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BRIEF COMMUNICATION

Theoretical models of reaction times arising from simple-choice tasks

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Abstract

In this work we present a group of theoretical models for reaction times arising from simple-choice task tests. In particular, we argue for the inclusion of a shifted version of the Gamma distribution as a theoretical model based on a mathematical result on first hitting times. We contrast the goodness-of-fit of those models with the Ex-Gaussian distribution, using data from recently published experiments. The evidence of the results obtained highlights the convenience of proposing theoretical models for reaction times instead of models acting exclusively as quantitative distribution measurements.

Keywords Cognitive process · Reaction times · Quantitative distribution measurements · Theoretical models · Simple-choice task tests

Understanding the processes and mechanisms underlying the distribution of response or reaction times (RTs) has been a major goal in experimental psychology [some classic and recent articles can be found in Woodrow (1911), Smith (1968), Wainer (1977), Meyer et al. (1988), Posner (2005), Balota and Yap (2011), Woods et al. (2015), Donkin and Brown (2018), Rousselet and Wilcox (2018)]. In general terms, the proposed models for RTs can be classified in two ways, not necessarily mutually exclusive: as quantitative distribution measurements, where the main aim is to capture major shape features of the empirical distribution of RTs; and/or as theoretical models, where the main aim is to derive candidate probability distributions under considerations of cognitive mechanisms [see also Anders et al. (2016)]. Therefore, the success of a proposed model depends on it qualifying as a good quantitative distribution measurement such that its parameters can account for quantities relating to the underlying cognitive process (although the way to evaluate this aspect remains not entirely clear). The aim of this work is to propose some theoretical models for some class of RTs and show their empirical performance as quantitative distribution measurements.

A model widely used as a quantitative distribution measurement for RTs is the Ex-Gaussian (EG) distribution. While it has shown good fits for empirical RT data, there is not a clear link between EG distribution's parameters and quantities interpretable from a cognitive point of view [see e.g. Ratcliff (1978), Rohrer and Wixted (1994), Matzke and Wagenmakers (2009), Palmer et al. (2011), Marmolejo-Ramos and Gonzalez-Burgos (2013), Marmolejo-Ramos et al. (2015a, b), Velez et al. (2015), Osmon et al. (2018)]. In fact, while the mathematical support of the EG density is the whole real line, RTs can take only non-negative values. This is a major example of a model used exclusively as a quantitative distribution measurement for RTs.

Regarding theoretical models, a further classification can be made according to the complexity of the task. In simple-choice (SC) tasks, a stimulus is presented one at a time and the participant makes one of two possible responses in order to categorize the stimulus (e.g. the participant sees a picture and she classifies it as positive or negative in valence). In multiple-choice (MC) tasks, a

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stimulus is presented one at a time and the participant makes one of three or more possible responses in order to categorize the stimulus (e.g. the participant sees a picture and she classifies it as positive, neutral, or negative in valence). There are other RT tasks that require a binary forced choice. In a spatial 2AFC (two-alternative forced choice) task, two stimuli are presented at the same time (e.g. two pictures side by side) and the participant is required to make a discrimination (e.g. which stimulus is more positive in valence) [see Ratcliff et al. (2018) for an example and definition]. In a temporal 2AFC task (aka, 2IFC [two-interval forced choice] task), two stimuli are presented sequentially. In these tasks "the signal is presented in one and only one of the intervals and the observer is required to report the interval (first vs second) in which the signal was presented" (Yeshurun et al. 2008, p. 1837). It is worth adding that while SC and MC tasks require decisions, judgments, choices, categorizations, or classifications of stimuli, 2AFC and 2IFC tasks require the discrimination of stimuli. In other words, while in an *n*-choice RT task a single stimulus must be assigned to one of *n* response categories, in an *m*-AFC task *m* stimuli are presented concurrently or sequentially and the observer must select the most XXX among them, where XXX is some predefined attribute (J. Miller, personal communication, January 5, 2019; see also Massaro 1989; Wichmann and Jäkel 2018).

A simple underlying cognitive process is roughly schematized as in Fig. 1. Mathematically, such a scheme can be described as:

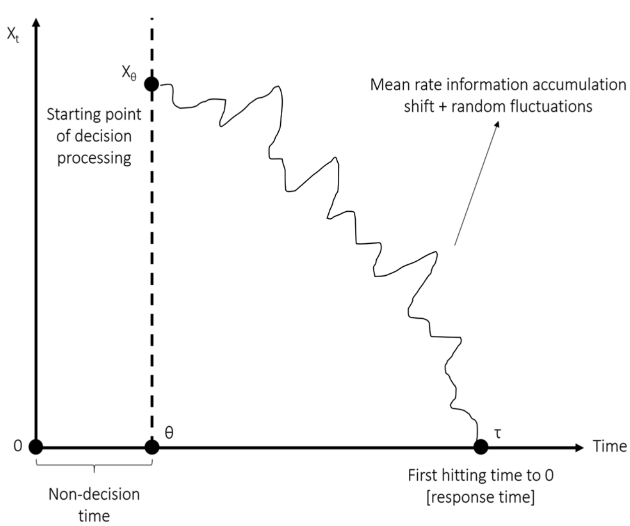

Fig. 1 An illustration of a cognitive processing underlying an RT task. The cognitive process, $\{X_t\}_{t\in\mathbb{R}_+}$, consists of a first time step, θ, in which the individual is primarily receiving the stimulus and no response can be produced during this time-lapse. Then, the decision stage commences by processing the information of task's difficulty plus random fluctuations. A response is finally produced when $\{X_t\}_{t\in\mathbb{R}_+}$ reaches the barrier at 0

$$X_t = X_\theta - \mu(t-\theta) + \varepsilon_{t-\theta}, \mathrm{t} \geq \theta, \tag{1}$$

and $X_t = 0$ for $t < \theta$. Here, $X_\theta > 0$ is the starting processing point, where θ is referred to as the "non-decision" parameter; μ is the mean rate information accumulation (which is influenced by either individual differences in the quality of information processing or by stimulus characteristics that reflect task difficulty), and $\{\varepsilon_t\}_{t\in\mathbb{R}_+}$ is an stochastic process representing random fluctuations. According to such elements, an RT can be defined as the following stopping time:

$$\tau := \inf\{t \geq \theta : X_t = 0\}. \tag{2}$$

Sometimes, the non-decision time also includes the time for processes after reaching the barrier (i.e. response execution). In our setting, for convenience, the non-decision time is moved to the beginning of the task's time line (i.e., into θ), otherwise if this two times are considered separately, say θ_1 at the beginning and θ_2 at the end, the resulting distribution for the RT becomes not identifiable (that is, different values of the parameters θ_1 and θ_2 do not necessarily generate different probability distributions for the corresponding RT).

The scheme and mathematical model previously described are similar to the one-choice extension of the Ratcliff's diffusion model (Ratcliff and Van Dongen 2011). In a sense, our model is simpler than that of Ratcliff and Van Dongen (2011) in that their model assumes that the drift of information accumulation and the non-decision time are random from trial to trial, whereas in our model this drift and non-decision time are fixed. However, we can take into account some variants with respect to the random fluctuations $\{\varepsilon_t\}_{t\in\mathbb{R}_+}$ and the behavior of the initial condition X_θ among trials.

We are going to consider Eqs. (1)–(2) as our model for RTs arising from SC task tests. Although our scheme boils down to a binary response (since we only consider one barrier), we can justify its use by approaching the estimation of the density of RTs according to the chosen response separately. That is, we estimate the conditional densities of the RTs, conditioned on each chosen response, as if they represented two different populations. Thus, the proposed mechanism of the underlying cognitive process according to each chosen response will be described through Eqs. (1) to (2) following the scheme in Fig. 1. This separation will imply that within each conditional event we use a one-barrier drift diffusion process whose mean rate information accumulation will be directed to the given response option. As a consequence of this we will have that the resulting densities for the RTs for each chosen response will be probability distributions of low-dimensional parameters. For MC, or alternative forced choice tasks, our scheme can be quite simplistic. Usually they require to take into account, among other considerations, multiple accumulators, double barriers or specified structures to account for magnitude difference effects between stimuli [see e.g. LaBerge (1962), Ratcliff (1978), Ratcliff and Tuerlinckx (2002), Usher et al. (2002), Horrocks and Thomson (2004), Brown and Heathcote (2008), Blurton et al. (2017), Ratcliff et al. (2018)]. However, we focus on SC tasks as they are found in most published studies involving RTs (Ratcliff and Van Dongen 2011).[1]

Now, regarding the random fluctuations $\{\varepsilon_t\}_{t\in\mathbb{R}_+}$ in Eq. (1), the resulting distribution of Eq. (2) varies. It is well-known that the distribution of (2) is a shifted Wald (SW) when X_θ is fixed and $\{\varepsilon_t\}_{t\in\mathbb{R}_+}$ corresponds to a Brownian motion. The SW has reported good fits for RT data obtained in SC tasks (see e.g. Matzke and Wagenmakers (2009), Anders et al. (2016)). Recently in Tejo et al. (2018), it was shown that a shifted version of the Birnbaum–Saunders (SBS) distribution can be theoretically derived for Eq. (2) when X_θ is fixed and the cognitive process represented by Eq. (1) presents a sharp trend towards the barrier, regardless of the distribution of $\{\varepsilon_t\}_{t\in\mathbb{R}_+}$ (although it has independent and identically distributed [IID] increments). That is, if we consider an equally spaced time partition $0 = t_0 < t_1 < t_2 < \ldots$ and $\{\varepsilon_{t_i} - \varepsilon_{t_{i-1}}\}_{i\in\mathbb{N}}$ forms an IID sequence of random variables such that $\mathbb{P}(-\mu + \varepsilon_{t_i} - \varepsilon_{t_{i-1}} < 0)$ is high, then the resulting distribution of Eq. (2) approaches a SBS distribution. Tejo et al. (2018) also showed that the SBS provided good fits to RT data from SC tasks. In this work we are going to derive another distribution of Eq. (2) when X_θ presents trial-to-trial randomness.

The Gamma distribution has been used in the modeling of RT data, reporting also good fits [see e.g. McGill and Gibbon (1965), Campitelli et al. (2017)]. We found that a shifted version of this distribution (which will be called here as shifted Gamma [SG] distribution) can be theoretically derived from Eqs. (1) to (2) when $\{\varepsilon_t\}_{t\in\mathbb{R}_+}$ is a Brownian motion and X_θ is random, based on the results from Jackson et al. (2009). There, the following inverse problem is analyzed: given some distribution F, find a probability density for X_θ such that the resulting distribution of Eq. (2) is F. In this regard, we adapt and apply here the following specific result: when the distribution F of Eq. (2) belongs to the Gamma family, under some conditions, a positive supported probability density function (PDF) is found for X_θ. In the "Appendix" we derive and present the SG distribution under this context. The assumption that the initial condition is considered as random from trial to trial reflects the fact that each individual subjected to an experimental test may present a different ability or predisposition to reach the expected response.

The data sets from two recent studies reporting SC tasks are used to feature the models discussed here. Specifically, we compare the fittings of the EG, SW, SBS and SG distributions to the SC RTs reported in Velasco et al. (2016) and Osmon et al. (2018).[2] The description of these data sets and the results obtained are detailed below.

- *Data from* Velasco et al. (2016). In two experiments Velasco et al. (2016) investigated the matching of tastes with shapes. In one of the tasks in their experiment 2, participants were shown the taste words "sour" or "sweet" one at a time, and were required to respond with either an angular or round shape. Congruent RT mappings were those correct responses in which the words "sour" and "sweet" were mapped to round and angular shapes, respectively (otherwise, the response was deemed incongruent). We compared the fits of the EG, SW, SBS and SG to the distribution of raw RTs in congruent trials for the sour-angular (DS1) and sweet-round (DS2) mapping's distributions (i.e., incongruent and practice RTs were not included).
- *Data from* Osmon et al. (2018). Osmon et al. (2018) collected the RTs of attention-deficit/hyperactivity disorder (ADHD) and non-ADHD participants to stimuli in three tasks; simple, SC, and cognitive control RT tasks. In the SC task, a circle appeared on the left or right side of a computer screen and participants had to spot the location of the circle by pressing the appropriate key located on the same side as the circle (left-or-right key). We compared the fits of the distributions considered here to the raw RTs obtained from ADHD (DS3) and non-ADHD (DS4) participants in the SC task.

Results from Velasco et al. (2016)'s study

In the case of the selected RTs in Velasco et al. (2016)'s study, the corresponding histograms are shown in Fig. 2. Despite the typical positive skewness observed for RTs, we

[1] Other RT tasks such as go/no-go tasks [see e.g. Nosek and Banaji (2001), McKenzie et al. (2001), Verbruggen and Logan (2008)] and simple RT tasks are not discussed herein.

[2] The R codes needed to replicate the results reported herein for the SG distribution are provided in the Supplementary Material. The corresponding R codes for the SW and the SBS distributions have been provided in the Supplementary Material of Tejo et al. (2018), and the R library *retimes* contains the codes for the application of the EG distribution. Implementations of the EG and other range of distributions are available in the R library *gamlss.dist*.

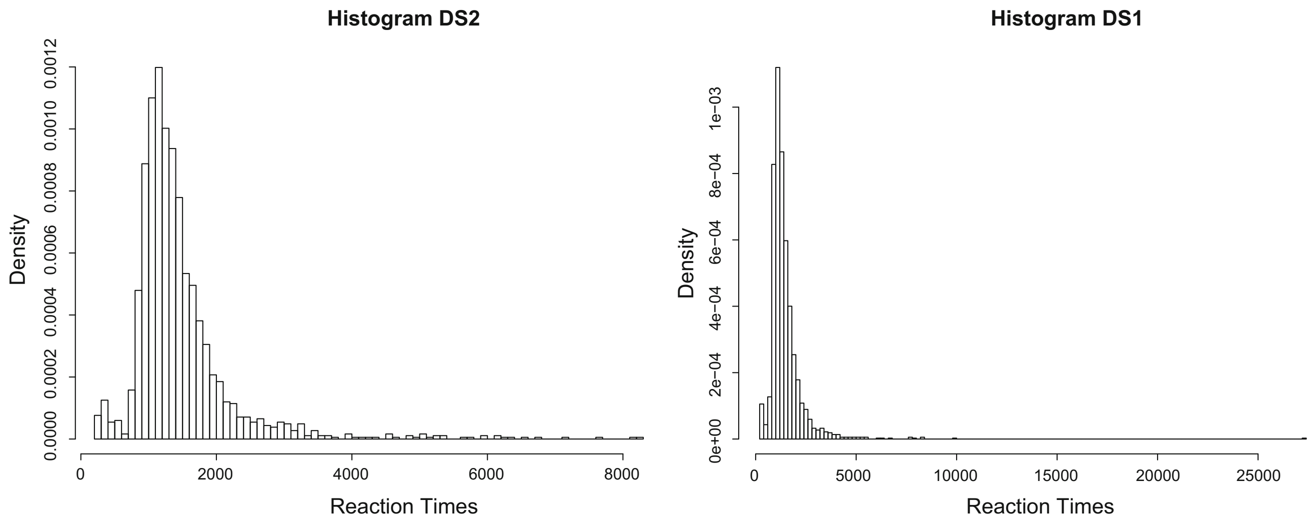

Fig. 2 Histograms for the observed RTs in the case of sour-angular (DS1) and sweet-round (DS2) mapping's distributions, in Velasco et al. (2016)'s study

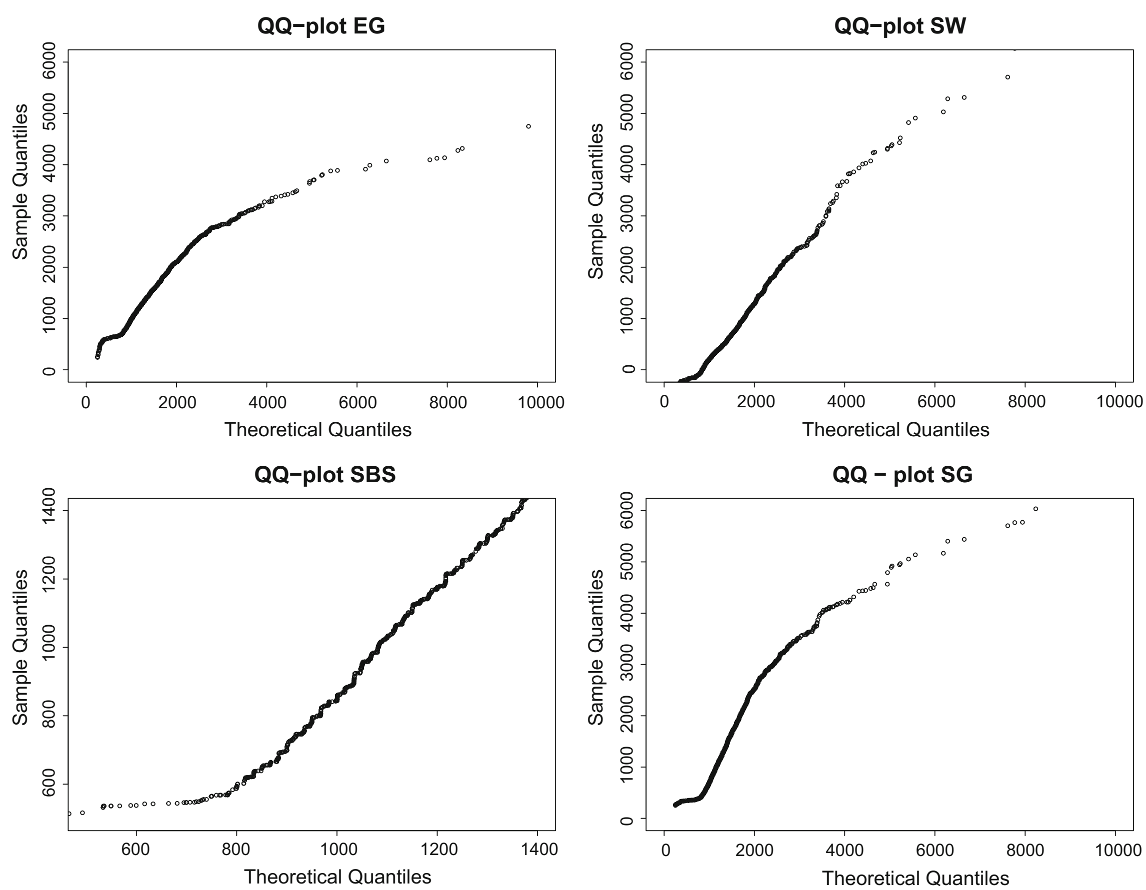

Fig. 3 The resulting QQ-plots for the sour-angular (DS1) mapping's distribution in Velasco et al. (2016)'s study. From top to bottom and left to right, the corresponding R-squared values were $\approx 0,837$, $\approx 0,883$, $\approx 0,835$ and $\approx 0,825$, respectively

can appreciate in both cases a small tail on the left which, in particular, makes a case for the introduction of the EG distribution as a quantitative distribution measurement. In Figs. 3 and 4 we reported the fits for the EG, SW, SBS and SG distributions, respectively, through quantile-quantile plots (QQ-plots). We can notice that, although better fits

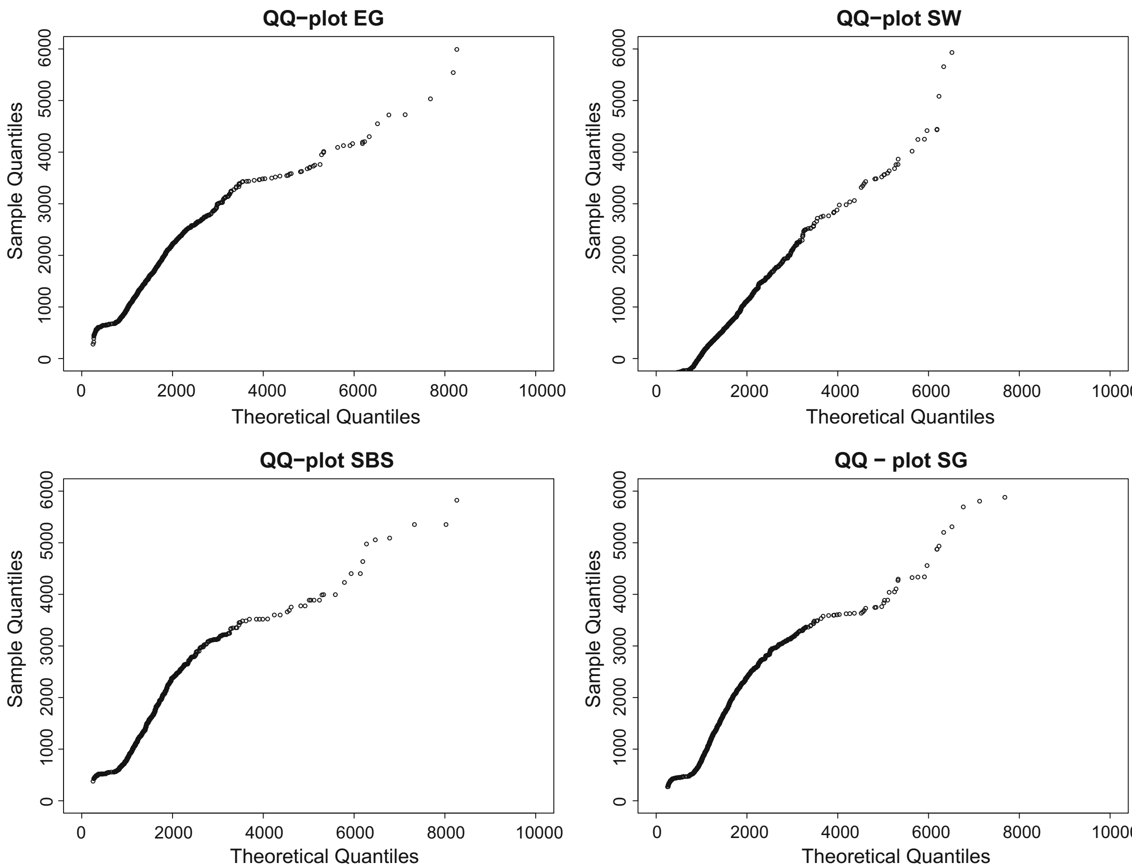

Fig. 4 The resulting QQ-plots for the sweet-round (DS2) mapping's distribution in Velasco et al. (2016)'s study. From top to bottom and left to right, the corresponding R-squared values were ≈ 0,959, ≈ 0,995, ≈ 0,948 and ≈ 0,944, respectively

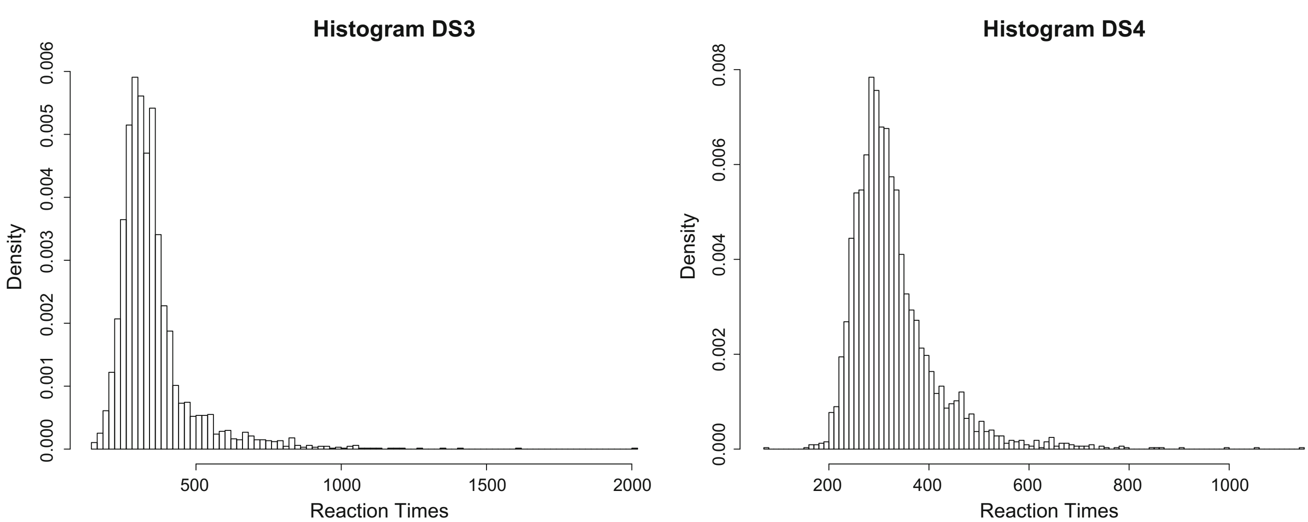

Fig. 5 Histograms for the observed RTs in the case of ADHD (DS3) and non-ADHD (DS4) participants in the SC task, in Osmon et al. (2018)'s study

resulted for DS2, their goodness-of-fit were quite adequate in both data sets. The corresponding coefficients of determination (R-squared values) did not vary notoriously within both data sets, and the SW and the SG distributions exhibited the highest and lowest R-squared values, respectively, in both cases (Fig. 5).

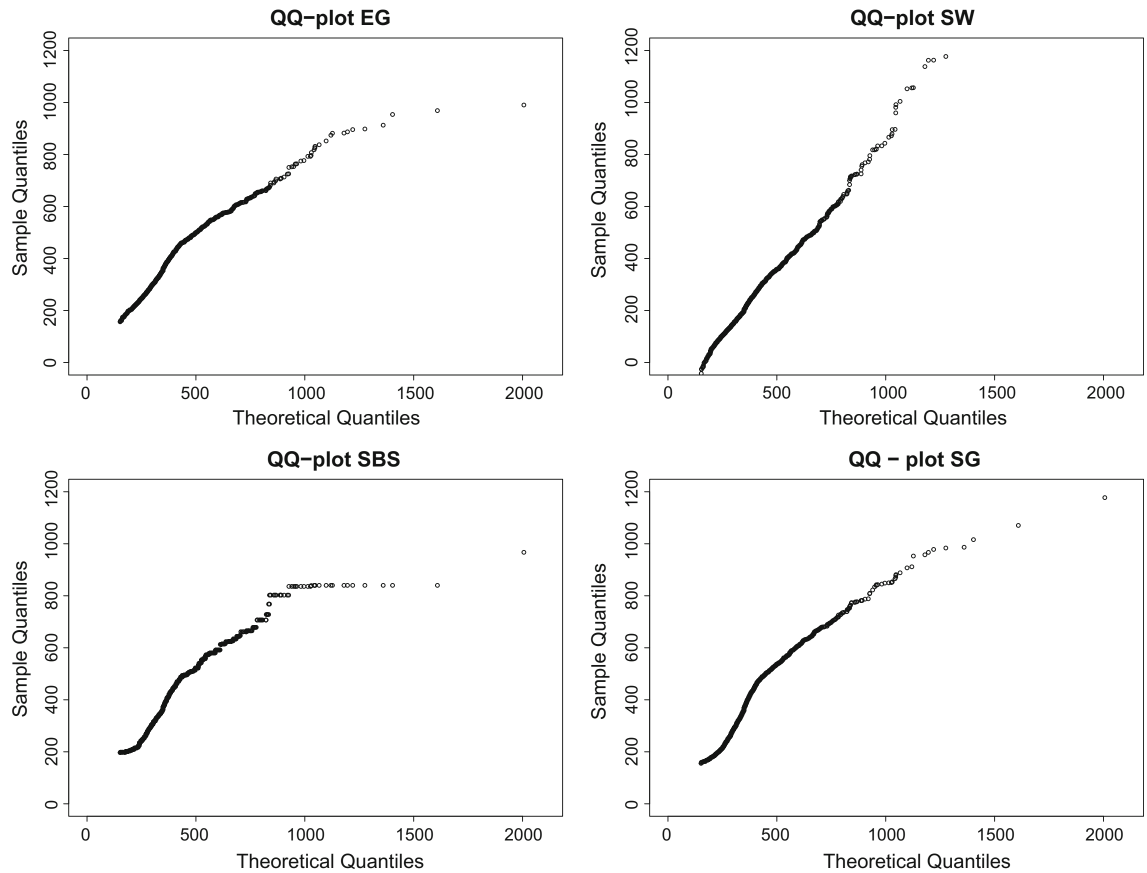

Fig. 6 The resulting QQ-plots for the case of ADHD (DS3) participants in the SC task in Osmon et al. (2018)'s study. From top to bottom and left to right, the corresponding R-squared values were $\approx 0,967$, $\approx 0,997$, $\approx 0,963$ and $\approx 0,958$, respectively

Results from Osmon et al. (2018)'s study

Regarding the selected RTs in Osmon et al. (2018)'s study, we found that the reported goodness-of-fit were even better than those of the previous study (see Figs. 6 and 7); just notice that the corresponding R-squared from the QQplot of the SW in DS4 was around 0,999. Again, the corresponding R-squared values did not vary notoriously within both data sets, but then again the SW and SG distributions showed the highest and lowest R-squared values, respectively.

Discussion

In both studies the theoretical models proposed for SC tasks (i.e. the SW, SBS and SG distributions) reported good fits. Thus, those models can provide information as to the potential mechanisms supporting the cognitive processes behind RT tasks. These results also suggest that SW, SBS and SG distributions could be better alternatives to distributions that are used exclusively for quantitative measurement (e.g. the EG distribution).

Regarding the quality of the fits, we believe that the R-squared values between the theoretical quantiles under each reference model versus the empirical quantiles are good enough for a first comparison among the models considered here. We believe the R-squared values are informative since such models have the same parametric dimension and, therefore, do not need to be corrected by the number of parameters. Of course, this criterion of goodness-of-fit and comparison is quite general, and there is no criterion of statistical significance that tells us whether the differences found among these R-squared values are significant or not. However, it could be of interest to obtain more robust models that can better capture the "outlying data" that generate the curvatures observed in the QQ-plots, although this may involve obtaining models that can not be derived from theoretical cognitive schemes.

As Tejo et al. (2018) showed for the SW and SBS distributions, given the maximum likelihood estimators (MLEs) of the corresponding parameters, it is possible to reconstruct a cognitive scheme whose resultant RT distribution is well represented by those distributions. In the provided R codes for the SG distribution (Supplementary Material), we can obtain the MLEs for the parameters of

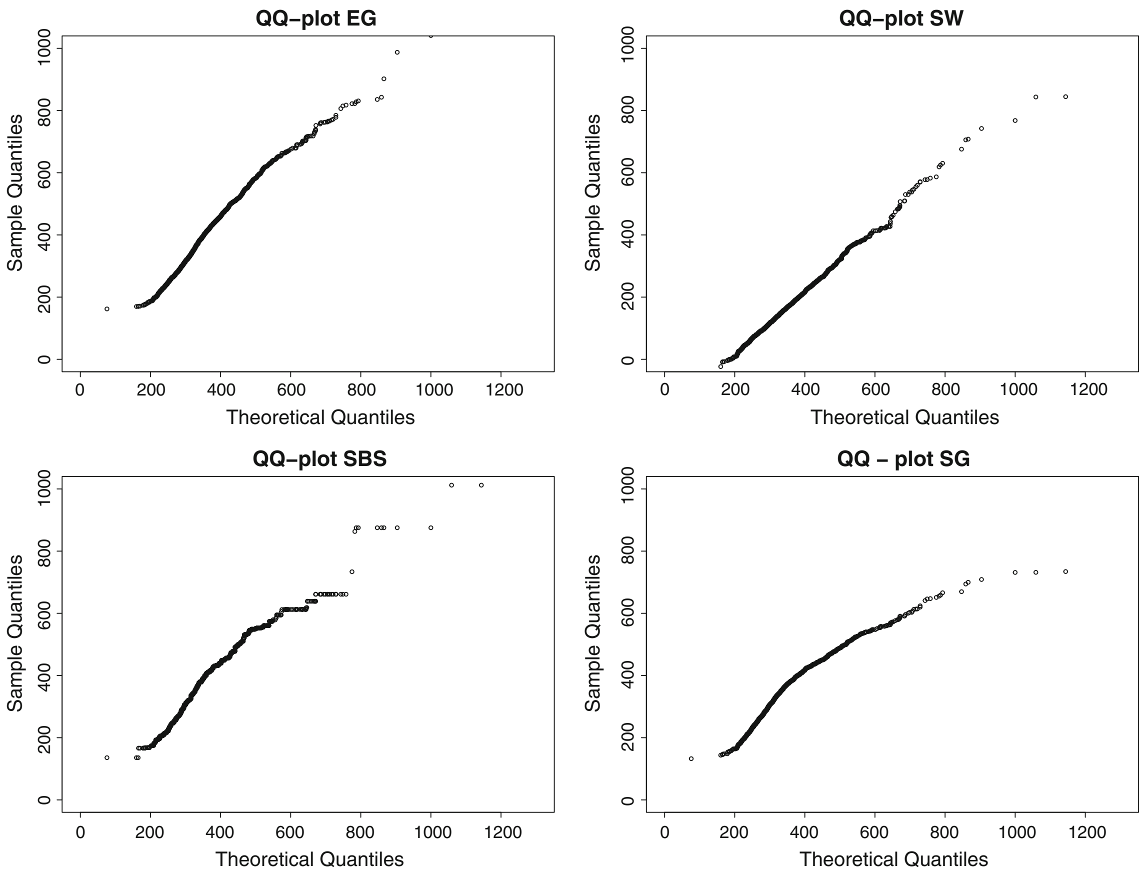

Fig. 7 The resulting QQ-plots for the case of non-ADHD (DS4) participants in the SC task in Osmon et al. (2018)'s study. From top to bottom and left to right, the corresponding R-squared values were $\approx 0,991$, $\approx 0,999$, $\approx 0,986$ and $\approx 0,965$, respectively

Eq. (3), $\widehat{\alpha}$, $\widehat{\beta}$ and $\widehat{\theta}$, and then proceed analogously to reconstruct an estimated cognitive scheme. To do this, following what is mentioned in Appendix, set first a $\tilde{\mu} \geq \sqrt{2\widehat{\beta}^{-1}}$ and construct the underlying estimated cognitive process as $\tilde{X}_t = X_{\widehat{\theta}} - \tilde{\mu}(t - \widehat{\theta}) + \varepsilon_{t-\widehat{\theta}}$ for $t \geq \widehat{\theta}$, such that $X_{\widehat{\theta}}$ has the distribution of the sum of two independent Gamma distributions with common (estimated) shape parameter $\widehat{\alpha}$ and (estimated) scale parameters $\tilde{\mu} - \sqrt{\tilde{\mu}^2 - 2\widehat{\beta}^{-1}}$ and $\tilde{\mu} + \sqrt{\tilde{\mu}^2 - 2\widehat{\beta}^{-1}}$, respectively. In this way, we can use the SG distribution to simulate RTs whose distribution results similar to those observed in the data sets considered here. Note that, for interpretability, we can take $\tilde{\mu} = \sqrt{2\widehat{\beta}^{-1}}$ to reconstruct the underlying cognitive process, being now $\sqrt{2\widehat{\beta}^{-1}}$ equal to the (estimated) mean rate information.

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