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RESEARCH ARTICLE

# Bistable perception of ambiguous images: simple Arrhenius model

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**Abstract**
Watching an ambiguous image leads to the bistability of its perception, that randomly oscillates between two possible interpretations. The relevant evolution of the neuron system is usually described with the equation of its "movement" over the nonuniform energy landscape under the action of the stochastic force, corresponding to noise perturbations. We utilize the alternative (and simpler) approach suggesting that the system is in the quasi-stationary state being described by the Arrhenius equation. The latter, in fact, determines the probability of the dynamical variation of the image being percepted (for example, the left Necker cube ↔ the right Necker cube) along one scenario or another. Probabilities of transitions from one perception to another are defined by barriers detaching corresponding wells of the energy landscape, and the relative value of the noise (analog of temperature) influencing this process. The mean noise value could be estimated from experimental data. The model predicts logarithmic dependence of the perception hysteresis width on the period of cyclic sweeping the parameter, controlling the perception (for instance, the contrast of the presented object). It agrees with the experiment and allows to estimate the time interval between two various perceptions.

**Keywords** Ambiguous images · Analytic theory · Bistable perception · Perception hysteresis

## Introduction

Bistable perception is manifested when an ambiguous image, admitting two interpretations, is presented to the subject. In that case the image perception oscillates with time in a random manner between those two possible interpretations (Huguet et al. 2014). Such a bistability arises for different types of modality (Moreno-Bote et al. 2007)—ambiguous geometrical figures (Necker 1832), figure-ground processes (Pressnitzer and Hupé 2006), etc. (cf. Leopold and Logothetis 1999; Long and Toppino 2004).

Why those oscillations occur? Concrete "microscopic" mechanism of that phenomenon is not known (see Sterzer et al. 2009), but various formal models are suggested based, mainly, on the idea of competitions between distinct neuron populations (engrams) (Lago-Fernández and Deco 2002; Laing and Chow 2002; Wilson et al. 2001). The fundamental attribute of the most part of similar models is the existence of fluctuations (the noise) which leads to random switching over different perceptions (Urakawa et al. 2017).

We exploit the popular model according to which the dynamical process of the bistable recognition might be reduced to traveling the ball along the energy landscape in the presence of the high enough "noise" (H. Haken, Principles of Brain Functioning, Springer 1996). Relatively deep wells of that landscape correspond to old neuronal patterns ("long-stored" in the memory), while new images, being subjected to identification, are landscape regions with higher energy. The image recognition by the brain is analogous to removing the ball in the nearest deeper well corresponding to some known engram. Then, the possible perception bistability is due to the fact that probabilities of transitions in different wells, corresponding to different images, differ weakly, while in the usual situation (with unambiguous image recognition) one of these probabilities significantly outweighs another one. Now, the main problem is to establish, which details of the system dynamics define characteristics of the bistable image recognition.

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## Energy function

Due to fluctuations, the system state changes randomly that results in the perception bistability. It is suggested (Moreno-Bote et al. 2007), that two neuron populations (two different neuron graphs, or two engrams) represent two possible interpretations of the stimulus. Those two populations "compete" with one another, changing the activity of their neurons. Such a model is based on introducing some energy function $U$ with two local minima, corresponding to both different image perceptions, and a barrier between these two states.

The temporal evolution of the neuron system is usually described with the equation of its "movement" over the nonuniform energy landscape under the action of the stochastic force, representing noise perturbations (Pisarchik et al. 2014; Runnova et al. 2016). We utilize the alternative (and simpler) approach suggesting that the system is in the quasi-stationary state which could be described by the Arrhenius equation (Stiller 1989). That would be true if the average energy $\langle \Phi \rangle$ of noise fluctuations was less than the height of the barrier separating two system states. Below, we will see that this suggestion is valid. But it is the aim of the work to show that this "limited" model, though much more simple, gives no less (in some cases–more) information than the more detailed model of Moreno-Bote type for describing bistable perception. In addition, our approach is analytical one, while other models result in numerical calculations and results only.

Usually, the energy function is written, by analogy with the phenomenological theory of phase transitions (Toledano and Toledano 1987), in the form of the power function of some state parameter whose changing corresponds to the dynamic transition of the system from one state to another. However drawing such a power form is justified only by the possibility to expand the function $U$, in the neighborhood of its minimum, in powers of the state parameter. Therefore, the form of that function could be selected arbitrary (mainly, for the ease of convenience) from the class of those, preferably simple functions, that describe the needed evolution of the two-well potential with changing the state parameter. Specifically, we write that function in the form[1]

$$U(\theta) = -U_0(\sin^2\theta + J\theta), \tag{1}$$

where $\theta$ is the generalized coordinate of the system state (the dynamical variable, or the order parameter), $U_0$ is the typical system "energy". Here $J(t)$ is the control parameter, generally time-dependent, that defines the system state. For instance, in the case of the Necker cube (see below) the image contrast could play the role of such a control parameter). We will be interested in the interval of changing the parameter $\theta_0$ that corresponds to those minima of the function $U(\theta)$ which are proximate to the point $\theta = 0$. At $J = 0$ these extremes are placed in points $\theta_1 = -\pi/2$, $\theta_2 = \pi/2$ (minima), $\theta_0 = 0$ (maximum). If $J \neq 0$, then the maximum shifts to the point, where $\sin 2\theta_0 = J$, and minima—to points $\theta_1 = -\pi/2 + \theta_0$, $\theta_2 = \pi/2 + \theta_0$ (see Fig. 1).

With rising the parameter $J$, the tilt of the energy landscape changes—the first minimum becomes shallower, the second one – more deep, and the barrier between them diminishes. Let, for instance, in the original state $J = -1$ and the system resides in the first deep minimum. Then, with rising the control parameter $J$ the system will move (due to fluctuations) from the state $\theta_1$ (where it has existed at $J = -1$) to the state $\theta_2$, clearing the reduced barrier with the top in the point $\theta_0$. In full that barrier disappears at $J = +1$ (see Fig. 2).

Under cyclic variation of the parameter $J$, the system does not have time to follow it, and, due to such an "inertia", the hysteretic dependence $\theta(J)$ arises, shown in the insert of Fig. 2 and associated with system transitions from one well to another over the detaching barrier of the finite height. In the example case, the transition occurs at $J = \pm 0.5$.

Barrier heights $\Delta_{12}$ and $\Delta_{21}$, obstructing, respectively, system transitions from the minimum $\theta_1$ to the minimum $\theta_2$ and the reverse, is readily found from Eq. (1):

$$\begin{aligned} \Delta_{12}/U_0 &= \sqrt{1 - J^2} + J \cdot (\arcsin J - \pi/2), \\ \Delta_{21}/U_0 &= \sqrt{1 - J^2} + J \cdot (\arcsin J + \pi/2) \end{aligned} \tag{2}$$

In the linear approximation

$$\Delta_{12}/U_0 \approx 1 - \pi J/2, \ \Delta_{21}/U_0 \approx 1 + \pi J/2. \tag{3}$$

Dependencies $\Delta_{12}(J)$, $\Delta_{21}(J)$ of those barriers on the control parameter are shown in Fig. 3 which demonstrates that, with monotonous variation $(J = -1) \to (J = +1)$, they are also monotonous and cross in the point $J = 0$. Somewhere in the vicinity of that point the transition occurs from one minimum to another. This is the phase transition with hysteresis whose width is, as usually, depends on the relation between the time $T$ of sweeping the control parameter and the characteristic time $\tau$ (see Eq. 4) of the phase transition.

Instead of the explicit accounting the noise influence we will use the well-known Arrhenius-Kramers formula (Kramers 1940) for the mean lifetime $\tau$ of the system in the certain quasi-stationary state which is determined by the

[1] The function of Eq. (1)-type is known in the theory of magnetics, where the first term describes the energy of magnetic anisotropy, and the second one—the energy of the magnetic moment in magnetic field [so called—Zeeman energy (Vonsovsky 1974)].

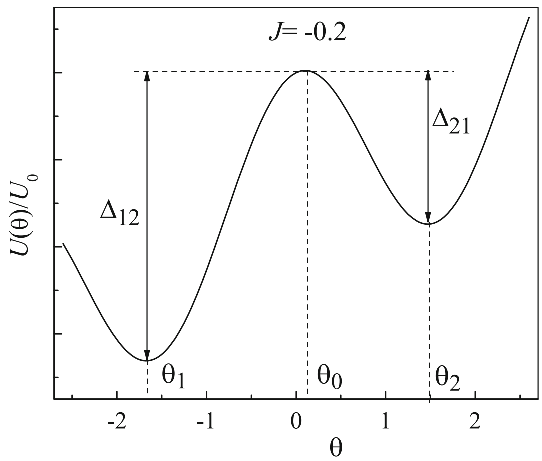


**Fig. 1** Extremes of the energy function (1) at $J = -0.2$

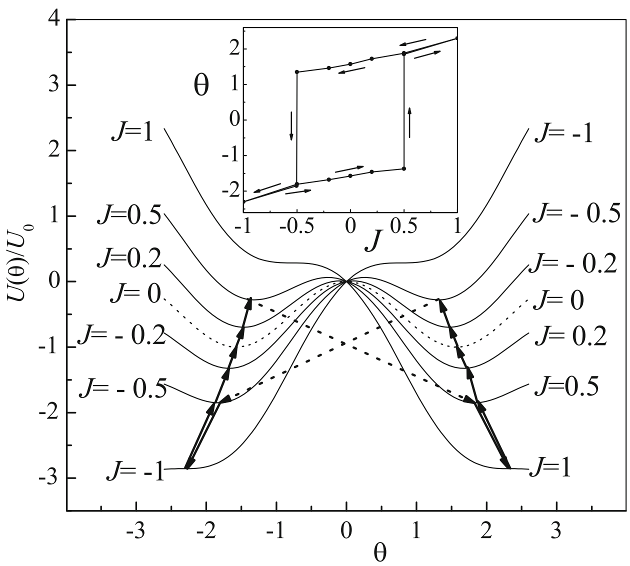


**Fig. 2** Energy landscape $U = U(\theta)$ at various values of the control parameter $J$. Arrows indicate the system evolution under cyclic changing that parameter within limits $-1 < J < 1$. The cycle corresponds to the hysteresis loop shown in the insert. It is accepted that jumps between states occur at $J = \pm 0.5$ by crossing the barrier of $0.5U_0$ -height

relation between the height $\Delta$ of the "energy" barrier and the mean value $\langle\Phi\rangle$ of the noise fluctuation energy (that value could be called the chemical temperature):

$$\tau = \tau_0 \exp(\Delta/\langle\Phi\rangle), \tag{4}$$

where $\tau_0$ is the constant which should be estimated (see below), and by reason of its general sense is the time between two successive attempts to clear the barrier. In fact, that relationship defines the probability of the system transition in one or another state.

The chemical or noise temperature $\langle\Phi\rangle$ is the chemical analog of thermal fluctuations (to which the thermal energy corresponds in the chemical kinetics).[2]

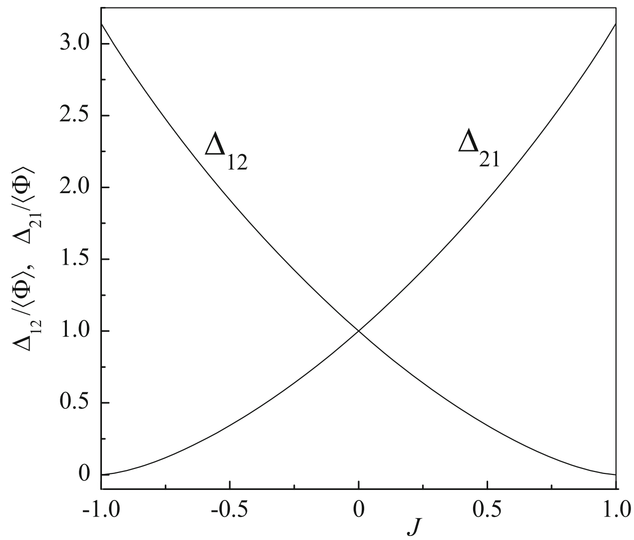


**Fig. 3** Dependencies of barriers heights for transitions $\theta_1 \to \theta_2$ and $\theta_2 \to \theta_1$ on the control parameter $J$

## Hysteresis

To estimate the width of the hysteresis loop for the dependence $\theta(J)$ (for instance, with varying the control parameter $J(t)$ with time), we will base on the assumption that the transitions $\theta_1 \to \theta_2$ and $\theta_2 \to \theta_1$ between minimums of the energy $U(\theta)$ occur not at the moment when the barrier between these two states disappears, but upon condition that the life-time $\tau$ of the current state (see. Eq. 4) diminishes (due to reducing the barrier height) so that becomes much less than the time $T$ of the $J$-parameter sweeping, that is under the condition

$$\tau = \tau_0 \exp\left(\frac{\Delta}{\langle\Phi\rangle}\right) = \gamma T, \quad \text{where} \quad \gamma \ll 1. \tag{5}$$

It follows from Eqs. (3), (5) that the transition $\theta_1 \to \theta_2$ occurs at

$$J = J_{1\to 2} = \frac{2}{\pi}\left[1 - \frac{\langle\Phi\rangle}{U_0}\ln(\gamma T/\tau_0)\right]. \tag{6}$$

By the system symmetry respective to transitions $\theta_1 \to \theta_2$ and $\theta_2 \to \theta_1$, the reverse transition occurs at

[2] In the statistical physics and electronics, the concept of the effective temperature $T_{\text{eff}}$ (or noise temperature) is often introduced that is one way of expressing the level of available noise power (see, for example, https://en.wikipedia.org/wiki/Noise_temperature). On the order of value, that temperature is the average fluctuation $\langle|\Phi|\rangle$ of the relevant quantity, expressed in energy units: $k_B T_{\text{eff}} = \langle|\Phi|\rangle$, where $k_B$ is the Boltzmann constant. In the text, by fluctuations we mean the deviation of ion or neurotransmitter concentrations in synaptic contacts. That is why we call this noise (and that temperature) as chemical or neural ones. This term is purely phenomenal, different processes could group together under this same heading. But, nevertheless, the electric potential of a membrane fluctuates in random manner (see Burns 1968).

$J = J_{2\to1} = -J_{1\to2}$, so that the whole width of the hysteresis loop equals

$$h = J_{2\to1} - J_{1\to2} = \frac{4}{\pi}\left[1 - \frac{\langle\Phi\rangle}{U_0}\ln(\gamma T/\tau_0)\right]. \tag{7}$$

## Necker cube: perception bistability

In the experiment (Runnova et al. 2016), the Necker cube (Necker 1832) has been presented as the ambiguous figure (see Fig. 4) with the contrast of three neighbor cube edges, meeting in its *left* middle corner, as the control parameter $-1 < J < 1$. The values $J = -1$ and $J = +1$ correspond, respectively, to luminosities $j = 0$ and $j = 255$ for pixels of those edges images with 8-bit gray scale. Thus, the contrast $J$ (the control parameter) has been defined by the relation $J = 2j/255 - 1$, where $j$ is the luminosity of those lines on the given scale. In such a case, the contrast of three middle cube edges, meeting in the *right* middle corner, equals $1 - 2J$, and the contrast of six visible outer cube edges equals to 1. In the symmetrical case $J = 0$, so that the parameter $J$ defines the deviation from the symmetry. For the pure left cube $J = -1$, and for the pure right cube $J = 1$.

In the course of the experiment, cube images with $N$ random values $J_i$ of the control parameter $(i = 1, 2, \ldots, N)$ have been presented many times. Subjects have been requested to press the left or the right button on the control panel, according to their initial impression—if the cube is "left" (Fig. 4a) or "right" (Fig. 4e). During the experiment (32 min) each cube with the fixed value of the control parameter $J_i$ has been randomly presented $K = 47$ times. Thus, in the course of the experiment $M = N \times K = 752$ images have been presented in sum.

For each value $J_i$ of the control parameter, the probability

$$P_L(J_i) = \frac{l(J_i)}{l(J_i) + r(J_i)} \tag{8}$$

of observing the left cube has been calculated. Here $l(J_i)$ and $r(J_i)$ are, respectively, numbers of pressing the left or the right button after presenting cubes with the value $J_i$ of the control parameter.

Shown in Fig. 5 experimental results are qualitatively similar for all subjects but differ quantitatively. For some observers, the perception of images as left cube ones transforms steeply into their perception as right cubes (near the "symmetry point" $J = 0$, where $P_L = 0.5$; see the upper panel of Fig. 5), while for others this conversion is smeared (see the lower panel of Fig. 5).

In Runnova et al. (2016) those results are associated with competing different neuron populations near the cusp point in the catastrophe theory with noise included (Poston and Stewart 1978). Our approach is much simpler one—we use the Arrhenius relation (4) for the system life-time in a metastable state that permits to describe correctly not only the dependency $P_L(J)$, but the hysteresis of the image perception under the cyclic variation of the control parameter (see below), as well.

We could identify the memorized patterns of the left and the right cubes with some long-formed wells of the energy landscape, while the new image to be recognized—with the virtual (recently formed) well (see Fig. 6). Recognizing the image in that model is the transfer of the system from the new well of the energy landscape, corresponding to the presenting image, into one of two other wells, corresponding in our case to engrams of the left and the right cubes. The direction of such a random, to some extent, transfer is defined by the fact that barriers between the initial and two final wells have different heights. The barrier between wells of more similar images is lower, and that leads to the preferred transfer from the well of the presented image into the well of more similar memorized one.

Let $\Delta_L$ and $\Delta_R$ be the heights of the barriers indicated. If the presented image is more similar to the left cube image, then $\Delta_L < \Delta_R$ and conversely. It is clear, the more the contrast of the presented cube differs from the zero contrast of the symmetrical image ($J = 0$, and $\Delta_L = \Delta_R$), the more the difference between barriers. Then the simplest linear relation between barriers heights and the contrast $J$ of the new image has the form

$$\frac{\Delta_L - \Delta_R}{\langle\Phi\rangle} = c \cdot J, \tag{9}$$

where $c$ is the individual constant to be experimentally determined.

The probability $P_L$ to recognize the cube as the left one (or the right one) depends on probabilities $p_L$, ($p_R$) of transferring from the well corresponding to the presented image into the well of the left (the right) cube. According to (4):

$$p_L \propto \exp(-\Delta_L/\langle\Phi\rangle), \quad p_R \propto \exp(-\Delta_R/\langle\Phi\rangle). \tag{10}$$

Hence, the total probability to see the left cube equals

$$P_L = \frac{p_L}{p_L + p_r} = \frac{1}{1 + \exp(cJ)}. \tag{11}$$

Figure 5 shows [together with experimental data Runnova et al. 2016] theoretical dependencies $P_L(J)$ calculated by Eq. (11) which, apparently, match well with the experiment when the numerical value of the parameter $c$ is properly chosen. The latter varies within the limits from $c \approx 20$ (the upper panel of Fig. 5) down to $c \approx 2$ (the lower

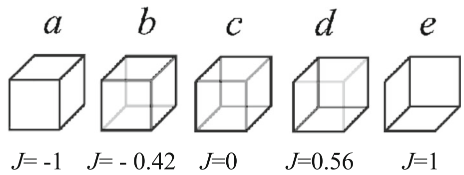


**Fig. 4** Images of Necker cubes with different contrasts being defined by the control parameter $J$ (Runnova et al. 2016)

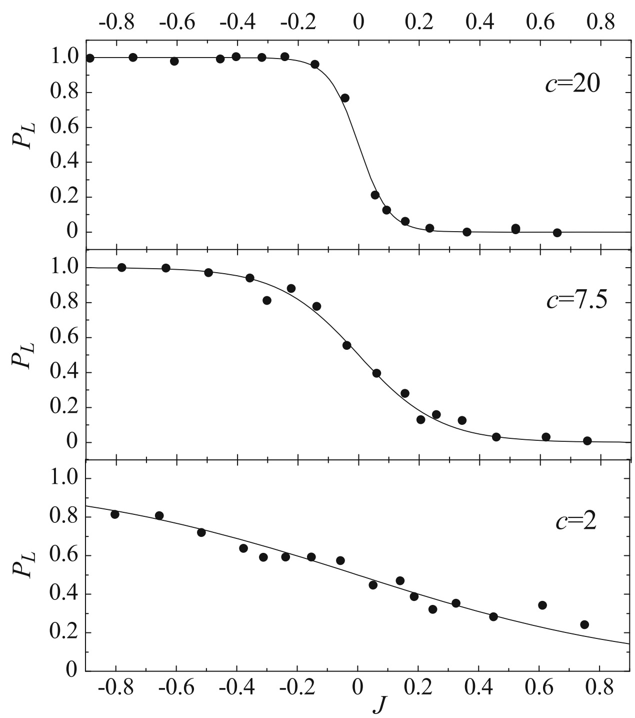


**Fig. 5** Typical experimental dependencies (Runnova et al. 2016) of probabilities $P_L(J)$ to percept the image as a left cube on the control parameter $J$ (points in three panels relate to three different observers). Solid curves are theoretical dependencies (11) with $c$-parameter values specified in each panel

panel of Fig. 5). It follows therefrom, that in the first case the noise is rather weak:

$$\frac{\langle\Phi\rangle}{\Delta_L - \Delta_R} = \frac{1}{cJ} \sim 0.1,$$

while in the last case the noise intensity is high enough: $\langle\Phi\rangle/(\Delta_L - \Delta_R) \sim 1$, and is comparable with barrier heights.

## Necker cube: hysteresis of perception

In Pisarchik et al. (2014), experiments with Necker cube are discussed which relate to the statistics of switching between two possible perceptions of the relevant image with varying the control parameter in time. At first, that parameter has been gradually changed (for the time $T$) in the straight-going direction (from $J = -1$ to $J = 1$), and then, for the same time, in the reversal direction (from $J = 1$ to $J = -1$), wherein the time $T$ of the contrast sweeping has been varied. Moments $\tau_f$, $\tau_b$ (for the forward and back sweeping the control parameter, correspondingly) have been registered, when the observer has *for the first time* switched from some image perception to another one. In the bistable system, such an over-switching takes place twice—at the forward and backward variation of that parameter[3]. Such a hysteresis phenomenon depends on the rate of varying the control parameter and is observed in different bistable systems.

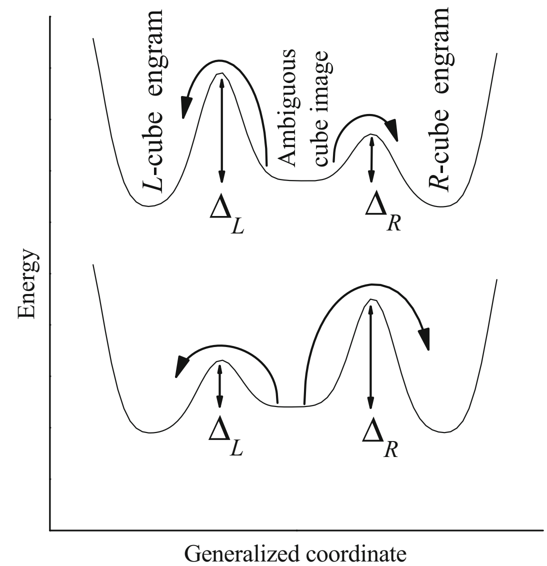


**Fig. 6** Coordinate dependency of the energy. The central minimum corresponds to the presented ambiguous image, lateral minima—to memorized images of the left and the right cubes. Arrows show potentially possible over-barrier "jumps" between wells: below—preferentially on the left, above—to the right

Hysteresis is the system property which lies in the fact that under varying external conditions the system state differs, more or less, from the state being equilibrium at the current conditions. The latter state is that, which could be reached in infinite time after onset of certain (further unchanged) conditions. Really, to arrive at the state which is close enough to the equilibrium one, the finite characteristic relaxation time $\tau$ is needed, so that the existence (or nonexistence) of hysteretic phenomena is defined by the relation of two times—the relaxation one and the experiment duration time $T$: there is the hysteresis if $T \lesssim \tau$, and the hysteresis is absent if $T \gtrsim \tau$.

The hysteresis (more exactly, the hysteresis width) could be conveniently characterized be the parameter

[3] In the presence of noises, another phenomenon could be observed—the random intermittence that is the simultaneous switching from one image perception to another one under the constant control parameter.

$$h = \left(\frac{\tau_f + \tau_b}{T}\right) - 1, \tag{12}$$

which goes to zero (and even becomes negative), when $\tau_f, \tau_b < T/2$, and is distinct from zero at low $T$, when $\tau_f, \tau_b > T/2$ and $h > 0$. Hysteresis loops for these two cases are gone in opposite directions—clockwise ($h < 0$) and anti-clockwise ($h > 0$). As it is seen from (7), the case $h < 0$ is realized under the condition

$$\frac{\langle\Phi\rangle}{U_0} > 1/\ln(\gamma T/\tau_0), \tag{13}$$

that corresponds to high enough (other factors being equal) intensity of fluctuations $\langle\Phi\rangle/U_0 \gtrsim 1$, provoking "advanced" transitions between energy minima over high barriers.

The logarithmic dependency predicted by our model agrees with the experiment (Pisarchik et al. 2014), that allows to estimate numerically some model parameters. Figure 7 presents two typical experimental dependencies of the hysteresis width (14) on $T$ (for two different subjects), which are properly approximated by straight lines in the logarithmic scale. For numerical estimates, it is convenient to introduce the dimensional constant $\tau_1 = 1$ s and rewrite Eq. (7) in the dimensionless form:

$$h = A - B\ln(T/\tau_1), \qquad A = \frac{4}{\pi}\left[1 - \frac{\langle\Phi\rangle}{U_0}\ln\left(\frac{\gamma}{\tau_0/\tau_1}\right)\right], \quad B = \left(\frac{4\langle\Phi\rangle}{\pi U_0}\right). \tag{14}$$

Parameters $A$ and $B$ are determined from linear dependencies of Fig. 7. For example, for the upper of those dependencies $A \approx 1$, $B \approx 0.5$. Herefrom, it follows at once

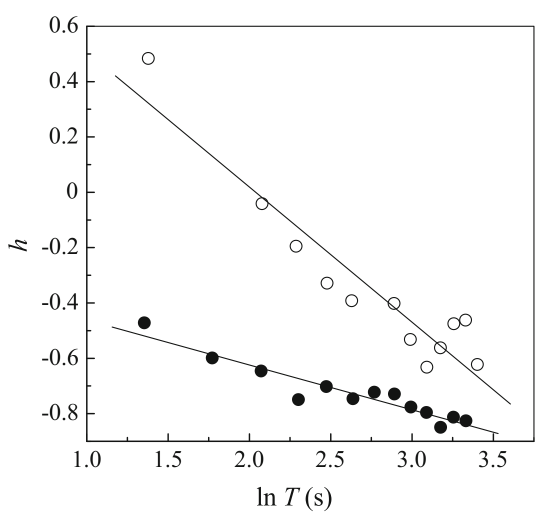


**Fig. 7** Typical experimental dependencies (points) of hysteresis width (12) on the duration $T$ of scanning the control parameter (Pisarchik et al. 2014). Straight lines are linear fits

$$\frac{\langle\Phi\rangle}{U_0} \approx 0.4, \quad \tau_0/\tau_1 \approx 2\gamma. \tag{15}$$

Thus, the relative intensity of fluctuations (for the concrete tested person) is rather high[4]. To compare, the lower dependency in Fig. 7 gives $\langle\Phi\rangle/U_0 \approx 0.15$. Thus, we see that in all cases the noise is relatively small, and, hence, the Arrhenius equation could be used.

As for the time $\tau_0$ or the parameter $\gamma$, directly coupled with it (see (15)), if one chooses, for instance, $\gamma = 0.3$, then $\tau_0 \sim 1$ s.

We could also consider some simpler model of transferring the system from one state into another one, suggesting that this transition occurs always (independently of the sweeping time $T$) at the moment, when the difference between the initial contrast ($J = -1$ at the moment $t = 0$) and the contrast at the switching moment ($t = t_f$) reaches some critical value $J_c$. For the linear sweeping in the forward direction,

$$J = -1 + t/T, \tag{16}$$

so that $J_c = t_f/T$, or

$$t_f = J_c \cdot T, \tag{17}$$

that corresponds to the simple rule: switching time is proportional to the sweeping time. That rule is in some extent confirmed by the experiment (Pisarchik et al. 2014)—see Fig. 8, where experimental dependencies $\tau_{f,b}(T)$ are presented. One could see that in spite of high data scattering those dependencies could be, in fact, considered as linear ones. They correspond to the value $J_c \approx 0.15$. Hence, in that model the switching should happen every time when the contrast difference reaches $\sim 15\%$. However, this over-simplified model predicts the constant hysteresis width $h \approx -0.7$ (see 12), that contradicts to the experiment.

## On distribution of dominance durations

In various experiments on observing ambiguous images, the random switching of two possible interpretations has been registered[5]). Some features of such a random process could be described with the distribution function $F(\tau_D)$ of dominance durations $\tau_D$ during which the preferential interpretation of presented images is remained unchanged. Typically, that function is markedly stretched up to the side of long times (righthand asymmetry) and is often approximated with the gamma- or log-normal distribution (Lehky

[4] Parameters $\langle\Phi\rangle/U_0$, $\gamma$, $\tau_0$ are individual ones and, in principle, could vary in broad limits.

[5] In electrical engineering, it is known as the *telegraph noise*.

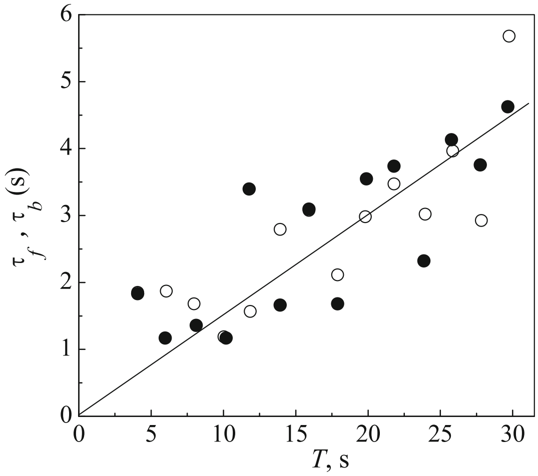


**Fig. 8** Experimental dependencies $\tau_{f,b}(T)$ [filled circle – $\tau_f$, open circle – $\tau_b$ (Pisarchik et al. 2014)]

1995; Levelt 1968). However, notice that the accuracy of experiments is too low to decide between those functions. In the framework of our model, the average value of the random quantity $\tau = \langle \tau_D \rangle$ is defined by the relation (4), and the distribution function could be found from following considerations.

The system transfer from the state with certain interpretation of the ambiguous image into the state with another interpretation, occurs in two stages: firstly, from, say, the left-cube well[6]—to the well of the presented image and then—to the right-cube well (or vice versa),

We need distribution functions of system lifetimes for each of those states. Let $w_{10}(\tau_D)$ be the probability that before transferring the system from well 1 to the well 0 it "lives" in the original well during the time $\tau_D$ Then the probability that it resides in this well during the time $\tau_D + d\tau_D$ equals $w_{10}(\tau_D + d\tau_D)$. The latter event is the complex one and consists of two consecutive independent events: (i) situating the system in the well 1 during the time $\tau_D$ (with the probability $w_{10}(\tau_D)$), and (ii) situating the system in the same well during the time $d\tau_D$ (with the probability $w(d\tau_D)$). Thus,

$$w_{10}(\tau_D + d\tau_D) = w_{10}(\tau_D) \cdot w_{10}(d\tau_D).$$

Then, $1 - w_{10}(d\tau_D)$ is the probability of running the system from the well 1 during the time $d\tau_D$, and it is proportional to $d\tau_D$. Let us denote the proportionality coefficient by $\tau_{10}^{-1}$ (for the transition $1 \to 0$). Then, $1 - w_{10}(d\tau_D) = d\tau_D/\tau_{10}$, or $w_{10}(d\tau_D) = 1 - d\tau_D/\tau_{10}$. Therefore,

$$w_{10}(\tau_D + d\tau_D) = w_{10}(\tau_D)(1 - d\tau_D/\tau_{10}).$$

On the other hand, $w_{10}(\tau_D + d\tau_D) = w_{10}(\tau_D) + (dw/d\tau_D)d\tau_D$, wherefrom the equation

$$dw_{10}(\tau_D)/d\tau_D = -w_{10}(\tau_D)/\tau_{10}$$

follows, whose solution is:

$$w_{10}(\tau_D) \propto e^{-\tau_D/\tau_{10}}, \tag{18}$$

where $\tau_{10}$ is the mean lifetime of the system in a given well (corresponding to the preferable image interpretation[7]).

More exact result could be obtained suggesting that the interchange of the image perceptions occurs not after the single transition $1 \to 0$, but after the double transition $1 \to 0 \to 2$ with the probability of the second stage being analogical to Eq. (18):

$$w_{02}(\tau_D) \propto e^{-\tau_D/\tau_{02}}. \tag{19}$$

The double event distribution function $F(\tau_D)$ for *sum* times equal to $\tau_D$ is[8]

$$F(\tau_D) = \int_0^{\tau_D} w_{10}(x) w_{02}(\tau_D - x) dx = \left(\frac{1}{\tau_{10} - \tau_{02}}\right)\left(e^{-\tau_D/\tau_{10}} - e^{-\tau_D/\tau_{02}}\right). \tag{20}$$

Plots of that function with various relationships between $\tau_{10}$ and $\tau_{02}$ are shown in Fig. 9. It is seen that this distribution function is markedly asymmetric [with $\tau_{10}/\tau_{02} \to 1$ it tends to the simple exponential function (18)]. However, for comparing with experiments the accuracy of the latter should be significantly increased.

Figure 10 shows that the obtained distribution function adequately describes experimental data obtained from registering switches of Necker cube perceptions (Merk and Schnakenberg 2002). The discrepancy at small dominance durations ($\tau_D \lesssim 1$ s) are likely associated with the finite response time of observers.

Analogical distribution functions $F(\tau_D)$, similar to (20), have been also observed for other ambiguous images (Huguet et al. 2014).

## Conclusions

Described above bistability models consider, in fact, dynamical processes of switching between different perceptions of an ambiguous image and the hysteresis of such

6 For convenience, in the present section we use the terminology relating to the bistable perception of the Necker cube, but conclusions are applicable to any bistable system.

7 Derivation of the obtained formulae and its form are identical to those being known for the distribution of gas particles over free path lengths.

8 The distribution obtained is known from the theory of two-stage radioactive decay.

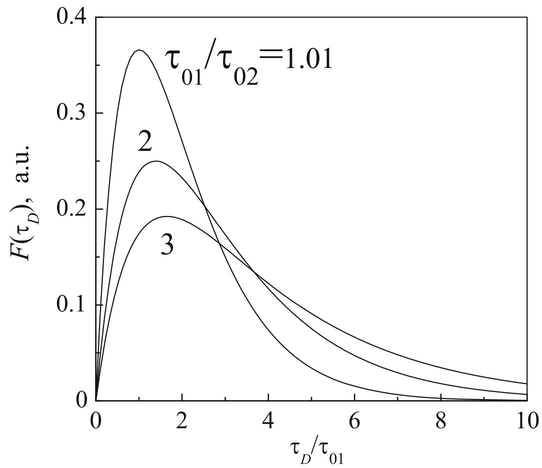


**Fig. 9** The distribution function of dominance durations $\tau_D$

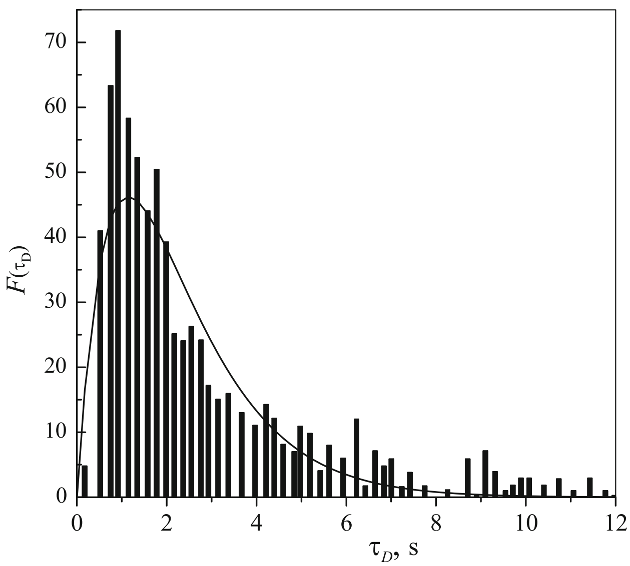


**Fig. 10** Approximation of experimental data (Merk and Schnakenberg 2002) (the histogram) by the distribution function (20)

a perception. On the other hand, no dynamical equations (such as $\dot{\theta} = -\partial U/\partial\theta$) are not used in our scheme. That is based on the Arrhenius–Boltzmann relation (4) which defines the probability of dynamical changing the percepted image type (for example, the left Necker cube $\leftrightarrow$ the right cube) under some scenario.

In the considered model, probabilities of transitions from one perception type to another are calculated. They are determined by barriers separating respective wells (of the depth $U_0$) of the energy landscape, and the noise level influencing that process. The latter is represented by the parameter $\langle\Phi\rangle$ whose relative value could be estimated from experimental data: $\langle\Phi\rangle/U_0 \approx 0.1 - 1$ (individually for various observers).

Predicted by the model, the logarithmic dependency of the perception hysteresis width on the period of cyclic sweeping the parameter controlling the perception (for instance, the contrast of the presented image), agrees with the experiment and allows to estimate the time $\tau$ of switching between two potentially possible perceptions of the ambiguous image: $\tau \sim 1$ s for $T = 30$ s.

The obtained distribution function of dominance durations is asymmetric one and is qualitatively similar to those which is used for describing experimental data.

Thus, in the framework of the described "non-dynamical" approach one could obtain some certain conclusions on dynamics characteristics of the bistable perception for ambiguous images.

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