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## ORIGINAL ARTICLE

# Estimation of running endurance by means of empirical models: A preliminary study

## *Estimation de l'endurance en course à pied par des modèles empiriques: étude préliminaire*

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### KEYWORDS

Critical velocity;  
Kennelly model;  
Power laws;  
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Running

### Summary

**Purpose.** – The relationships between distance ( $D_{lim}$ ), speed ( $S$ ) and exhaustion time ( $t_{lim}$ ) can be described by different empirical models: power laws ( $D_{lim} = kt_{lim}^g$ , and  $S = kt_{lim}^{(g-1)}$  in Kennelly's model); critical speed ( $S_{crit}$ ) ( $D_{lim} = a + S_{crit} t_{lim}$ ) and logarithmic model ( $S = 1 - E \ln t_{lim}$  in Péronnet–Thibault model).

**Methods.** – These models have been applied to the performance of nine physical education students. In the first session, they performed the Montreal Track Test whose speed at the last stage (sMTT) is assumed to correspond to maximal aerobic speed. Thereafter, they performed 3 exhausting running exercises at 90, 100 and 110% sMTT, in different sessions with randomized order. The values of  $g$  and  $k$  were computed as ( $g_1, k_1$ ) from  $t_{lim}$  at 90 ( $t_{lim90}$ ), 100 ( $t_{lim100}$ ) and 110% sMTT ( $g_1, k_1$ ) and also from  $t_{lim90}$  and  $t_{lim100}$  ( $k_2, g_2$ ). The values of  $S_{crit}$  were normalized to sMTT.

**Results.** – Significant correlations ( $r > 0.999$ ) of the relationships between the logarithms of  $D_{lim}$  and  $t_{lim}$  were observed.  $S_{crit}$  was not significantly correlated with  $g_1, g_2$  and  $E$ . In contrast, the correlations between the individual values of  $g_1, g_2$  and  $E$  were highly significant.

**Conclusion.** – The high relationship between the logarithms of  $D_{lim}$  and  $t_{lim}$  suggested that power laws can be applied to subjects not-specialized in endurance. However, further studies with different protocols are necessary to determine if the power-law model is more accurate and useful than the logarithmic model in subjects who are not specialized in endurance.

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**MOTS CLÉS**

Vitesse critique ;  
Modèle de Kennelly ;  
Lois de puissance ;  
Modèle de  
Péronnet–Thibault ;  
Course à pied

**Résumé**

**Objectif.** – Les relations entre la distance ( $D_{lim}$ ), la vitesse ( $S$ ) et le temps d'épuisement ( $t_{lim}$ ) peuvent être décrites par différents modèles empiriques: lois de puissance ( $D_{lim} = kt_{lim}^g$  et  $S = kt_{lim}^{(g-1)}$ , modèle de Kennelly), vitesse critique ( $S_{crit}$ ) ( $D_{lim} = a + S_{crit} t_{lim}$ ) et modèle logarithmique ( $S = 1 - E \ln t_{lim}$ , modèle de Péronnet–Thibault).

**Méthodes.** – Ces modèles ont été appliqués aux performances de 9 étudiants en éducation physique. Dans la première séance ils réalisèrent le « Montreal Track Test » dont la vitesse au dernier palier (sMTT) est proche de la vitesse maximale aérobie. Ensuite, ils réalisèrent 3 exercices de course jusqu'à épuisement à 90, 100 et 110 % sMTT, dans différentes sessions dans un ordre randomisé. Les valeurs de  $g$  et  $k$  furent calculées à partir de  $t_{lim}$  à 90 ( $t_{lim90}$ ), 100 ( $t_{lim100}$ ) et 110 % sMTT ( $g_1, k_1$ ) et aussi uniquement à partir de  $t_{lim90}$  and  $t_{lim100}$  ( $k_2, g_2$ ). Les valeurs de  $S_{crit}$  furent rapportées à sMTT.

**Résultats.** – Des corrélations significatives ( $r > 0,999$ ) entre les logarithmes de  $D_{lim}$  et  $t_{lim}$  furent constatées.  $S_{crit}$  n'était pas significativement corrélé avec  $g_1, g_2$  et  $E$ . Au contraire, les corrélations entre les valeurs individuelles de  $g_1, g_2$  et  $E$  étaient très significatives.

**Conclusion.** – Les corrélations élevées entre les logarithmes de  $D_{lim}$  et  $t_{lim}$  suggèrent que les lois de puissance peuvent être appliquées aux sujets non spécialisés en endurance. Cependant, des études ultérieures avec différents protocoles sont nécessaires pour déterminer si le modèle lois-de-puissance est plus précis et utile que le modèle logarithmique chez des sujets non spécialisés en endurance.

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**Introduction**

In addition to biomechanical and biochemical models [1–8] different empirical models of the running performances have been proposed: power laws, hyperbolic and logarithmic models. In 1966, Ettema [9] applied the concept of critical power [10] to world records in running:

$$D_{lim} = a + bt_{lim}$$

Where  $t_{lim}$  corresponded to the world record for a given distance ( $D_{lim}$ ). Slope  $b$  was considered as a critical speed ( $S_{crit}$ ) equivalent to critical power:

$$D_{lim} = a + S_{crit} t_{lim}$$

In 1981, the interest for the concept of critical power or critical speed was renewed after a study on the application of the critical power to cycling exercises [11]. Nowadays, the critical-speed model is probably the most studied although the  $D_{lim}$ – $t_{lim}$  relationship is not linear. Indeed, the slope of the  $D_{lim}$ – $t_{lim}$  relationship depends on the range of  $t_{lim}$  and is higher for sprint races than for endurance races [9]. Many years before (1906), Kennelly had proposed power laws [12] for the relationship between speed ( $S$ ), distance ( $D$ ) and time ( $t$ ) in a descriptive model for world running records:

$$D = kt^g$$

$$S = kt^{g-1}$$

where  $g$  is an exponent.

As in the study by Kennelly, power laws have also recently been applied to world records [13–15]. Exponent  $g$  is a dimensionless parameter that can be considered as an endurance index. Indeed,  $g$  is close to 1 in elite endurance runners and lower in subjects who are not specialized in endurance running [16–18].

Another not-linear empirical model (logarithmic model) has been proposed for the relationship between speed ( $S$ ) and  $t_{lim}$  for running exercises beyond 7 min by Péronnet and Thibault [1]:

$$S/MAS = 100 - E \ln(t_{lim}/t_{MAS})$$

Where  $E$  is an endurance index and  $t_{MAS}$  was assumed to be equal to 7 min. However, the value of  $t_{MAS}$  is debated [2].

The models of Kennelly [12–15] and Péronnet–Thibault [1] have already been tested in elite athletes. Moreover, these two models were highly correlated in a study on elite athletes [16]. Both the logarithmic model and Kennelly's model can describe with accuracy [17] the performances of elite endurance runners whose values of  $g$  are high (0.90–95). On the other hand, the correlation between  $g$  (Kennelly model) and  $E$  (Péronnet–Thibault model) could be weak in subjects who are not endurance runners and whose values of  $g$  are lower than 0.85 [18]. The present study aimed to compare, the different indices of running endurance ( $S_{crit}$ ,  $g$ ,  $E$ ) in subjects who are not endurance runners in order to estimate the most useful index.

**Materials and methods****Participants**

The  $D_{lim}$ – $t_{lim}$  and  $S$ – $t_{lim}$  relationships were studied in nine physical-education students ( $24.7 \pm 1.9$  years;  $75.6 \pm 6.5$  kg;

1.79 ± 0.05 m). All procedures were explained to them and a written consent document approved by the institutional review board was completed before the procedures were started. The experimental protocol was carried out according to the guidelines of the declaration of Helsinki.

### Protocol of running exercises

The running exercises were performed on a 100m indoor track with cones placed every 12.5 meters. Running speed was imposed by a beeper controlled by a personal computer. The subjects must be within 2 m from the cones at each sound and they were considered as exhausted when they were unable to accelerate to compensate a 2-meter delay. In the first session, MAS was estimated by means of the Montreal Track Test [19] that consists of a graded-test (two-minute stages with speed increments equal to 1 km.h<sup>-1</sup>) until exhaustion. MAS is assumed to be close to the running speed (sMTT) of the last completed stage. In each of the following sessions, the participants performed an exhausting running exercise at either 90 or 100 or 110% sMTT, in a random order.

Heart rate was recorded during the exercise with a heart rate monitor (Polar RS400, Polar Electro Oy, Kempele, Finland) with a sampling frequency every 5 seconds. The higher value at the end of the exercise has been registred.

Blood lactate was measured using a portable blood lactate analyzer (Lactate Pro, Arkray, Tokyo, Japan). The lactate analyzer was calibrated before each testing session. This blood lactate analyzer has been reported to be reliable and valid [20]. Capillary blood samples (5 μL) were collected from the fingertip of a prewarmed hand for lactate concentration immediately and 2 min after exhaustion. The higher value was taken account. These capillary blood samples were considered as arterialized blood samples.

The rate of perceived exertion (RPE) was recorded immediately at the end of the exercise using the Borg 15-point category scale ranging from 6 (resting state) to 20, (maximal exertion) [21].

### Computations and statistics

The values of  $g$  and  $k$  of the individual power laws were computed from the linear regression between the logarithms of  $D_{lim}$  and  $t_{lim}$ :

$$D_{lim} = kt_{lim}^g$$

$$\ln(D_{lim}) = \ln(k) + g\ln(t_{lim})$$

$$Y = \ln(D_{lim})$$

$$X = \ln(t_{lim})$$

$$Y = m + nX$$

where  $k = \exp(\ln(k)) = \exp(m)$  and  $g = n$ .

The values of  $k$  and  $g$  ( $k_1$  and  $g_1$ ) were computed from the values of  $D_{lim}$  corresponding to  $t_{lim}$  at 90 ( $t_{lim90}$ ), 100 ( $t_{lim100}$ ) and 110% sMTT. To verify that the  $D_{lim}-t_{lim}$  power law is an accurate model,  $k$  and  $g$  were also computed ( $k_2$  and  $g_2$ ) from the values of  $D_{lim}$  corresponding to  $t_{lim}$  at 90 ( $t_{lim90}$ ) and 100 ( $t_{lim100}$ ) without taking into account  $D_{lim110}$  and  $t_{lim110}$ . If the individual power laws computed from  $D_{lim}$  and  $t_{lim}$  are perfect, the individual values of  $k_1$  and  $g_1$  should be equal  $k_2$  and  $g_2$ , respectively.

$S_{crit}$  was estimated by computing the regression between  $D_{lim}$  and  $t_{lim}$ :

$$D_{lim} = a + bt_{lim}$$

Where  $b$  was equal to  $S_{crit}$ .

Parameter  $E$  of the logarithmic model proposed by Péronnet and Thibault [1] was adapted to sMTT:

$$S/sMTT = 1 - \text{Eln}(t_{lim}/t_{limsMTT}) = 1 - \text{Eln}(t_{lim}/t_{lim100})$$

The value of  $t_{lim110}$  was not used in the computation of  $E$  as this model is used to describe the  $S-t_{lim}$  relationship for  $t_{lim} \geq t_{MAS}$ .

$$S_{90}/sMTT = 1 - \text{Eln}(t_{lim90}/t_{limsMTT}) = 0.90$$

Therefore,  $E$  was equal to:

$$E = (1 - 0.90) / \ln(t_{lim90}/t_{lim100}) = 0.1 / \ln(t_{lim90}/t_{lim100})$$

The individual differences between  $g_1$  and  $g_2$  were computed as following:

$$\text{Differencein\%} = 200|g_1 - g_2| / (g_1 + g_2)$$

The individual differences between  $k_1$  and  $k_2$  was computed with the same method.

### Statistics

Given the small number of participants, a Wilcoxon test for paired samples was used to analyze statistical differences. Wilcoxon tests and linear regressions were carried out using Sigma-Stat software (Systat Software, Germany).

### Results

The average values of  $t_{lim}$ , HR, [Lact] and RPE at 90, 100 and 110% sMTT are presented in Table 1. The difference between RPE at 100 and 110% sMTT was the only significant difference ( $P = 0.008$ ) for HR, [Lact] and RPE.

### Power law model

The correlation coefficients of the individual regressions between  $\ln(D_{lim})$  and  $\ln(t_{lim})$  were high ( $r \geq 0.999$ ). The mean values of exponents  $g_1$  and  $g_2$  were not significantly different and the individual absolute differences between  $g_1$  and  $g_2$  were moderate ( $4.8 \pm 4.1\%$ ). The mean values of  $k_1$  and  $k_2$

**Table 1** Mean values  $\pm$  SD of  $t_{lim}$ , heart rate, blood lactate and rate of perceived exertion (RPE) for the 3 running velocities (90, 100 and 110% sMTT).

	90% sMTT	100% sMTT	110% sMTT
$t_{lim}$ (s)	624 $\pm$ 127	338 $\pm$ 61	217 $\pm$ 37
Heart rate (bpm)	190 $\pm$ 8.3	190 $\pm$ 7.8	190 $\pm$ 11.2
Lactate (mmol.L <sup>-1</sup> )	12.7 $\pm$ 1.8	12.6 $\pm$ 1.54	13.7 $\pm$ 1.9
RPE	15.7 $\pm$ 2.3	15.3 $\pm$ 1.5	16.7 $\pm$ 1.7

**Table 2** Individual values, mean (M) and standard deviation (SD) of the parameters of the different models, computed either from  $t_{lim90}$ ,  $t_{lim100}$  and  $t_{lim110}$  ( $k_1$ ,  $g_1$  and  $S_{crit}$ .) or from  $t_{lim90}$  and  $t_{lim100}$  ( $k_2$ ,  $g_2$  and E). The values of  $S_{crit}$  are normalized to sMTT.

Subject	sMTT km.h <sup>-1</sup>	Power laws				Logarithmic	Hyperbolic
		$k_1$	$g_1$	$k_2$	$g_2$	E	$S_{crit}$
1	16.00	9.43	0.872	7.8	0.902	0.093	0.852
2	14.50	23.9	0.701	19.8	0.731	0.255	0.752
3	16.75	24.7	0.714	40.2	0.634	0.348	0.836
4	15.50	12.2	0.826	21.0	0.740	0.247	0.808
5	16.50	12.5	0.826	11.1	0.844	0.148	0.860
6	13.75	10.8	0.825	8.0	0.871	0.122	0.874
7	15.75	12.0	0.832	12.7	0.823	0.168	0.806
8	14.00	11.4	0.810	9.7	0.837	0.155	0.730
9	14.00	11.0	0.816	10.6	0.821	0.170	0.875
M	15.19	14.2	0.802	15.7	0.800	0.190	0.821
SD	1.15	5.8	0.057	10.4	0.083	0.079	0.052

(Table 2) were not significantly different but the individual absolute differences between  $k_1$  and  $k_2$  were not negligible ( $22.8 \pm 17.7\%$ ). As  $k_1$  and  $k_2$ , exponents  $g_1$  and  $g_2$  were significantly correlated:

$$k_2 = -5.387 + 1.481k_1, r = 0.828; P = 0.006$$

$$g_2 = -0.176 + 1.216g_1, r = 0.826; P = 0.006$$

**Logarithmic model**

Parameter E was significantly correlated with  $g_1$  (Fig. 1, black dots):

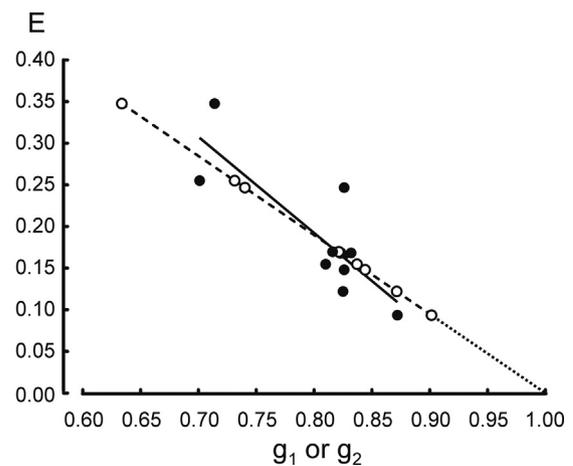
$$E = -1.116 + 1.154g_1, r = 0.826; P = 0.006$$

The relationship between the individual values of E and  $g_2$  was perfectly linear (Fig. 1, empty circles):

$$E = 0.949 - 0.949g_2, r = 1$$

**Critical speeds**

The average value of  $S_{crit}/sMTT$  ( $0.821 \pm 0.052$ , Table 1) was not significantly different ( $P > 0.373$ ) from  $g_1$  ( $0.802 \pm 0.057$ ) and  $g_2$  ( $0.800 \pm 0.083$ ). However, the individual values of



**Figure 1** Relationship between either E and  $g_1$  (black dots, solid line) or E and  $g_2$  (empty circles, dashed line).

$S_{crit}/sMTT$  were not significantly correlated with the individual values of  $g_1$ ,  $g_2$  and E:

$$S_{crit}/sMTT = 0.531 + 0.361g_1, r = 0.391; P = 0.298$$

$$S_{crit}/sMTT = 0.689 + 0.165g_2, r = 0.263; P = 0.494$$

$$S_{crit}/sMTT = 0.854 - 0.174E, r = 0.263; P = 0.494$$

## Discussion

The results of the present study suggested that both models of Kennelly and Péronnet–Thibault can be applied to subjects who are not endurance runners. Indeed the parameter  $E$  of the Péronnet–Thibault model was significantly correlated with the parameters  $g_1$  and  $g_2$  of the Kennelly model. Similarly, the value of  $S_{\text{crit}}/s\text{MTT}$  was not significantly different from  $g_1$  and  $g_2$ . The differences between the individual values of  $g_1$  and  $g_2$  were moderate and the correlation between  $g_1$  and  $g_2$  were significant. On the other hand, the individual differences between  $k_1$  and  $k_2$  were not negligible although the mean values of  $k_1$  and  $k_2$  were not significantly different.

In a previous study [18] value of  $S_{\text{crit}}$  was close to the value of  $g$  for the usual range of  $t_{\text{lim}}$  between 3–15 min, as in the present study. Similarly, the average value of  $S_{\text{crit}}/s\text{MTT}$  was not significantly different from  $g_1$  and  $g_2$  in the present study. But the correlations between the individual values of  $S_{\text{crit}}$  and  $g_1$ ,  $g_2$  or  $E$  were not significant probably because the range of  $S_{\text{crit}}$  was relatively small (from 0.730 to 0.875, Table 1) and  $S_{\text{crit}}$  depends on  $t_{\text{lim}}$  [9]. In theory,  $S_{\text{crit}}$  represents the fastest speed that can be maintained a very long time. However, on a treadmill, the subjects are generally only able to maintain  $S_{\text{crit}}$  for less than 30 min and the running velocities that could be maintained for 60 minutes on a treadmill are largely overestimated by  $S_{\text{crit}}$  [22]. An overestimation should be higher in subject whose  $D_{\text{lim}}-t_{\text{lim}}$  relationship is more curved. Maximal lactate steady state is usually sustainable for 30 to 60 min [23–25]. In a study on endurance runners [26], the running speed at lactate steady-state ( $4.80 \text{ m}\cdot\text{s}^{-1}$ ), estimated according to the protocol proposed by Chassain [27], was significantly correlated ( $r=0.97$ ) and not significantly different from the value of  $S_{\text{crit}}$  ( $4.83 \text{ m}\cdot\text{s}^{-1}$ ), excepted in one subject. In this latter study, the highest individual values of  $t_{\text{lim}}$  were longer than 30 min.

The curvature of the  $D_{\text{lim}}-t_{\text{lim}}$  relationship depends on the decrease in the fraction of maximal aerobic metabolism that can be sustained during long lasting exercises. The  $D_{\text{lim}}-t_{\text{lim}}$  relationship is almost linear and  $g$  is close to 1 in elite endurance runners [16, 18]. The value of  $g_1$  was lower (0.802) for the subjects of the present study who were not specialized in endurance exercises. The results of the present study suggest that the Kennelly's model could describe the running performances of subjects who are not specialized in endurance running because: i) the correlation coefficient between  $\ln(D_{\text{lim}})$  and  $\ln(t_{\text{lim}})$  were high ( $\geq 0.999$ ); ii) the mean values of  $g_1$  and  $g_2$  were not significantly different and the individual values of  $g_1$  and  $g_2$  were significantly correlated. However, the individual differences between the parameters of the power laws in Tables 1 and 2 were not negligible (4.8% for the differences between  $g_1$  and  $g_2$  and 22.8% for the difference between  $k_1$  and  $k_2$ ). Parameter  $k$  whose difference between  $k_1$  and  $k_2$  is important is less interesting than parameter  $g$  that is an endurance index. The difference between  $g_1$  and  $g_2$  could be the expression of the lack of precision of the Kennelly's model and/or the result of submaximal performances.

The relationship between  $E$  and  $g_2$  (Fig. 1, empty circle) was perfectly linear ( $r=1$ ). But it can be mathematically

demonstrated (Appendix 1) that the relationship between the individual values of  $E$  and  $g$  is perfectly linear when  $E$  and  $g$  are computed from  $t_{\text{lim}100}$  and only one another values of  $t_{\text{lim}}$  corresponding to the same fraction of  $s\text{MTT}$  (or  $\text{MAS}$ ). This perfect linear relationship between  $E$  and  $g_2$  also explained that the coefficients of correlation were the same ( $r=0.263$ ) for the relationships between  $S_{\text{crit}}/s\text{MTT}$  and either  $E$  or  $g_2$ . The results of the protocol of the present investigation suggest the possibility of the application of Kennelly's model in subjects who are not elite endurance runners. Moreover, the results obtained with the present protocol showed that  $g_1$  was highly correlated with  $E$  (Fig. 1). However, the accuracy of the Péronnet–Thibault model could not be evaluated with the protocol used in the present study because  $E$  was computed with only two values of  $t_{\text{lim}}$  ( $t_{\text{lim}100}$  and  $t_{\text{lim}90}$ ).

In the future, it would be interesting to test what model (Kennelly or Péronnet–Thibault) is the most accurate by studying the running performances with other experimental protocols including at least 3 speeds, lower or equal to 100%  $s\text{MTT}$ . For example, either a 0.80–0.90–100%  $s\text{MTT}$  protocol or a 0.90–0.95–100%  $s\text{MTT}$  protocol. Moreover, a protocol with repetitions of the exhausting exercises at each running speed would probably improve the validity of the performances. After this future study, endurance could be evaluated with the endurance parameter of the best model ( $E$  or  $g$ ) applied to the results of a simplified protocol consisting of only 2 running speeds. Indeed, critical speed, Kennelly model and Péronnet–Thibault model can be computed with only two values of  $t_{\text{lim}}$ . Such a protocol would include running speeds at 85 and 100%  $s\text{MTT}$  in subjects who are not endurance runners. The aim of the slower running exercise (85%  $s\text{MTT}$ ) would be to reach a value of  $t_{\text{lim}}$  at least between 20 and 40 min. In these subjects it is likely that  $S_{\text{crit}}$  would be closer to the running speed at lactate steady-state with such a protocol including a lower speed (85%  $s\text{MTT}$ ) and excluding speeds higher than 100%  $s\text{MTT}$ . In endurance runners, a protocol including running speeds at 90 and 100%  $s\text{MTT}$  would be performed. In elite endurance runners, a 95%  $s\text{MTT}$  speed would probably be preferable to a 90% running speed. Moreover, several repetitions at each speed would be performed to improve the validity of the data.

## Conclusion

Parameters  $g_1$ ,  $g_2$  and  $E$  were significantly correlated. Therefore, the results of the present study suggested that Kennelly's model can be applied to the individual performances of subjects who are not elite endurance runners. However, the data of the present study did not enable to decide if the model of Kennelly [12] is more accurate and more useful than the logarithmic model proposed by Péronnet and Thibault [1]. Therefore, further studies with different protocols are necessary for the confirmation of the results of the present study and the comparison of the accuracy of the power law and logarithmic models.

## Disclosure of interest

The authors declare that they have no competing interest.

## Appendix A. Appendix A

For  $t_{lim100}$  and  $t_{lim90}$  the  $S$ - $t_{lim}$  equation in the power law model is:

$$S_{100}/sMTT = k t_{lim100}^{(g-1)}/sMTT = 1$$

$$S_{90}/sMTT = k t_{lim90}^{(g-1)}/sMTT = 0.9$$

Therefore:

$$(t_{lim90}/t_{lim100})^{(g-1)} = 0.9 \ln(t_{lim90}/t_{lim100})^{(g-1)} = (g-1)$$

$$\ln(t_{lim90}/t_{lim100}) = \ln(0.9) \ln(t_{lim90}/t_{lim100}) = \ln(0.9)/(g-1)$$

$$\text{As } E = 0.1/\ln(t_{lim90}/t_{lim100}) \text{ and } \ln(0.9) = -0.1054$$

$$E = 0.1 (g-1)/(-0.1054) = -0.949g + 0.949$$

## References

- [1] Péronnet F, Thibault G. Mathematical analysis of running performance and world running records. *J Appl Physiol* 1989;67:453–65.
- [2] Di Prampero PE, Capelli C, Pagliaro P, Antonutto G, Girardis M, Zamparo P, et al. Energetics of best performances in middle-distance running. *J Appl Physiol* 1993;74:2318–24.
- [3] Henry FM. Time-velocity equations and oxygen requirements of all-out and steady-pace running. *Res Q* 1954;25:164–77.
- [4] Morton RH. A three component model of human bioenergetics. *J Math Biol* 1986;24(4):451–66.
- [5] Ward-Smith AJ. A mathematical theory of running, based on the first law of thermodynamics, and its application to the performance of world-class athletes. *J Biomech* 1985;18:337–49.
- [6] Ward-Smith AJ. The bioenergetics of optimal performances in middle-distance and long-distance track running. *J Biomech* 1999;32:461–5.
- [7] Wilkie DR. Equations describing power input by humans as a function of duration of exercise. In: Cerretelli P, Whipp BJ, editors. *Exercise bioenergetics and gas exchange*. Amsterdam: Elsevier; 1980. p. 75–80.
- [8] Wilkie DR. Man as a source of mechanical power. *Ergonomics* 1960;3:1–8.
- [9] Ettema JH. Limits of human performance and energy-production. *Int Zeitschrift Angew Physiol Einschl Arbeitsphysiol* 1966;22:45–54.
- [10] Monod H, Scherrer J. The work capacity of a synergic muscular group. *Ergonomics* 1965;8:329–38.
- [11] Moritani T, Nagata A, DeVries HA, Muro M. Critical power as a measure of physical work capacity and anaerobic threshold. *Ergonomics* 1981;24:339–50.
- [12] Kennelly AE. An approximate law of fatigue in the speeds of racing animals. *Proc Am Acad Arts Sci* 1906;42:275–331.
- [13] Katz JS, Katz L. Power laws and athletic performance. *J Sports Sci* 1999;17:467–76.
- [14] Savaglio S, Carbone V. Human performance: scaling in athletic world records. *Nature* 2000;404:244.
- [15] Carbone V, Savaglio S. Scaling laws and forecasting in athletic world records. *J Sports Sci* 2001;19:477–84.
- [16] Vandewalle H. Mathematical modeling of the running performances in endurance exercises: comparison of the models of Kennelly and Péronnet-Thibault for World records and elite endurance runners. *Am J of Eng Res* 2017;6:317–23.
- [17] Vandewalle H, Zinoubi B, Driss T. Modelling of running performances: comparison of power laws and Endurance Index. In: 14th Annual Conference of The Society of Chinese Scholars on Exercise Physiology and Fitness (SCSEPF). 2015.
- [18] Zinoubi B, Vandewalle H, Driss T. Modeling of running performances in human: comparison of power laws and critical speed. *J Strength Cond Res* 2017;31:1859–67.
- [19] Léger L, Boucher R. An indirect continuous running multistage field test: the Université de Montréal track test. *Can J Appl Sport Sci* 1980;5:77–84.
- [20] Pyne DB, Boston T, Martin DT, Logan A. Evaluation of the Lactate Pro blood lactate analyser. *Eur J Appl Physiol* 2000;82(1–2):112–6.
- [21] Borg GAV. Psychophysical base of perceived exertion. *Med Sci Sports Exerc* 1982;14(5):377–81.
- [22] Pepper ML, Housh TJ, Johnson GO. The accuracy of the critical velocity test for predicting time to exhaustion during treadmill running. *Int J Sports Med* 1992;13:121–4.
- [23] Baron B, Noakes TD, Deckerle J, Moullan F, Robin S, Matran R, et al. Why does exercise terminate at the maximal lactate steady state intensity? *Br J Sports Med* 2008;42.
- [24] Fontana P, Boutellier U, Knöpfli-Lenzin C. Time to exhaustion at maximal lactate steady state is similar for cycling and running in moderately trained subjects. *Eur J Appl Physiol* 2009;107:187–92.
- [25] Beneke R, Leithäuser RM, Ochental O. Blood lactate diagnostics in exercise testing and training. *Int J Sports Physiol Perform* 2011;6:8–24.
- [26] Sid-Ali B, Vandewalle H, Chair K, Moreaux A, Monod H. Lactate steady state velocity and distance-exhaustion time relationship in running. *Arch Int Physiol Biochim Biophys* 1991;99:297–301.
- [27] Chassain AP. Méthode d'appréciation objective de la tolérance de l'organisme à l'effort : application à la mesure des puissances critiques de la fréquence cardiaque et de la lactatémie. *Sci Sports* 1986;1:41–8.