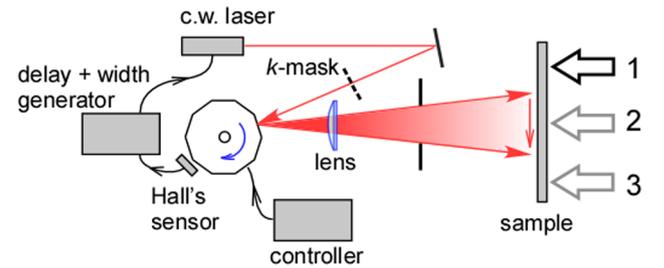


Fig. 2. Experimental setup for generation of NGV Lamb wave by moving laser.

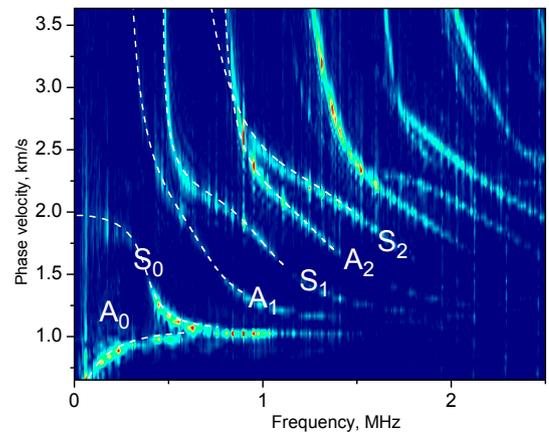


Fig. 3. Experimental dispersion curves in Polystyrene and calculated dispersion curves (dashed lines).

### 3. Experimental approach

As we have discussed before, in order to generate NGV wave by a moving laser source, its velocity must exceed the phase velocity at the ZGV point, which can be very high comparing with the sound speed. The minimal possible value of the phase velocity at the ZGV point is  $V_{ZGV} = 3/2V_L = 3V_T$  attained for  $\nu = 1/3$  ( $\kappa = 2$ ). To reduce  $V_{ZGV}$  to the experimentally convenient values, one must choose acoustically slow material with the Poisson’s ratio close to 1/3. In our experiment polystyrene plate with  $V_{ZGV}$  about 4 km/s was used. Thickness of 2.3 mm was chosen for efficient generation of symmetric modes in plates.

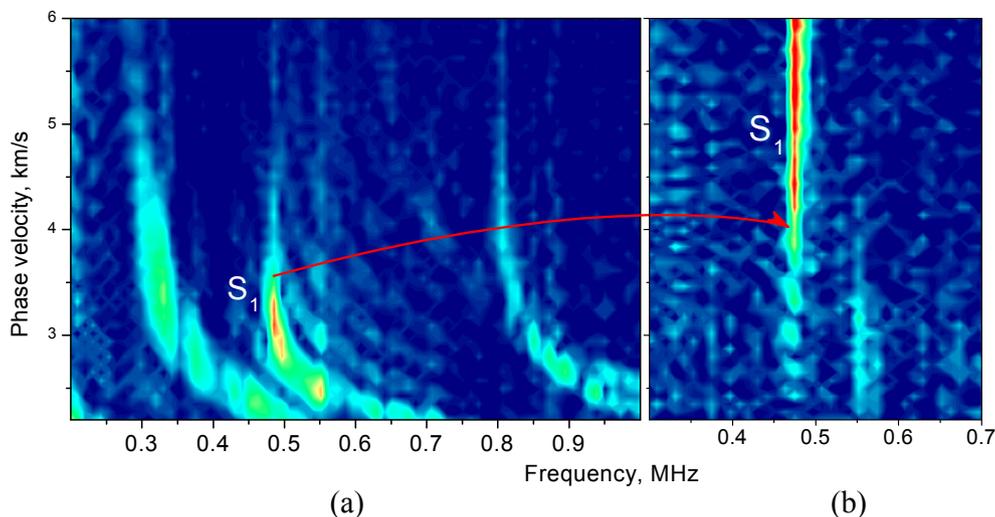


Fig. 4. Dispersion curves detected at (a) position 3 and (b) position 1 in Fig. 2.

In order to implement the thermoelastic source moving at the desired speed, the beam of a powerful (up to 1 kW) c.w. fiber laser was reflected at a mirror mounted on the laser scanning system as shown in Fig. 2 [13]. This system, based on a digital controller with a phase-lock and a DC motor, provides a precisely controllable angular speed within the range 100–400 revolutions per second with an accuracy of 0.02%. Characteristic length of the beam arm measured from the rotation mirror to the sample was 100 cm, and typical pass length on the sample was 10 cm. This determines the geometric variation of the spot velocity, which can be estimated as 0.013%, neglecting the effect of the lens. Position of the mirror was measured by the Hall's sensor, whose output was used to trigger the functional generator. This generator computed the necessary time delay and the width of the pulse which switched the radiation of the c.w. laser so that the laser was on during the necessary interval only. The spot shape at the sample surface has been formed by a combination of a long-focal spherical lens which provided the size of the spot, and a periodic mask which shaped the laser spot at the sample surface. As we were focusing on efficient generation of the  $S_1$  mode around its ZGV point, the periodic mask was applied to the laser beam to make the spatial distribution of the power flux in the form of 3 fringes with the period close to that of the ZGV wavelength. Before the sample surface, the laser beam hit the photodiode which was used to trigger the oscilloscope and thus determined the time frame. Detection was implemented on the opposite surface by a contact wide-band transducer based on the piezoelectric PVDF foil. The transducer can be installed in different positions with respect to the generation area: behind the laser source run (position 1), within the generation area (position 2), or ahead of the spot run (position 3).

#### 4. Results and discussions

After scanning the speed (typical step about 5–10 m/s), we obtain 2D diagram of acoustical field in the time-speed domain. For each velocity the Fourier transform of the corresponding waveform with respect to time was calculated and thus the dispersion curves were obtained. In the case of the transducer installed within the generation area (position 2), the obtained set of dispersion curves in polystyrene is shown in Fig. 3. Here the laser spot had a shape of a 3 mm wide strip, which provided a broad range of the wavenumber, sufficient for generation of multiple modes.

By fitting the theoretical calculated dispersion curves into the experimental result in Fig. 3, parameters  $V_L = 2306$  m/s and  $V_T = 1132$  m/s of the sample material were obtained. For such material  $\kappa > 2$  and NGV exists in the “even”  $S_1$  mode given by Eq. (3) with

$n = 1$ , originated from the transversal thickness resonance with  $\omega h|_{k=0} = \pi V_T$ . Using the evaluated material parameters of the sample material, theoretical value for the ZGV point is calculated as  $V_{ZGV} = 4110$  m/s at  $kh \approx 0.8$ .

The fact that the phase velocity at the ZGV point is always greater than any acoustical velocity of the material ( $V_{ZGV} \geq 3/2V_L$ ) helps to avoid simultaneous generation of lower plate modes  $A_0$  and  $S_0$ . This makes the method of mode selection through fixed  $\omega/k$  more effective in comparison with the fixed  $k$  method [14], where significant portion of energy goes into unwanted  $A_0$  and  $S_0$  modes.

To generate ZGV and NGV Lamb waves, the quasi-periodic laser spot was used with the fringes spacing of 9 mm, corresponded to the  $kh$  value near the optimum value 0.8–1. The transducer could be placed in three different positions with respect to the generation area: within the generation area (position 2 in Fig. 2), ahead of the spot run (position 3 in Fig. 2) or behind the laser source run (position 1 in Fig. 2). Dispersion curves obtained in those two locations 3 and 1 are shown in Fig. 4(a) and (b) respectively. As the velocity of the laser spot remains below  $V_{ZGV}$ , the  $S_1$  branch with the frequency range around 490 kHz is seen in the position ahead of the generation area, as shown in Fig. 4a. When the spot velocity approaches  $V_{ZGV}$ , the wave detected at position 3 vanishes gradually, and appears in the backward point for the velocities beyond  $V_{ZGV}$ , as shown in Fig. 4(b), where only the backward waves (NGV) can exist. Thus one can switch the propagation direction of the mode possessing NGV simply by changing the generation conditions.

#### 5. Summary

In summary, we have demonstrated an effective method for generating waves with negative group velocity. The method allows for switching the propagation regime from forward to backward. Combination of specifying the phase velocity and the wave vector range by using optimal beam width provides selective generation of the desired acoustic mode. Material conditions for the NGV Lamb wave existence were obtained in the concise form of Eqs. (2)–(5).

#### Acknowledgments

This work is supported by National Natural Science Foundation of China (Grant No. 11604153), Russian Foundation for Basic Research (Grant No. 19-02-00682), Natural Science Foundation of Jiangsu Province, China (No. BK20160818).

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