



Research article

Estimating confined aquifer parameters using a simple derivative-based method

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ARTICLE INFO

Keywords:

Earth sciences
Hydrology
Confined aquifer
Derivative-based method
Aquifer parameters
Pumping tests

ABSTRACT

The confined aquifer parameters, transmissivity and storage coefficient, are commonly determined using the pumping tests. Several methods have been developed to estimate confined aquifer parameters using pumping tests, but different methods suffer from different drawbacks. Those methods that use the truncated Theis well function $w(u)$, apply just early or late drawdowns, depending on the case, to estimate the aquifer parameters. Those methods, such as Theis (1935), that use non-truncated well function $w(u)$, can apply all drawdown data for aquifer parameter estimation but may still suffer from subjectivities such as personal judgment in curve matching, time-consuming procedure and requiring values for Theis well function, $w(u)$, and its argument $u \left(u = \frac{r^2 S}{4Tt} \right)$. The aim of this study is to present a new method to overcome the aforementioned drawbacks and subjectivities involved in available published methods. In this paper, a simple derivative-based method is presented to estimate confined aquifer parameters applicable for entire drawdowns during the pumping period. The time derivative of drawdowns relate non-linearly with pumping time t , and therefore, aquifer parameters are estimated using developed equations based on the least squares optimization approach. The method is applied to three sets of synthetic, published and field data and results show that the estimation accuracy is acceptable. The drawdown time interval measurement has a marginal effect on parameters estimation due to the analytical basis of derivative calculations. The method does not require construction of graphs, and numerical calculations may be performed on a calculator to determine the aquifer parameters on site. It does not require curve matching, initial guess of the parameters and values of $w(u)$ and u .

1. Introduction

The transmissivity (T) and storage coefficient (S) of the aquifers control the movement and extraction of groundwater in the geological formations [1]. These hydraulic parameters of the aquifers, which are key inputs for groundwater modeling and management, widely estimated using pumping tests [2, 3]. Theis (1935) developed an equation to determine the transmissivity and storage coefficient in a homogenous, isotropic, infinite areal extent confined aquifer using the measured drawdowns in an observation well located at a specific distance from a fully-penetrated tube well that is pumped in a constant rate [4]. He proposed a curve-matching approach to determine the aquifer parameters, but this approach is time-consuming, requiring to know argument u and well function ($w(u)$); and it involves errors due to the personal

judgment in determination of the best match between the observed and theoretical curves, especially when only early drawdowns are measured [1, 5, 6, 7, 8, 9, 10]. Some researchers have developed different simple solutions for the Theis equation to overcome these drawbacks [2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

The aquifer parameters are determined using a fitted straight line through drawdown versus logarithmic time plot for $u < 0.01$ in Cooper and Jacob solution, which was developed in 1946. This solution is simple and widely used, but it suffers from the subjective judgment in fitting a straight line through those data that satisfies $u < 0.01$, requiring the trial and error. The method applicability is limited by the short duration of the pumping test and large distance of observation well from the pumping well [5, 6, 7, 12, 15].

Chow (1952) proposed a solution in which a semi-logarithmic plot is

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prepared similarly to the Cooper-Jacob's solution. A tangent at any arbitrary point is drawn and value of $F(u)$, a function defined by him, is calculated using the corresponding drawdown and drawdown difference per log cycle of time. Then the values of u and $W(u)$ are obtained from a special chart prepared by him and aquifer parameters are subsequently calculated. The method drawbacks are personal judgment in curve fitting, especially in drawing the tangent [5] and requirement of special chart to calculate the values of u and $W(u)$ [6, 7, 12].

Khan (1982) proposed a solution for the Cooper-Jacob's method using the regression analysis of the time and drawdown data. It does not require construction of a graph, avoiding the error due to the personal judgment in curve fitting. The Khan's method, in spite of lack of this drawback [6], still involves drawbacks as described in the Cooper-Jacob's method. The main question is considering the fraction of drawdown data to meet the condition for $u < 0.01$, while any graph is not constructed.

Rai (1985) proposed a numerical solution based on the Chow's analytical method in which two or three successive drawdowns are selected to calculate aquifer parameters [12]. The estimated aquifer parameters are acceptable when the method applied for the very-small time interval measurements during the pumping test [7]. It also requires a specific graph or table to calculate $f(u)$ and u values for each drawdown and its derivative [9, 12].

Sen (1986) developed a slope-matching solution for the Theis's equation where the aquifer parameters for each time interval measurement are calculated using the slope of the logarithmic plot of drawdown versus time [7, 13]. The procedure requires calculating the values of u and corresponding $w(u)$ using a prepared chart by Sen.

El Khatib (1987) suggested a solution based on the slope-matching method in which the logarithm of tds/dt is plotted versus $1/t$, then the values of T and S are calculated using the intercept and slope of the fitted straight line, respectively [14]. The method requires the computation of derivatives of discrete drawdowns with acceptable accuracy for reliable estimation of the parameters [2, 9].

Yeh (1987) used the non-linear least squares and finite-difference Newton's method to determine the aquifer parameters which requires an initial guess for T and S [7]. The convergence problem may arise by applying deviated initial guess values [3]. Additionally, their finite-difference implementations may give different results because the derivative expressions are not based on an analytical approach [3, 9].

Singh used early drawdown data ($u \geq 0.01$) to determine the aquifer parameters using different methods [8, 9, 10, 15, 18]. He used a derivative-based robust optimization method to identify the aquifer parameters under boundaries effect [8] in which the convergence problem is common [3]. Singh (2000, 2002) proposed solutions based on the peak determination of unimodal curves drawing through the early drawdown data without and under boundaries effect, respectively [10, 15]. He proposed a diagnostic unimodal curve solution, for early drawdowns, in year 2008 which requires curve matching with much less subjectively [18]. Singh in his first publication in year 2001, used a temporal derivative of early drawdowns, similar to the El Khatib's solution, to calculate the aquifer parameters using the least-square optimization technique [9]. Both Singh's methods proposed in years 2000 and 2001 were reduced to the Cooper-Jacob's method for late drawdowns ($u \leq 0.01$) [9, 15]. The method's advantages include saving of time and money, no data corruption due to aquifer boundaries (except the methods developed later in year 2001 and that of 2002), no curve matching (except the method developed in year 2008), no initial guess of the parameters (except the method developed later in year 2001) or no special care to check for argument u (except the method developed in year 2001). Regarding the peak determination-based solutions, the personal judgment is involved due to drawing a smooth curve through a few points of early drawdowns. The accuracy of the estimated aquifer parameters depends on the accurate determination of the peak [15]. A necessary condition for a peak to be identified is that at least one point should have $u > 0.4348$ [10, 15, 18]. Therefore, the peak identification depends on the aquifer hydraulic parameters, pumping rate and distance of observation well from

pumping well [2].

Cimen (2008) proposed a slope-matching method in which the aquifer parameters are calculated for $u \leq 0.01$ via the trial and error procedure [16]. It is necessary to continue the iterative computation until the relative error between the successive parameter estimations becomes less than 5% [16]. The parameters are accurately estimated only when the time interval measurements are very small during the pumping period [2]. Cimen (2009) also fitted a straight line through late drawdowns ($u \leq 0.01$) using the Excel spreadsheet and calculated T and S by the slope and intercept of the fitted line, respectively [2].

Copty et al. (2011) calculated the aquifer parameters for each pumping time using a technique based on the ratio of drawdown to the logarithmic derivative of drawdown in which the well function values and subsequently aquifer parameters are determined [19]. Sahin (2016) used a radial basis function collection method to determine the aquifer parameters for each pumping time. The logarithmic differences between successive drawdown values of field data are identical to the same differences in the type curve when the perfect matching is achieved [20].

Avci et al. (2013) presented diagnostic curves based on the incremental area method to identify the aquifer system and parameters [17]. In their study, it was noted that the effectiveness of the proposed method depends on field data quality to ensure recognizable diagnostic plot generation and hydraulic parameter estimation and background data on the hydrogeology to eliminate non-uniqueness of the aquifer system identification. These restrictions are also applicable to the curve matching procedures as well as logarithmic time derivative-based diagnostic plot methodology [17]. Finally, different optimization techniques such as genetic algorithm, extended Karman filter-based and artificial neural network are applied by researchers to estimate aquifer parameters [21, 22, 23, 24, 25, 26].

The aim of this paper is to present a simple method based on the time derivative of drawdown to estimate the aquifer parameters without any restricted applicability during the pumping test and overcome to the difficulties and subjectivities involved in available published methods.

2. Methodology

Theis (1935) proposed an equation for the drawdown \hat{s} at an observation well due to the constant pumping rate of a fully-penetrated tube well in a homogeneous, isotropic and infinite areal extent confined aquifer.

$$\hat{s} = \frac{Q}{4\pi T} W(u) \quad (1)$$

$$W(u) = -0.577 - \ln(u) + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} \dots \quad (2)$$

$$u = \frac{r^2 S}{4Tt} \quad (3)$$

where in Eqs. (1), (2), and (3), \hat{s} is observed drawdown at observation well (m), Q the constant pumping rate (m^3/s), r the distance of the observation well from pumping well (m), t the time since pumping starts, T the aquifer transmissivity, S the storage coefficient of the aquifer and $W(u)$ is Theis well function.

The time derivative of drawdown is as follows:

$$\frac{\partial s}{\partial t} = \frac{Q}{4\pi T t} \left(1 - u + \frac{u^2}{2!} - \frac{u^3}{3!} + \frac{u^4}{4!} \dots \right) = \frac{Q}{4\pi T t} e^{-u} \quad (4)$$

Let's $\varphi = \frac{\partial s}{\partial t}$, $A = \frac{Q}{4\pi T}$ and $u = \frac{B}{t}$ where $B = \frac{r^2 S}{4T}$, Eq. (4) becomes:

$$\varphi = \frac{A}{t} e^{-\frac{B}{t}} \quad (5)$$

The parameter φ in Eq. (5) relates non-linearly with pumping time t where A and B are constant values. The least squares optimization

approach can be applied to find the constant values of A and B in Eq. (5) in which the objective function (F) is minimizing the predicted and observed temporal derivative of drawdowns (Eq. (6)).

$$\text{minimize } F = \sum_{i=1}^n \left(\varphi_i - \frac{A}{t_i} e^{-\frac{B}{t_i}} \right)^2 \quad (6)$$

where n is the total number of time derivative of drawdowns (φ).

The derivative of objective function (F) with respect to the constant values of A and B is zero $\left(\frac{\partial F}{\partial A} = 0 \text{ and } \frac{\partial F}{\partial B} = 0 \right)$. Therefore, the values of A and B are calculated using Eqs. (7) and (8), respectively.

$$A = \exp \left[\frac{\sum \left(\frac{1}{t} \right)^2 (\sum \ln \varphi + \sum \ln t) - \sum \frac{1}{t} \left(\sum \frac{\ln \varphi}{t} + \sum \frac{\ln t}{t} \right)}{n \sum \left(\frac{1}{t} \right)^2 - \left(\sum \frac{1}{t} \right)^2} \right] \quad (7)$$

$$B = \frac{n \ln A - \sum \ln \varphi - \sum \ln t}{\sum \frac{1}{t}} \quad (8)$$

After the constant values of A and B are calculated, the aquifer parameters T and S are estimated using Eqs. (9) and (10), respectively.

$$T = \frac{Q}{4\pi A} \quad (9)$$

$$S = \frac{BQ}{\pi A r^2} \quad (10)$$

The parameters φ and t at each pumping test must be calculated from measured drawdown and pumping time data. For two consecutive time records t_i and t_{i+1} , the derivative time, \bar{t}_i , is calculated using Eq. (11):

$$\bar{t}_i = \frac{t_i + t_{i+1}}{2} \quad i = 1, 2, 3, \dots, N_t - 1 \quad (11)$$

where N_t is the number of time records.

For each drawdown \hat{s}_i , the parameter φ_i can be calculated using Eq. (12):

$$\varphi_i = \frac{s_{i+1} - s_i}{t_{i+1} - t_i} \quad i = 1, 2, 3, \dots, N_d - 1 \quad (12)$$

in which the N_d is the number of drawdown records.

In this paper, the proposed derivative calculations are developed based on the analytical approach. Therefore, the time interval of measured drawdowns has a marginal effect on the parameters estimation.

Estimating the aquifer parameters, T and S , using the proposed method requires the following procedure:

- 1 Calculate the parameter \bar{t}_i using Eq. (11) for each pumping time record.
- 2 Calculate the parameter φ_i using Eq. (12) for each drawdown record.
- 3 Calculate the constant values of A and B using Eqs. (7) and (8), respectively.
- 4 Calculate the aquifer parameters, T and S , using Eqs. (9) and (10), respectively.

3. Results

Three pumping test data sets are used in this study to validate the applicability and accuracy of the proposed method. Set 1 is synthetic data in which the drawdown is theoretically calculated at an observation well near a tube well with constant pumping rate inside a confined aquifer. Set 2 is the published data from page 166 of Todd and Larry [27]. This

published data has been frequently used in the literature to compare the results of different methods in estimating aquifer parameters. Set 3 is field data due to measuring the drawdown at an observation well within a confined aquifer.

3.1. Set 1: synthetic data

A homogenous and isotropic confined aquifer is assumed to be sandwiched between two impermeable clay layers (Fig. 1). The aquifer thickness and initial piezometric head over the aquifer are 25 and 50 m, respectively. The aquifer storage coefficient is selected as 0.05 and the hydraulic conductivity (K) of the aquifer as 100 m/d (0.069444 m/min), leading to the transmissivity of 2500 m²/d (1.7361 m²/min). A rectangular area with dimensions of 4 km is selected to satisfy the condition of infinite areal extent for the aquifer. The fully-penetrated pumping tube well is situated in the center which pumped by a constant rate of 14400 m³/d (10 m³/min). An observation well is located at a distance of 50 m from the pumping well (Fig. 1). The groundwater flow is numerically simulated using MODFLOW based on the finite-difference method in which the horizontal flow system (areal extent) is divided into 5-m dimensional grid cells (Fig. 1). The stress period for groundwater simulation is set to be 0.5 min.

The measured drawdowns may contain a noise during real pumping tests due to measurement errors, heterogeneity or discharge variations [20, 28]. The normally distributed observational errors (noise) up to 5% are randomly added to the theoretical drawdowns to mimic the natural conditions in synthetic pumping test data (Table 1) [29]. The derivative calculations for aquifer parameters estimation are also presented in

Table 1. The calculated parameters are $\sum \left(\frac{1}{t} \right) = 0.3256$, $\sum \left(\frac{1}{t} \right)^2 = 0.025$, $\sum \ln t = 79.82$, $\sum \frac{\ln t}{t} = 0.9687$, $\sum \ln \varphi = -100.34$, and $\sum \frac{\ln \varphi}{t} = -1.682$. Therefore, the constant values of A and B are 0.385 and 16.128, respectively. Accordingly, the aquifer parameters of T and S are 2977 m²/d (2.067 m²/min) and 0.053, respectively.

The result of the proposed method is compared to ten published methods. The root-mean-square error (rms error) of drawdown (Eq. (13)) is used to compare the estimation accuracy of the proposed method with available methods [30].

$$\text{rms error} = \sqrt{\frac{\sum_{i=1}^N (S_i^* - S_i)^2}{N}} \quad (13)$$

where the s^* and s are the predicted and measured drawdowns, respectively and N is the number of measured drawdowns.

Four methods of Theis, Chow, Sen and El Khatib associated with the proposed method are applicable for the entire pumping period to estimate the aquifer parameters, but remaining methods are applicable only for specific pumping times (Table 2). However the methods of Cooper-Jacob, Khan and Cimen [2, 16] are applicable for $t > 1800$ min, the aquifer parameters are estimated using a portion of data which are graphically fitted on a straight line. The Singh's methods [9, 15] are applicable for $t < 1800$ min. The aquifer parameter estimation using these methods which are applicable for specific pumping times is time-consuming and tedious because a trial and error is required to identify the fraction of pumping data which meet the applicability of the method, i.e., specified condition for argument u .

The methods of Theis and Sen have minimum (0.028) and maximum (0.335) rms errors, respectively (Table 2). The rms error of the proposed method is 0.122. The minimum and maximum T values are estimated to be 1989 and 4424 m²/d using methods of the Singh [15] and Sen, respectively (Table 2). The estimated hydraulic conductivity (K) also varies from 79.56 m/d in Singh's [15] method to 176.96 m/d in Sen's method (Table 2). The proposed method estimates the K value as 119.08 m/d. The S value is estimated by 0.053 using the proposed method within a range of estimation by 0.037–0.071 using other methods

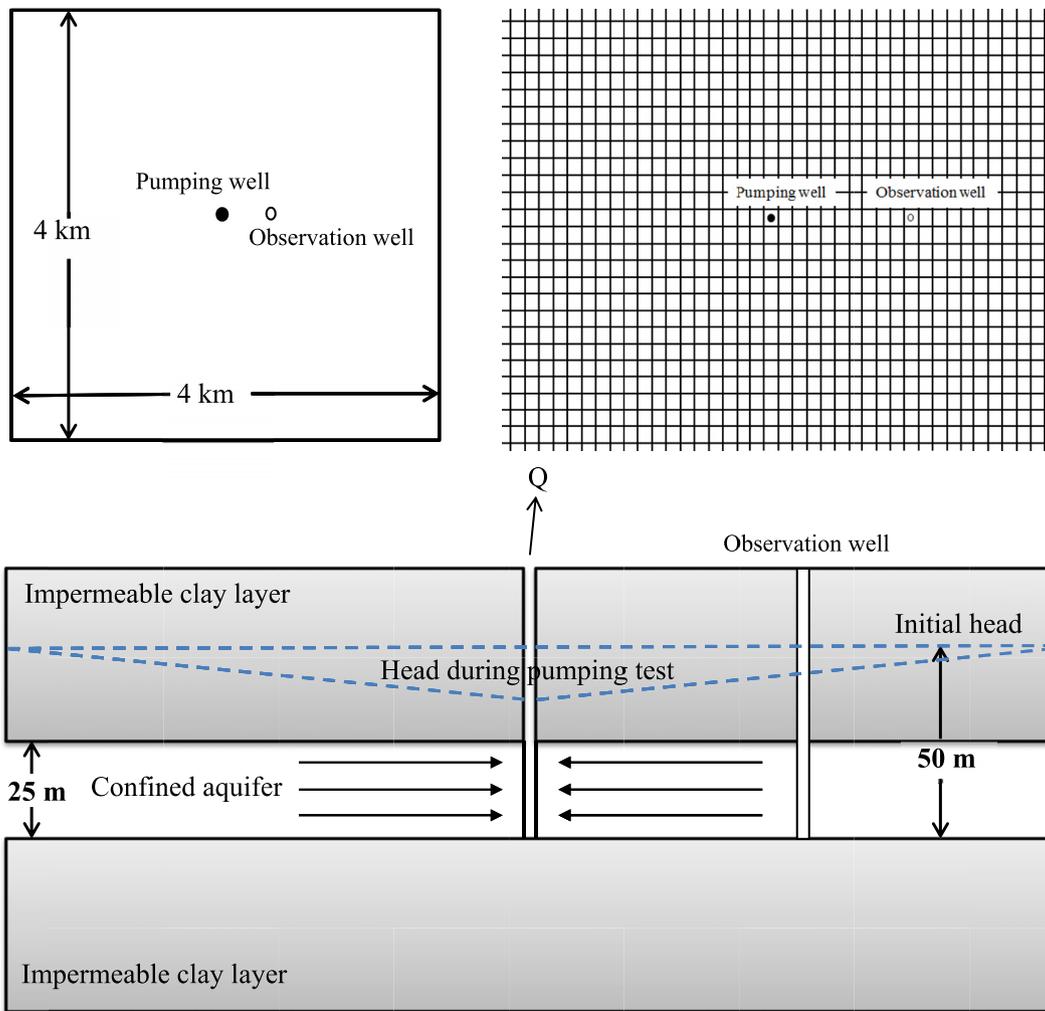


Fig. 1. The schematic diagram of the confined aquifer and locations of the pumping and observation wells in groundwater numerical modeling induced by the pumping test.

Table 1
The synthetic pumping test data (Set 1) and related derivative calculations.

i	$t(\text{min})$	$s(\text{m})$	\bar{t}_i	$\frac{1}{\bar{t}_i}$	$\left(\frac{1}{\bar{t}_i}\right)^2$	$\text{Ln } \bar{t}_i$	$\frac{\text{Ln } \bar{t}_i}{\bar{t}_i}$	φ_i	$\text{Ln } \varphi_i$	$\frac{\text{Ln } \varphi_i}{\bar{t}_i}$
1	5	0.0027	7.5	0.1333333	0.0177778	2.014903	0.2686537	0.00556	-5.1921572	-0.6922876
2	10	0.0305	15	0.0666667	0.0044444	2.7080502	0.1805367	0.0084	-4.7795236	-0.3186349
3	20	0.1145	25	0.04	0.0016	3.2188758	0.128755	0.00917	-4.691818	-0.1876727
4	30	0.2062	40	0.025	0.000625	3.6888795	0.092222	0.008335	-4.7872918	-0.1196823
5	50	0.3729	60	0.0166667	0.0002778	4.0943446	0.0682391	0.004095	-5.4979886	-0.0916331
6	70	0.4548	85	0.0117647	0.0001384	4.4426513	0.0522665	0.00566	-5.1743314	-0.0608745
7	100	0.6246	125	0.008	0.000064	4.8283137	0.0386265	0.00288	-5.849965	-0.0467997
8	150	0.7686	175	0.0057143	3.265E-05	5.164786	0.0295131	0.001874	-6.2796801	-0.0358839
9	200	0.8623	225	0.0044444	1.975E-05	5.4161004	0.0240716	0.001656	-6.4033502	-0.0284593
10	250	0.9451	275	0.0036364	1.322E-05	5.6167711	0.0204246	0.002782	-5.8845852	-0.0213985
11	300	1.0842	350	0.0028571	8.163E-06	5.8579332	0.016737	0.000347	-7.9661858	-0.0227605
12	400	1.1189	450	0.0022222	4.938E-06	6.1092476	0.0135761	0.001955	-6.2373651	-0.0138608
13	500	1.3144	550	0.0018182	3.306E-06	6.3099183	0.0114726	0.000155	-8.7720854	-0.0159492
14	600	1.3299	650	0.0015385	2.367E-06	6.4769724	0.0099646	0.000388	-7.8545052	-0.0120839
15	700	1.3687	850	0.0011765	1.384E-06	6.7452363	0.0079356	0.000671	-7.3067414	-0.0085962
16	1000	1.57	1250	0.0008	6.4E-07	7.1308988	0.0057047	0.0004688	-7.6653343	-0.0061323
17	1500	1.8044	-	-	-	-	-	-	-	-
Σ	-	-	-	0.3256389	0.0250138	79.823882	0.9686992	-	-100.34291	-1.6827095

(Table 2). The Theis's method is considered as a benchmark solution and estimated aquifer parameters using other methods are compared to those values in Theis's solution. The percentage errors of T, S and K parameters are shown in Table 2. The T error varies from 5.64% in Chow's method to

75.76% in Sen's method. The T error is 18.28% in the proposed method. The range of the K error is also similar to the range of T error. The S error also varies from zero (Chow's method) to 42% (Singh's method) in which the S error is 6% in the proposed method. The results of the synthetic data

Table 2

The estimated aquifer parameters and relevant errors using proposed method and different published methods in synthetic data set (Set 1).

Method	Applicability	T (m ² /d)	T Error (%)	K (m/d)	K Error (%)	S	S Error (%)	rms error
Proposed method	All data	2977	18.28	119.08	18.28	0.053	6.00	0.122
Theis, 1935	All data	2517	0.00	100.68	0.00	0.05	0.00	0.028
Cooper-Jacob, 1946	$t > 1800$ min	2843	12.95	113.72	12.95	0.037	26.00	0.061
Chow, 1952	All data	2659	5.64	106.36	5.64	0.05	0.00	0.044
Khan, 1982	$t > 1800$ min	2843	12.95	113.72	12.95	0.037	26.00	0.061
Sen, 1986	All data	4424	75.76	176.96	75.76	0.056	12.00	0.335
El Khatib, 1987	All data	2997	19.07	119.88	19.07	0.055	10.00	0.136
Singh, 2000	$t < 1800$ min	1989	20.98	79.56	20.98	0.048	4.00	0.181
Singh, 2001	$t < 1800$ min	3423	36.00	136.92	36.00	0.071	42.00	0.273
Cimen, 2008	$t > 1800$ min	2956	17.44	118.24	17.44	0.049	2.00	0.097
Cimen, 2009	$t > 1800$ min	2843	12.95	113.72	12.95	0.037	26.00	0.061

set show that the proposed method can be used to estimate the aquifer parameters reliably without time-restricted applicability and requiring special charts.

3.2. Set 2: published data

This data set was published on page 166 of Todd and Larry [27]. A fully-penetrated tube well in a confined aquifer is discharged with a constant rate of 2500 m³/d (1.736 m³/min). The drawdowns are measured during the pumping period of 240 min at an observation well located 60 m away from the pumping well (Table 3) [27]. The calculated parameters are $\sum \left(\frac{1}{t}\right) = 3.565$, $\sum \left(\frac{1}{t}\right)^2 = 1.517$, $\sum \ln t = 70.8$, $\sum \frac{\ln t}{t} = 4.137$, $\sum \ln \varphi = -113.55$ and $\sum \frac{\ln \varphi}{t} = -10.717$ for this pumping test (Table 3).

The constants *A* and *B* are 0.1744 and 0.2336, respectively, leading to the estimated *T* and *S* values by 1141 m²/d and 0.00020, respectively (Table 4). The aquifer parameters are also calculated using ten published methods (Table 4). The Singh's method [15] is not applicable to estimate the aquifer parameters in this data set because the peak of the fitted curve through early drawdowns ($t < 23$ min) cannot be determined. A necessary condition for a peak to be identified is that at least one point should

have $u > 0.4348$ [10, 15, 18]. Though the argument *u* in early draw-downs ($t < 23$ min) is greater than 0.01, the maximum value is still less than 0.4348.

The rms error is 0.017, 0.05, 0.017 and 0.053 in methods of Chow, Sen, EL Khatib and Singh [9], respectively, and is less than 0.01 in proposed method and other methods. The lowest (0.00018) and highest (0.00022) *S* values are estimated using Khan's [6] and Singh's [9] methods, respectively. Different methods estimate a variety of *T* values. The minimum (1138 m²/d) and maximum (1233 m²/d) values are estimated using Theis's [4] and Sen's [13] methods, respectively. The methods of Cooper-Jacob, El Khatib and the present study (proposed method) estimate the *T* value as about 1141 m²/d but the other methods estimate it with some departure from this value. The hydraulic conductivity (*K*) of the aquifer is not estimated in this data set due to unknown aquifer thickness in the problem. The percentage errors of *T* and *S* parameters are shown in Table 4. The *T* error varies from 0.26% in proposed method to 8.35% in Sen's method. The proposed method has the lowest *T* error in this data set. The *S* error also varies from zero in Sen's, Chow's and proposed methods to 10% in Khan's and Singh's methods. The errors comparison between proposed and ten published methods shows that the proposed method reliably estimates the aquifer parameters.

Table 3

The published pumping test data (Set 2) from page 166 of Todd and Larry [27] and related derivative calculations.

<i>i</i>	<i>t</i> (min)	<i>s</i> (m)	\bar{t}_i	$\frac{1}{\bar{t}_i}$	$\left(\frac{1}{\bar{t}_i}\right)^2$	$\ln \bar{t}_i$	$\frac{\ln \bar{t}_i}{\bar{t}_i}$	φ_i	$\ln \varphi_i$	$\frac{\ln \varphi_i}{\bar{t}_i}$
1	1	0.2	1.25	0.8	0.64	0.2231436	0.1785148	0.14	-1.9661129	-1.5728903
2	1.5	0.27	1.75	0.5714286	0.3265306	0.5596158	0.3197805	0.06	-2.8134107	-1.6076633
3	2	0.3	2.25	0.4444444	0.1975309	0.8109302	0.3604134	0.08	-2.5257286	-1.1225461
4	2.5	0.34	2.75	0.3636364	0.1322314	1.0116009	0.3678549	0.06	-2.8134107	-1.0230584
5	3	0.37	3.5	0.2857143	0.0816327	1.252763	0.3579323	0.04	-3.2188758	-0.9196788
6	4	0.41	4.5	0.2222222	0.0493827	1.5040774	0.3342394	0.04	-3.2188758	-0.7153057
7	5	0.45	5.5	0.1818182	0.0330579	1.7047481	0.3099542	0.03	-3.5065579	-0.637556
8	6	0.48	7	0.1428571	0.0204082	1.9459101	0.2779872	0.025	-3.6888795	-0.5269828
9	8	0.53	9	0.1111111	0.0123457	2.1972246	0.2441361	0.02	-3.912023	-0.4346692
10	10	0.57	11	0.0909091	0.0082645	2.3978953	0.2179905	0.015	-4.1997051	-0.3817914
11	12	0.6	13	0.0769231	0.0059172	2.5649494	0.1973038	0.015	-4.1997051	-0.3230542
12	14	0.63	16	0.0625	0.0039063	2.7725887	0.1732868	0.01	-4.6051702	-0.2878231
13	18	0.67	21	0.047619	0.0022676	3.0445224	0.1449773	0.0083333	-4.7874917	-0.2279758
14	24	0.72	27	0.037037	0.0013717	3.2958369	0.122068	0.0066667	-5.0106353	-0.1855791
15	30	0.76	35	0.0285714	0.0008163	3.5553481	0.1015814	0.005	-5.2983174	-0.1513805
16	40	0.81	45	0.0222222	0.0004938	3.8066625	0.0845925	0.004	-5.5214609	-0.1226991
17	50	0.85	55	0.0181818	0.0003306	4.0073332	0.0728606	0.005	-5.2983174	-0.096333
18	60	0.9	70	0.0142857	0.0002041	4.2484952	0.0606928	0.0015	-6.5022902	-0.0928899
19	80	0.93	90	0.0111111	0.0001235	4.4998097	0.0499979	0.0015	-6.5022902	-0.0722477
20	100	0.96	110	0.0090909	8.264E-05	4.7004804	0.0427316	0.002	-6.2146081	-0.0564964
21	120	1	135	0.0074074	5.487E-05	4.9052748	0.0363354	0.0013333	-6.6200732	-0.0490376
22	150	1.04	165	0.0060606	3.673E-05	5.1059455	0.0309451	0.001	-6.9077553	-0.0418652
23	180	1.07	195	0.0051282	2.63E-05	5.2729996	0.027041	0.001	-6.9077553	-0.0354244
24	210	1.1	225	0.0044444	1.975E-05	5.4161004	0.0240716	0.0006667	-7.3132204	-0.0325032
25	240	1.12	-	-	-	-	-	-	-	-
Σ	-	-	-	3.5647244	1.5170357	70.804256	4.1372889	-	-113.55267	-10.717451

Table 4

The comparison between estimated aquifer parameters and relevant errors using proposed method and different methods in published data set (Set 2).

Method	Applicability	T (m ² /d)	T Error (%)	K (m/d)	K Error (%)	S	S Error (%)	rms error
Proposed method	All data	1141	0.26	-	-	0.00020	0	0.009
Theis, 1935	All data	1138	0.00	-	-	0.00020	0	0.007
Cooper-Jacob, 1946	$t > 23$ min	1144	0.53	-	-	0.00019	5	0.005
Chow, 1952	All data	1160	1.93	-	-	0.00020	0	0.017
Khan, 1982	$t > 23$ min	1150	1.05	-	-	0.00018	10	0.009
Sen, 1986	All data	1233	8.35	-	-	0.00020	0	0.05
El Khatib, 1987	All data	1144	0.53	-	-	0.00021	5	0.017
Singh, 2000	$t < 23$ min	-	-	-	-	-	-	-
Singh, 2001	$t < 23$ min	1206	5.98	-	-	0.00022	10	0.053
Cimen, 2008	$t > 23$ min	1148	0.88	-	-	0.00019	5	0.006
Cimen, 2009	$t > 23$ min	1148	0.88	-	-	0.00019	5	0.006

3.3. Set 3: field data

The alluvial Shiraz's aquifer is located in Fars providence, southern Iran (Fig. 2). It consists of Quaternary coarse alluviums (sand and gravel) mainly surrounded by Oligo-Miocene carbonates (karstic Asmari and

Jahrum Formations), Miocene marls (Razak Formation) and Plio-Pleistocene conglomerates (Bakhtyari Formation). The groundwater flow direction in Shiraz's aquifer is generally NW-SE. This aquifer is unconfined in northern and central regions but convert to the confined type in south and southeastern regions due to the presence of some

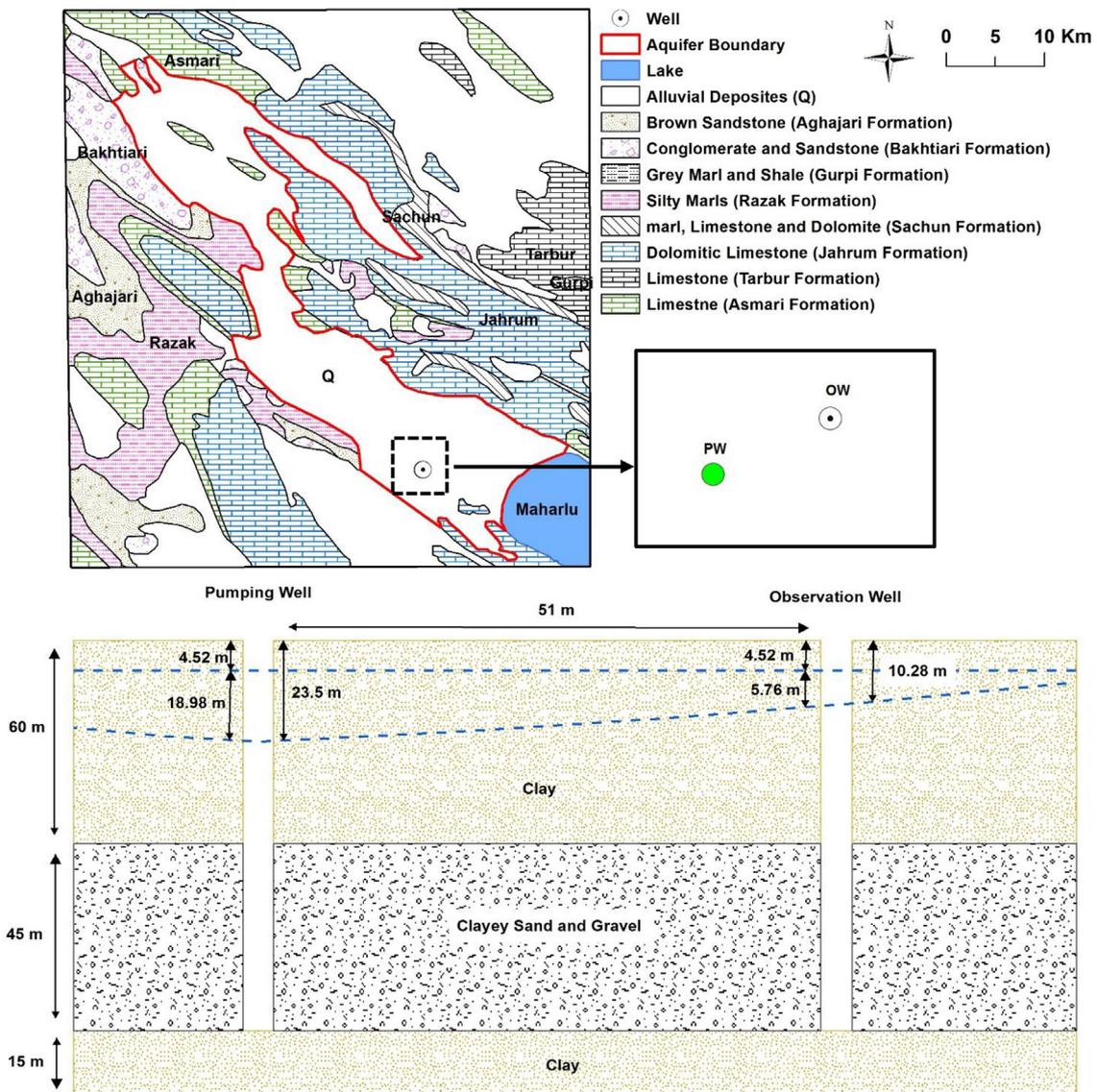


Fig. 2. The geological map of the alluvial Shiraz's aquifer, locations of the pumping (PW) and observation (OW) wells, aquifer lithology and water table depth during the pumping test in field data set.

interbedded clay layers [31]. The thickness of the alluvial aquifer varies from 10 to 500 m over the study area, resulting in a spatially-averaged thickness of 115 m [32]. A pumping test is performed at the south-eastern of the Shiraz's aquifer, Vazirabad region, where the aquifer is confined (Fig. 2). The aquifer consists of 45-m thick clayey sands and gravels in this region, underlying by a 60-m thick clay layer. The fully-penetrated pumping well and piezometer are drilled up to 120 m and screened for the portion of the aquifer. The water table depth is about 4.52 m around the pumping well before pump turns on and increases up to 23.5 and 10.28 m in pumping well and piezometer, respectively, at the end of pumping test, leading to corresponding drawdown of 18.98 and 5.76 m, respectively [33].

The pumping test with a constant discharge rate of 3888 m³/d (2.7 m³/min) is performed during in 270 min [33]. The drawdown is measured at an observation well located at 51 m from the pumping well

(Fig. 2 and Table 5). The calculated parameters are $\sum \left(\frac{1}{t}\right) = 2.577$,

$$\sum \left(\frac{1}{t}\right)^2 = 0.828, \sum \ln t = 99.39, \sum \frac{\ln t}{t} = 4.028, \sum \ln \varphi = -$$

100.225, and $\sum \frac{\ln \varphi}{t} = -4.537$ for the field data (Table 5). Therefore, the constants A and B are 1.036 and 0.726, respectively, leading to estimating T and S values as 299 m²/d and 0.00023, respectively (Table 6).

The rms error is greater than 0.1 in all methods, except in Theis's [4] and Cimen's [16] methods in which the rms error is 0.047 and 0.091, respectively. The rms error varies from 0.047 in Theis's [4] to 0.82 in Singh's [15] method. The proposed method has an rms error of 0.179. The S value is estimated in a wide range using different methods. The Singh's method [15] estimates minimum value (0.00016) and methods of El Khatib [14] and Singh [9] estimate maximum value (0.00025). The T value is also estimated in a wide range, varying from 246 m²/d in Singh's [15] method to 333 m²/d in Singh's [9] method. The estimated hydraulic conductivity (K) also varies from 5.466 m/d in Singh's [15] method to 7.4 m/d in Singh's [9] method (Table 6). The T error varies from 1.97% in proposed method to 19.34% in Singh's [15] method. The range of the K error is also similar to the range of T error. The S error also varies from 5.56% in Sen's method to 38.89% in El Khatib's and Singh's [9] methods. The S error is 27.78% in the proposed method.

4. Discussion

The estimated aquifer parameters, i.e., T and S , represent an average value over the cone of depression. Therefore, the estimated values in a homogenous and isotopic aquifer are independent of size and location of the cone as it evolves. The aquifer is not homogenous and isotropic in natural conditions and estimated parameters represent an average value over the cone of depression and influenced by the location, size and degree of heterogeneity as the cone of depression evolves [3, 34, 35]. The studies indicate that the T and S parameters in heterogeneous aquifers vary with time at early pumping times, but stabilize at late pumping times [3, 34, 35]. This may an advantage for the methods that use the late-drawdowns of the pumping test for parameter estimation, however, they omit representative characteristics of the depression cone at early pumping times. The drawdown data at late pumping times may suffer from boundary effect [9]. Though, the methods that use early drawdown data of the pumping test may be not affected by a hydrogeological boundary, but may be affected by gravel pack, well screen and storage, and the drawdown may not follow the Theis solution [9]. In the lack of the aforementioned issues, the estimated aquifer parameters still show the representative characteristics of the depression cone only near the pumping well due to early-drawdown data analysis. In summary, the estimated aquifer parameters using either early or late drawdown data may suffer from different drawbacks and omit the representative characteristics of a portion of the depression cone. Therefore, the aquifer parameters that are estimated using all drawdown data seems to be more reliable in natural heterogeneous conditions due to incorporation of all

representative characteristic of the depression cone.

The rms error and percentage errors of T and S in Tables 2, 4, and 6 indicate that these errors are generally less in those methods that use all drawdown data than those use late or early drawdown data for aquifer parameter estimation. However, the rms error and percentage errors of T and S among the methods that use all drawdowns also differ due to the method of solution. The errors comparison between methods that use early drawdowns with those use late drawdowns indicate that the errors are lower in the late drawdown-based methods. Therefore, the methods use early drawdowns for aquifer parameter estimation may be not trustable for reliable results.

The rms errors and percentage errors of the aquifer parameters (T and S) in published data set (Set 2) is very low for all methods, comparing to the other two data sets. This is probably due to the lower degree of heterogeneity of the aquifer in data set 2 and higher degree of heterogeneity in other data sets. The synthetic real drawdown data (drawdown without noise) in data set 1 is used to estimate the aquifer parameters using proposed method and other published methods to assess the effect of heterogeneity of the aquifer on parameter estimation. The synthetic drawdown data without noise shows the real homogenous and isotropic aquifer. The aquifer parameters (T and S) are estimated using all methods and results indicate that the estimated values are identical in all methods with an error of estimation less than 0.5%. The 0.5% error is due to the rounding numbers in numerical calculations and also requiring exact values for the defined functions ($w(u)$, $F(u)$, u) in some methods. When an aquifer is homogenous and isotropic, the aquifer parameters are independent of the size and location of the depression cone and subsequently pumping time. Therefore, all methods, independent of using either early, late or all drawdowns, estimate the same values for aquifer parameters.

5. Conclusions

The pumping tests are commonly used to estimate the transmissivity and storage coefficient of the aquifers which are principal parameters in aquifer modeling and management. In this study, a simple derivative-based method is presented based on the least squares optimization technique to estimate the confined aquifer parameters. The presented method has the following advantages:

- i The aquifer parameters can be estimated via numerical calculations performed in a table and further using the developed equations. It does not need any especial instrument or computer for parameter estimation because the numerical calculations can be simply performed in a calculator, making able the on-site parameter estimation.
- ii The method is applicable for the entire drawdown data during the pumping period and not restricted to specific pumping times. Consequently, it does not require to check the condition for argument u .
- iii It does not require curve matching, construction of a graph, initial guess of the parameters and values of the $w(u)$ and u .
- iv The rms error of estimation is acceptable for the proposed method comparing to the available methods.
- v The time interval of measurements has a marginal effect on the parameter estimation due to the analytical basis of the derivative calculations.

Declaration

Author contribution statement

M. Naderi: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

Table 5
The field pumping test data (Set 3) and related derivative calculations.

<i>i</i>	<i>t</i> (min)	<i>s</i> (m)	\bar{r}_i	$\frac{1}{\bar{r}_i}$	$\left(\frac{1}{\bar{r}_i}\right)^2$	$Ln \bar{r}_i$	$\frac{Ln \bar{r}_i}{\bar{r}_i}$	φ_i	$Ln \varphi_i$	$\frac{Ln \varphi_i}{\bar{r}_i}$
1	1	0.58	1.5	0.6666667	0.4444444	0.4054651	0.2703101	0.51	-0.6733446	-0.4488964
2	2	1.09	2.5	0.4	0.16	0.9162907	0.3665163	0.27	-1.3093333	-0.5237333
3	3	1.36	3.5	0.2857143	0.0816327	1.252763	0.3579323	0.22	-1.5141277	-0.4326079
4	4	1.58	4.5	0.2222222	0.0493827	1.5040774	0.3342394	0.2	-1.6094379	-0.3576529
5	5	1.78	5.5	0.1818182	0.0330579	1.7047481	0.3099542	0.17	-1.7719568	-0.322174
6	6	1.95	7	0.1428571	0.0204082	1.9459101	0.2779872	0.095	-2.3538784	-0.3362683
7	8	2.14	9	0.1111111	0.0123457	2.1972246	0.2441361	0.12	-2.1202635	-0.2355848
8	10	2.38	11	0.0909091	0.0082645	2.3978953	0.2179905	0.08	-2.5257286	-0.2296117
9	12	2.54	13.5	0.0740741	0.005487	2.6026897	0.1927918	0.0633333	-2.7593435	-0.2043958
10	15	2.73	16.5	0.0606061	0.0036731	2.8033604	0.1699006	0.0633333	-2.7593435	-0.1672329
11	18	2.92	20	0.05	0.0025	2.9957323	0.1497866	0.055	-2.9004221	-0.1450211
12	22	3.14	24	0.0416667	0.0017361	3.1780538	0.1324189	0.04	-3.2188758	-0.1341198
13	26	3.3	28	0.0357143	0.0012755	3.3322045	0.1190073	0.0325	-3.4265152	-0.1223755
14	30	3.43	32.5	0.0307692	0.0009467	3.4812401	0.1071151	0.032	-3.4420194	-0.1059083
15	35	3.59	37.5	0.0266667	0.0007111	3.6243409	0.0966491	0.026	-3.6496587	-0.0973242
16	40	3.72	42.5	0.0235294	0.0005536	3.7495041	0.0882236	0.026	-3.6496587	-0.0858743
17	45	3.85	47.5	0.0210526	0.0004432	3.8607297	0.0812785	0.02	-3.912023	-0.0823584
18	50	3.95	55	0.0181818	0.0003306	4.0073332	0.0728606	0.018	-4.0173835	-0.0730433
19	60	4.13	65	0.0153846	0.0002367	4.1743873	0.0642213	0.017	-4.0745419	-0.0626853
20	70	4.3	75	0.0133333	0.0001778	4.3174881	0.0575665	0.014	-4.2686979	-0.056916
21	80	4.44	85	0.0117647	0.0001384	4.4426513	0.0522665	0.011	-4.50986	-0.0530572
22	90	4.55	95	0.0105263	0.0001108	4.5538769	0.0479355	0.011	-4.50986	-0.0474722
23	100	4.66	110	0.0090909	8.26E-05	4.7004804	0.0427316	0.01	-4.6051702	-0.0418652
24	120	4.86	130	0.0076923	5.92E-05	4.8675345	0.0374426	0.009	-4.7105307	-0.0362349
25	140	5.04	150	0.0066667	4.44E-05	5.0106353	0.0334042	0.007	-4.9618451	-0.033079
26	160	5.18	170	0.0058824	3.46E-05	5.1357984	0.0302106	0.006	-5.1159958	-0.0300941
27	180	5.3	195	0.0051282	2.63E-05	5.2729996	0.027041	0.006	-5.1159958	-0.0262359
28	210	5.48	225	0.0044444	1.98E-05	5.4161004	0.0240716	0.0043333	-5.4414182	-0.0241841
29	240	5.61	255	0.0039216	1.54E-05	5.5412635	0.0217304	0.005	-5.2983174	-0.0207777
30	270	5.76	-	-	-	-	-	-	-	-
Σ	-	-	-	2.577395	0.8281389	99.392779	4.0277201	-	-100.22555	-4.5367845

Table 6
The estimated aquifer parameters and relevant errors using proposed method and different published methods in field data set (Set 3).

Method	Applicability	T (m ² /d)	T Error (%)	K (m/d)	K Error (%)	S	S Error (%)	rms error
Proposed method	All data	299	1.97	6.644	1.97	0.00023	27.78	0.179
Theis, 1935	All data	305	0.00	6.777	0.00	0.00018	0.00	0.047
Cooper-Jacob, 1946	<i>t</i> > 56 min	287	5.90	6.377	5.90	0.00023	27.78	0.111
Chow, 1952	All data	289	5.25	6.422	5.25	0.0002	11.11	0.106
Khan, 1982	<i>t</i> > 56 min	287	5.90	6.377	5.90	0.00023	27.78	0.111
Sen, 1986	All data	323	5.90	7.177	5.90	0.00019	5.56	0.199
El Khatib, 1987	All data	298	2.30	6.622	2.30	0.00025	38.89	0.251
Singh, 2000	<i>t</i> < 56 min	246	19.34	5.466	19.34	0.00016	11.11	0.82
Singh, 2001	<i>t</i> < 56 min	333	9.18	7.400	9.18	0.00025	38.89	0.513
Cimen, 2008	<i>t</i> > 56 min	289	5.25	6.422	5.25	0.00022	22.22	0.091
Cimen, 2009	<i>t</i> > 56min	287	5.90	6.377	5.90	0.00023	27.78	0.111

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

Acknowledgements

The author is thankful for the support of Zanjan Institute for Advanced Studies in Basic Sciences (IASBS), Iran. The help of Fars Regional Water Authority in providing the field pumping test data is

appreciated.

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