



## Research article

# Comparison of two-lifetime models of solid-state lighting based on sup-entropy

Omar A. Kittaneh<sup>a</sup>, M.A. Majid<sup>b,\*</sup><sup>a</sup> Natural Sciences, Mathematics and Technology Unit, Effat University, Jeddah, 21478, Saudi Arabia<sup>b</sup> Electrical and Computer Engineering Department, Effat University, Jeddah, 21478, Saudi Arabia

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## ABSTRACT

On the basis of the efficiency function introduced by Kittaneh and Beltagy [18], we compare the performance of censored samples from lognormal and Weibull distributions as two possible fitting models of solid-state lighting (SSL) luminaire lifetime. The validity of the efficiency function is demonstrated through several correlations with the accuracy in estimating the mean lifetime to fail.

## 1. Introduction

There is a significant revolution undergoing in the field of lighting. We are transitioning from incandescent light, which we have so grown used to, to solid-state lighting (SSL), and energy is one of the grand challenges facing us in this era. Lighting accounts for 19% of the worldwide electricity consumption, and one possible way to reduce electricity consumption is to adopt SSL technology [1]. Solid-state lighting, with LEDs as an active media, has been used in a variety of applications such as automotive headlights, residential lighting, industrial lighting, displays, etc. [2, 3, 4]. As compared to conventional lighting, the SSL has an advantage in luminous efficacy, energy-saving, physical robustness, and life. Residential and industrial utilization requires LEDs to survive in extreme operating conditions. SSL lighting luminaires may have a much longer lifetime, but it is not practical to gather data for an extended period. Thus, it is essential to develop some rapid reliability assessment techniques to predict the luminaire failure data accurately. The traditional reliability assessment techniques like accelerated lifetime test (ALT), highly accelerated lifetime test (HALT), and accelerated degradation test (ADT), are always time and cost consuming during operation [5]. Furthermore, assessors randomly presume the termination time of the experiment, which might be excessive (late termination) or useless (early termination).

The intentional termination of lifetime tests would lead to a portion of

the sample tested; this portion is called a censored sample. Therefore, information would come from censored samples instead of complete ones, which in turn provide minimal information about the distribution as compared to complete. Previously many measures (especially information measures or entropies) were applied in quantifying the amount of information in censored samples, for example, Hollander et al. [6] measured the information in censored samples from discrete distributions using Shannon entropy [7]. However, for continuous cases, Shannon entropy lacks many properties that its discrete version has. Therefore, some authors suggested the variance as an alternative measure of uncertainty [8]. On the other hand, some authors used consistent measures of information like Kullback-Leibler divergence as a measure of discrimination between two life-time models [9, 10, 11] and others tried to modify or change the primary definition of entropy. Among all, we mention the cumulative residual entropy [12], the average entropy [13] and in particular, the sup-entropy [14]. The sup-entropy has been recently used as a quantifier of the efficiency of the censored sample for several survival distributions such as exponential distribution [15, 16], Pareto [17], Weibull [18, 19]. However, none of these works applied the measure on real data nor provided a clear physical meaning of this important quantity.

This work applies sup-entropy in comparing the efficiency functions of two frequently employed fitting models of the solid-state lifetimes, Weibull and lognormal. The analysis showed the high correlation

\* Corresponding author.

E-mail address: [moabdulmajid@effatuniversity.edu.sa](mailto:moabdulmajid@effatuniversity.edu.sa) (M.A. Majid).

between the efficiency values and the accuracy of estimating the mean lifetime to fail based on the two distributions. Furthermore, it is found that Weibull distribution is preferable for late and lognormal for early or medium termination of the experiment. The paper organized as follows: Section 2 derives the sup-entropy of the complete and censored samples from the lognormal distribution. Also, it explicitly introduces the efficiency and size of the censored sample as functions of the termination time. In addition, it considers a real SSL failure data and computes the maximum likelihood estimates of the distribution parameters corresponding to the two models. In Section 3, a comparative study is conducted based on the estimations obtained in Section 2. The paper is concluded in Section 4.

**2. Theory**

The sup-entropy [14] has the following formula

$$A(X) = -E\left(\log\frac{f(X)}{\delta}\right), \text{ where } \delta = \sup_x f(x). \tag{1}$$

The sup-entropy (1) is proved to be positive, additive, and consistent. Most importantly, Kittaneh and Beltagy [18] also showed that the sup-entropy of type I censored sample converges to the sup-entropy of the complete as the termination time goes to infinity for any probability distribution. Consequently, the sup-entropy possesses the primary properties to serve as a quantifier of the amount of information in censored samples. In the following, the sup-entropy of type I right censored sample is obtained for the lognormal distribution; refer to [18] for the case of Weibull.

**Theorem 1.1.** If  $X_1, X_2, \dots, X_r$  denote the first r ordered statistics of a random sample of size  $n$  from the lognormal distribution with the parameters  $\mu$  and  $\sigma$ , and that is type I censored on the right at  $t$ , where  $t$  is above the mode of the distribution, then the sup-entropies of complete and censored samples are, respectively, given by:

$$A_{com} = \frac{n}{2}(1 + \sigma^2), \tag{2}$$

and

$$A_{cen} = \left[ A_{com} - n \frac{\varphi(\log t)}{\phi(\log t)} (\sigma + \log t) \right] \phi(z), \tag{3}$$

where  $z = \frac{\log t - \mu}{\sigma}$ ,  $\varphi$  and  $\phi$  are the probability density function (PDF) and cumulative distribution function (CDF) of the standard normal distribution, respectively.

**Proof.** Kittaneh and Beltagy [18] proved that the sup entropy of type I censored sample on the right at time  $t$  from a probability distribution with PDF  $f(x)$  is in general given by:

$$A_{cen} = -n \int_0^t G(x) dx, \tag{4}$$

when  $t > a$ , where  $a$  is the mode of the probability distribution, and

$$A_{com} = \lim_{t \rightarrow \infty} A_{cen} = -n \int_0^\infty G(x) dx, \tag{5}$$

where

**Table 1**  
The failure times of the first twelve units in the Hammer test [20].

|       |     |     |     |     |     |     |     |     |     |     |     |      |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| $r$   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12   |
| $t$   | 293 | 293 | 336 | 456 | 547 | 586 | 754 | 800 | 888 | 926 | 969 | 1176 |
| $r/n$ | 6%  | 12% | 18% | 24% | 30% | 35% | 41% | 47% | 53% | 59% | 65% | 71%  |

$$G(x) = f(x) \log[f(x) / f(a)]. \tag{6}$$

The PDF, CDF and the mode of the lognormal distribution are respectively given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}x} \exp\left[-(\log x - \mu)^2 / (2\sigma^2)\right], \tag{7}$$

$$F(x) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left[\frac{(\log x - \mu)}{\sqrt{2}\sigma}\right], \tag{8}$$

$$a = \exp[\mu - \sigma^2]. \tag{9}$$

In view of Eq. (6), by substituting (7)–(9) into (4) and letting  $y = \log x$ , (4) can be written as

$$A_{cen} = nF(t) \left[ E(Y) + \frac{1}{2\sigma^2} E(Y - \mu)^2 + \frac{1}{2}\sigma^2 - \mu \right], \tag{10}$$

such that  $Y$  is the truncated normal distribution over the interval  $(-\infty, \log t]$ .

After computing the expected values in (10) using the truncated Gaussian density, it immediately reduces to (3). Finally, by letting  $t$  go to infinity, (3) converges to (2).

The efficiency of a censoring scheme based on a given entropy measure is the ratio of the value of the entropy in the censoring scheme to its value in the complete. Therefore, the efficiency function of type I censored sample based on sup-entropy is given by

$$e(t) = \frac{A_{cen}}{A_{com}}, \tag{11}$$

Kittaneh and Beltagy [18] numerically estimated the efficiency function of the Weibull distribution, which has the PDF

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right]; x \geq 0$$

which has no explicit form. Whereas, for the lognormal case, the efficiency function is simply the ratio between Eqs. (3) and (2). In both cases (Weibull and lognormal), if the desired efficiency level is  $\varepsilon$  where  $0 < \varepsilon < 1$  then the corresponding termination time  $t$  can be numerically computed by equating  $e(t)$  in Eq. (11) with  $\varepsilon$ . However, this is not always possible as Eq. (11) contains the distribution parameters which are usually unknown, and require prior information.

If  $F(t)$  is the CDF of the desired model at time  $t$ , then the corresponding estimated censored sample size that fails on or before the time  $t$  is  $r = nF(t)$ , where  $n$  is the size of the complete sample. Accordingly, the estimated censored sample size for the two models are

$$\text{Lognormal: } r = \frac{n}{2} \left( 1 + \operatorname{erf}\left(\frac{\log t - \mu}{\sqrt{2}\sigma}\right) \right), \tag{12}$$

$$\text{Weibull: } r = n(1 - \exp(-(t/\beta)^\alpha)), \tag{13}$$

The experimental data described in the Hammer test [20] was selected to compare the two models. The test conducted on  $n = 17$  units from seven different commercial luminaire models until the failure of the twelfth luminaire ( $r = 12$ ). The remaining five units still operating at the test termination time  $t = 1470$  hours. The failure times of each unit is shown in Table 1.

The maximum likelihood estimates (MLEs) of the parameters of the two selected models were computed using the data of Table 1. The MLEs estimating equations and their derivations for both distributions (and almost for all probability distributions that frequently appear in reliability studies) are available in [21]. The MLE estimating equations of the lognormal parameters from the complete samples are

$$\hat{\mu}_{com} = \frac{1}{n} \sum_{i=1}^n \log x_i, \tag{14}$$

$$\hat{\sigma}_{com} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\log x_i - \hat{\mu}_{com})^2} \tag{15}$$

The corresponding estimating equations from censored samples with size  $r$  can be written as

$$\hat{\mu}_{cen} = \frac{1}{r} \sum_{i=1}^r \log x_i + \frac{n-r}{r} \frac{\varphi(z_{cen})}{1 - \varphi(z_{cen})} \hat{\sigma}_{cen}, \tag{16}$$

$$\hat{\sigma}_{cen} = \sqrt{\frac{1}{r} (\log t - \hat{\mu}_{cen}) \sum_{i=1}^r (\hat{\mu}_{cen} - \log x_i) + \frac{1}{r} \sum_{i=1}^r (\log x_i - \hat{\mu}_{cen})^2}, \tag{17}$$

where  $z_{cen} = \frac{\log t - \hat{\mu}_{cen}}{\hat{\sigma}_{cen}}$ .

In view of Eqs. (16) and (17), if the lognormal distribution with parameters  $\mu$  and  $\sigma$  is used as the fitting model to the Hammer data, then the maximum likelihood estimates of  $\mu$  and  $\sigma$  from the censored data are  $\hat{\mu}_{cen} = 6.7991$  and  $\hat{\sigma}_{cen} = 0.7536$ . On the other hand, the MLE estimating equations of the Weibull parameters from the complete samples are

$$\frac{\sum_{i=1}^n x_i^{\hat{\alpha}_{com}} \log x_i}{\sum_{i=1}^n x_i^{\hat{\alpha}_{com}}} - \frac{1}{\hat{\alpha}_{com}} = \frac{1}{n} \sum_{i=1}^n \log x_i, \tag{18}$$

$$\hat{\beta}_{com} = \left( \frac{1}{n} \sum_{i=1}^n x_i^{\hat{\alpha}_{com}} \right)^{1/\hat{\alpha}_{com}}. \tag{19}$$

The corresponding MLE estimating equations from the censored samples are

$$r = (n-r)(t/\hat{\beta}_{cen})^{\hat{\alpha}_{cen}} \log(t/\hat{\beta}_{cen}) - \sum_{i=1}^r \left[ 1 - (x_i/\hat{\beta}_{cen})^{\hat{\alpha}_{cen}} \right] \log(x_i/\hat{\beta}_{cen})^{\hat{\alpha}_{cen}}, \tag{20}$$

$$r = (n-r)(t/\hat{\beta}_{cen})^{\hat{\alpha}_{cen}} + \sum_{i=1}^r (x_i/\hat{\beta}_{cen})^{\hat{\alpha}_{cen}}. \tag{21}$$

Therefore, from Eqs. (20) and (21), if the Weibull distribution with parameters  $\alpha$  and  $\beta$  is used as the fitting model of the data, then the maximum likelihood estimates of  $\alpha$  and  $\beta$  from the censored data are  $\hat{\alpha}_{cen} = 1.6870$  and  $\hat{\beta}_{cen} = 1198.5455$ . For parameter estimation from more general censoring techniques, we would refer readers to Singh & Tripathi [22] and Banerjee & Kundu [23] for lognormal and Weibull distributions, respectively.

### 3. Results and discussion

In this section, an intensive simulation was performed by generating 5000 samples from both distributions of sample sizes  $n (= 20, 30, \text{ and } 50)$ . The parameters' values used in this study are the MLEs computed from the experimental Hammer test data. The purpose is to investigate a relationship between the efficiency function and the accuracy in estimating the mean lifetime to fail. Different censored samples are generated corresponding to several pre-assumed reasonable termination times  $t (= 700\text{--}3900 \text{ h with step size } 400 \text{ h})$ . The simulation process implemented as follows:

- i) Generate random samples (complete samples) of size  $n (= 20, 30, 50)$  from both distributions *Lognormal* ( $\mu = 6.7991, \sigma = 0.7536$ ) and *Weibull* ( $\alpha = 1.6870, \beta = 1198.5455$ ).

**Table 2**  
Lognormal VS. Weibull.

|     |      | Lognormal with $\mu = 6.7991$ and $\sigma = 0.7536$ |            |            |       |       | Weibull with $\alpha = 1.6870$ and $\beta = 1198.5455$ |            |            |       |       |
|-----|------|---|------------|------------|-------|-------|--|------------|------------|-------|-------|
| $n$ | $t$  | $\epsilon_L$  | $MT_{com}$ | $MT_{cen}$ | $e_L$ | $r/n$ | $\epsilon_W$   | $MT_{com}$ | $MT_{cen}$ | $e_W$ | $r/n$ |
| 20  | 700  | 37.1%   | 1195       | 1457       | 40.2% | 37.1% | 16.8%  | 1070       | 1385       | 30.9% | 33.3% |
|     | 1100 | 60.7%   | 1195       | 1263       | 15.6% | 60.8% | 19.5%  | 1072       | 1393       | 30.0% | 57.8% |
|     | 1500 | 75.2%   | 1200       | 1232       | 9.4%  | 75.2% | 32.3%  | 1068       | 1323       | 23.4% | 76.9% |
|     | 1900 | 84.0%   | 1197       | 1223       | 6.6%  | 84.0% | 52.1%  | 1067       | 1234       | 15.2% | 88.7% |
|     | 2300 | 89.4%   | 1196       | 1216       | 4.8%  | 89.4% | 71.3%  | 1070       | 1162       | 8.2%  | 95.0% |
|     | 2700 | 92.8%   | 1199       | 1211       | 3.6%  | 92.7% | 85.1%  | 1070       | 1114       | 3.9%  | 98.1% |
|     | 3100 | 95.0%   | 1198       | 1209       | 2.6%  | 95.0% | 93.3%  | 1071       | 1089       | 1.6%  | 99.3% |
|     | 3500 | 96.5%   | 1199       | 1206       | 2.0%  | 96.4% | 97.3%  | 1070       | 1076       | 0.5%  | 99.8% |
|     | 3900 | 97.4%   | 1202       | 1209       | 1.5%  | 97.4% | 99.0%  | 1066       | 1068       | 0.2%  | 99.9% |
| 30  | 700  | 37.1%   | 1189       | 1337       | 28.0% | 37.2% | 16.8%  | 1070       | 1392       | 30.7% | 33.3% |
|     | 1100 | 60.7%   | 1194       | 1234       | 12.0% | 60.6% | 19.5%  | 1072       | 1397       | 30.3% | 57.7% |
|     | 1500 | 75.2%   | 1194       | 1217       | 7.9%  | 75.2% | 32.3%  | 1069       | 1324       | 23.6% | 76.9% |
|     | 1900 | 84.0%   | 1199       | 1212       | 5.5%  | 84.0% | 52.1%  | 1069       | 1237       | 15.3% | 88.7% |
|     | 2300 | 89.4%   | 1199       | 1209       | 4.0%  | 89.3% | 71.3%  | 1071       | 1165       | 8.5%  | 95.0% |
|     | 2700 | 92.8%   | 1197       | 1207       | 3.0%  | 92.7% | 85.1%  | 1070       | 1116       | 4.1%  | 98.0% |
|     | 3100 | 95.0%   | 1196       | 1203       | 2.3%  | 95.0% | 93.3%  | 1071       | 1092       | 1.8%  | 99.3% |
|     | 3500 | 96.5%   | 1199       | 1204       | 1.7%  | 96.4% | 97.3%  | 1070       | 1078       | 0.7%  | 99.8% |
|     | 3900 | 97.4%   | 1195       | 1199       | 1.4%  | 97.4% | 99.0%  | 1071       | 1074       | 0.2%  | 99.9% |
| 50  | 700  | 37.1%   | 1192       | 1256       | 18.3% | 37.1% | 16.8%  | 1069       | 1397       | 30.8% | 33.2% |
|     | 1100 | 60.7%   | 1196       | 1217       | 9.3%  | 60.6% | 19.5%  | 1071       | 1401       | 30.8% | 57.9% |
|     | 1500 | 75.2%   | 1192       | 1203       | 5.8%  | 75.4% | 32.3%  | 1069       | 1327       | 23.9% | 76.8% |
|     | 1900 | 84.0%   | 1199       | 1208       | 4.2%  | 83.9% | 52.1%  | 1071       | 1240       | 15.5% | 88.5% |
|     | 2300 | 89.4%   | 1191       | 1196       | 3.1%  | 89.5% | 71.3%  | 1070       | 1163       | 8.5%  | 95.0% |
|     | 2700 | 92.8%   | 1192       | 1197       | 2.4%  | 92.9% | 85.1%  | 1070       | 1116       | 4.2%  | 98.0% |
|     | 3100 | 95.0%   | 1194       | 1198       | 1.9%  | 95.0% | 93.3%  | 1071       | 1090       | 1.8%  | 99.3% |
|     | 3500 | 96.5%   | 1192       | 1195       | 1.5%  | 96.5% | 97.3%  | 1072       | 1079       | 0.6%  | 99.8% |
|     | 3900 | 97.4%   | 1192       | 1193       | 1.1%  | 97.4% | 99.0%  | 1069       | 1072       | 0.2%  | 99.9% |

- ii) The censored samples for both distributions are obtained by choosing the elements of complete samples generated in step i) that are less than or equal to  $t(=700:400:3900)$ . The censored sample size ( $r$ ) and percentage( $r/n$ ) are computed.
- iii) The MLEs of the two distributions' parameters are computed from each complete sample and its corresponding censored sample using the suitable MLE formula as in section 2.
- iv) The MLEs of the mean lifetime to fail for each distribution at each termination time are computed from both complete ( $MT_{com}$ ) and censored ( $MT_{cen}$ ) samples using the following well-known formulas:

Lognormal:  $MT = exp(\mu + \sigma^2 / 2)$  (22)

Weibull:  $MT = \beta \text{ Gamma}(1 + 1 / \alpha)$  (23)

- v) The relative error in estimating the mean lifetime to fail using censored instead of complete samples is denoted by  $e_L$  for lognormal and  $e_W$  for Weibull case and is computed for each iteration as follows

$$e = \frac{|MT_{com} - MT_{cen}|}{MT_{com}}$$

- vi) Steps (i-v) are repeated 5000 times; the average of the 5000 results are calculated for each quantity ( $r/n, MT_{com}, MT_{cen}$  and  $e$ ) and reported in Table 2.
- vii) The values of the efficiencies  $\epsilon_L$  and  $\epsilon_W$  are computed for the lognormal and Weibull cases, respectively, using definition (11).  $\epsilon_L$  is easily computed using Theorem 1.1, whereas,  $\epsilon_W$  is computed numerically.

We have to mention here that the MLEs of the distributions' parameters for both models are not included in Table 2 as they are of different types and therefore not comparable. However, the mean lifetime to fail has a clear physical meaning which applies to both cases. The degree of agreement between the proposed efficiency measure of censored samples and their accuracy in estimating the mean lifetime to fail is a reliable indicator on its goodness in describing the quality of censored samples.

We can see from Table 2 that the efficiencies ( $\epsilon_L$  and  $\epsilon_W$ ) are free of the complete sample size ( $n$ ) and increase as the termination time increases starting from zero (when  $t = \text{zero}$ ) up to one (when  $t = \infty$ ), while the relative errors ( $e_L$  and  $e_W$ ) in estimating the mean lifetime to fail decrease as they should be. The censored samples from lognormal have higher efficiency values than the Weibull when the termination time is small or moderate even though the censored sample percentages ( $r/n$ ) are similar to some extent. For example, when the termination time is 1500 h, the censored samples of lognormal are smaller, yet they are more efficient than those of Weibull regardless of the complete sample size. This is precisely reflected in the relative error in estimating the mean lifetime to fail where  $e_L < e_W$ , keeping in mind that the relative errors  $e_L$  and  $e_W$  are computed as the averages of 5000 relative errors corresponding to each  $MT_{com}$  and  $MT_{cen}$ . In general, lognormal is the better fit if one decides to terminate the lifetime test early or moderately, whereas, Weibull is preferable for late or very late terminations as clear from the values of  $\epsilon_L$  and  $\epsilon_W$ . This exactly agrees with an argument proposed by Nelson [24], where the predictions under the Weibull distributions tend to be more pessimistic at the lower tails than the lognormal. A clearer picture can be found in Fig. 1 where the efficiency functions for both distributions are plotted against the termination time.

Based on the above observations, we recommend using the lognormal distribution instead of Weibull for the Hammer data [20] as they stopped the experiment at a moderate time of 1470 h. In conclusion, the efficiency is very consistent with the accuracy in estimating the mean lifetime, which is clear by comparing the efficiency  $\epsilon = \epsilon_L$  or  $\epsilon_W$  with the relative error of estimating the mean lifetime  $e = e_L$  or  $e_W$  between the two

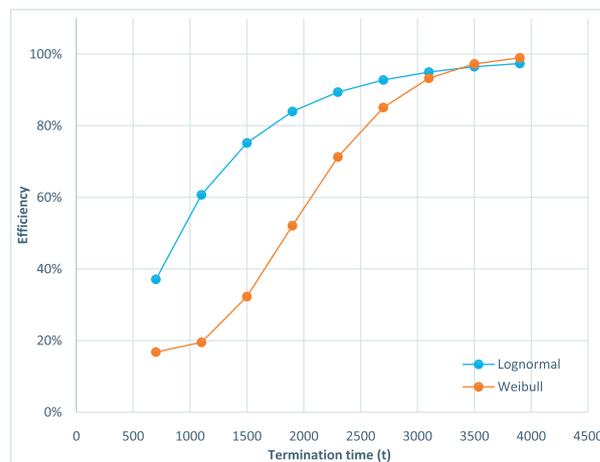


Fig. 1. The efficiency of the lognormal and Weibull distribution.

distributions. However, the degree of consistency between the efficiency and accuracy needs more investigation; in particular, one to measure the percentage of agreement between these two important quantities. For this purpose, we have prepared Table 3.

In Table 3, we have computed the difference in efficiency between the two distributions  $\Delta\epsilon = \epsilon_L - \epsilon_W$ .  $\Delta\epsilon = 0$  means that the censored samples from the two distributions have the same efficiency.  $\Delta\epsilon > 0$  means that the censored sample of lognormal is more efficient than that of the Weibull and vice versa. On the other hand, we have computed the difference in the relative error of estimating the mean lifetime to fail  $\Delta e = e_L - e_W$ . Again,  $\Delta e = 0$  means that the censored samples from the two distributions commit the same error in estimating the mean lifetime to fail.  $\Delta e < 0$  means that the censored sample of lognormal commits less error than that of the Weibull and vice versa. Therefore, it is necessary to compare  $\Delta\epsilon$  and  $\Delta e$ . It is clear that the two quantities are consistent only if they have different signs, that is  $(\Delta\epsilon)(\Delta e) < 0$ . Therefore, to quantify the goodness of the efficiency function in specifying which censored sample is better, we define a new percentage and call it the figure of merit  $\Omega$  as

Table 3 Comparison.

| $n$  | $t$  | $\Delta\epsilon$ | $\Delta e$ | $\Omega$ | $\rho$ |
|------|------|------------------|------------|----------|--------|
| 20   | 700  | 20.3%            | 9.3%       | 78%      | -77%   |
|      | 1100 | 41.2%            | -14.4%     |          |        |
|      | 1500 | 42.9%            | -14.1%     |          |        |
|      | 1900 | 31.9%            | -8.6%      |          |        |
|      | 2300 | 18.2%            | -3.4%      |          |        |
|      | 2700 | 7.7%             | -0.3%      |          |        |
|      | 3100 | 1.7%             | 1.0%       |          |        |
|      | 3500 | -0.9%            | 1.4%       |          |        |
|      | 3900 | -1.6%            | 1.3%       |          |        |
|      | 30   | 700              | 20.3%      |          |        |
| 1100 |      | 41.2%            | -18.3%     |          |        |
| 1500 |      | 42.9%            | -15.7%     |          |        |
| 1900 |      | 31.9%            | -9.8%      |          |        |
| 2300 |      | 18.2%            | -4.4%      |          |        |
| 2700 |      | 7.7%             | -1.0%      |          |        |
| 3100 |      | 1.7%             | 0.5%       |          |        |
| 3500 |      | -0.9%            | 1.1%       |          |        |
| 3900 |      | -1.6%            | 1.1%       |          |        |
| 50   |      | 700              | 20.3%      | -12.5%   | 89%    |
|      | 1100 | 41.2%            | -21.5%     |          |        |
|      | 1500 | 42.9%            | -18.1%     |          |        |
|      | 1900 | 31.9%            | -11.4%     |          |        |
|      | 2300 | 18.2%            | -5.4%      |          |        |
|      | 2700 | 7.7%             | -1.7%      |          |        |
|      | 3100 | 1.7%             | 0.1%       |          |        |
|      | 3500 | -0.9%            | 0.8%       |          |        |
|      | 3900 | -1.6%            | 0.9%       |          |        |

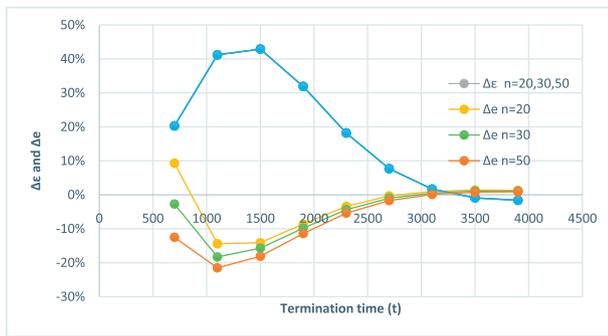


Fig. 2. Shows the difference in efficiency ( $\Delta\epsilon$ ) and the difference in the relative error of estimating mean lifetime ( $\Delta e$ ) versus the termination times ( $t$ ).

the ratio between the number of cases when  $(\Delta\epsilon)(\Delta e) < 0$  to the total number of cases. Formally, it is defined to be as follows

$\Omega = \frac{N((\Delta\epsilon)(\Delta e) < 0)}{N}$ , where  $N((\Delta\epsilon)(\Delta e) < 0)$  is the number of cases where  $(\Delta\epsilon)(\Delta e) < 0$  and  $N$  is the total number of cases. For example, for  $n = 20$ , termination time  $t = 700\Delta\epsilon = 20\% > 0$ , hence the efficiency function ( $\epsilon$ ) says that lognormal censored sample is better than that of Weibull, but, the relative error says the opposite as  $\Delta e = 9\% > 0$ . In this case, the efficiency gives wrong information, whereas, in most of the cases the efficiency agrees with the relative error, and the percentages of agreement ( $\Omega$ ) are 78%, 89%, and 89%, for  $n = 20, 30$ , and  $50$ , respectively. We should mention here that the results presented in the three tables are only for a small part of the data. Considering the whole data from  $t = 700$  to  $4000$  with step size  $100$ , it is observed that the percentages of agreement go up to 90%, 95%, and 96%, for  $n = 20, 30$ , and  $50$ , respectively.

In Fig. 2, we summarize a few observations noted earlier. As can be seen from the figure that when  $\Delta\epsilon$  is big in magnitude  $\Delta e$  is also big and vice versa. For long termination times the values of  $\Delta\epsilon$  and  $\Delta e$  converge to zero indicating that both models behave similarly for all sample sizes (both have high efficiencies and low relative errors). Surprisingly, there is a strong linear correlation between  $\Delta\epsilon$  and  $\Delta e$ , and the correlation gets stronger as the complete sample size increases. This is clear from Table 3 by looking at the last column where the correlation coefficient  $\rho$  is computed and found to be  $-77\%$ ,  $-96\%$  and  $-97\%$ , for  $n = 20, 30$ , and  $50$ , respectively.

#### 4. Conclusion

In this work, we derived the explicit form of the efficiency of type I censored sample of lognormal distribution based on sup-entropy. The efficiency function of the lognormal is compared with that of the Weibull distribution for SSL experimental data. The efficiency function showed a significant consistency with the accuracy in estimating the mean lifetime to fail; confirmed through an intensive simulation study. For small and moderate termination times, censored samples of the lognormal were more efficient in estimating the mean lifetime to fail whereas Weibull for late termination, and this was reflected in the relative errors of estimation. This allows efficiency function based on sup-entropy to be an appropriate quantifier of the amount of information carried by the censored sample about the distribution under consideration.

#### Declarations

##### Author contribution statement

Omar A. Kittaneh, M. A. Majid: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data;

Contributed reagents, materials, analysis tools or data; Wrote the paper.

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#### Competing interest statement

The authors declare no conflict of interest.

#### Additional information

No additional information is available for this paper.

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