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RESEARCH ARTICLE

Consensus of uncertain multi-agent systems with input delay and disturbances

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Abstract

In this paper, the problem of robust consensus for multi-agent systems affected by external disturbances is discussed. A novel consensus control is developed by using a feedback controller based on disturbance rejection and Smith predictor scheme. Specifically, the disturbance rejection performance of the uncertain multi-agent systems is improved according to the estimation of equivalent-input-disturbance and the effect of time delay in the control system is reduced via Smith predictor scheme by shifting the delay outside the feedback loop. Furthermore, by combining Lyapunov theory, matrix inequality techniques and properties of Kronecker product, a robust feedback controller for each agent is designed such that the desired consensus of the uncertain multi-agent systems affected by external disturbances can be ensured. Finally, to illustrate the validity of the designed control scheme, two numerical examples with simulation results are provided.

Keywords Multi-agent systems · Consensus · Equivalent-input-disturbance · Smith predictor

Introduction

For the past few years, the problem of multi-agent systems has received great attention from various scientific communities due to its broad applications in various areas such as cooperative unmanned air vehicles, scheduling of automated highway systems and air traffic control (Zhu et al. 2018; Ursino et al. 2018; Zhang et al. 2017). An important issue arising in multi-agent systems is to develop an appropriate control based on local information which enables all agents to reach an agreement on certain quantities of interest which is known as the consensus problem (Zhang et al. 2016). Recently, there has been much attention on study of reaching consensus for multi-agent systems because of its numerous applications in the research fields such as wireless sensor networks, biological networks, world wide web and artificial intelligence (Szalkai et al. 2017). Specifically, the main objective of this study is to ensure that a common goal can be reached by a group of identical agents by carrying out a consensus protocol which is based on the local state information with their neighbours (Xi et al. 2014; Sakthivel et al. 2018). Over the past few years, the consensus problems of multi-agent systems has been developed as an important and hot research topic, which have received considerable amount of attention from many researchers with different techniques (Hou et al. 2016; Liu et al. 2017). Wu et al. (2016) derived a set of linear matrix inequalities (LMIs) based sufficient conditions for consensusability of nonlinear dynamical multi-agent systems by using aperiodic sampled-data controllers, where the resulting closed-loop system was reformulated as a time-varying delay continuous system.

All researches mentioned above assumes an ideal system model without having any disturbance. On the other hand, disturbances such as stochastic noises, external interferences (Sun et al. 2016), communication delays (Revathi et al. 2016; Shi and Yang 2018), random losses (Lin et al. 2018, 2016) and model uncertainties (Wang

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et al. 2018) are some of the inevitable factors in multi-agent systems. These inevitable factors may degrade the consensus performance. Moreover, the existence of perturbations in multi-agent systems may deteriorate the system performance and can even destroy consensus. Hence, it is of utmost importance to investigate their consensus on the study of multi-agent systems and appropriately design protocol to achieve robust consensus. In Zhao et al. (2017), the consensus protocol problem of networked multi-agent systems with coupling dynamics and external disturbances is studied, where a set of LMI based conditions is developed for obtaining a sliding mode feedback control law based on H_∞ to acheive the consensus. Based on dynamic gain technique, a nonlinear disturbance observer is proposed to compensate the disturbances acting on the multi-agent systems in Zhang and Liu (2018). A novel finite-time consensus control to show fast response and strong robustness to various disturbances is discussed in Tian et al. (2018). Recently, to improve the performance of disturbance rejection for dynamical control system, a new technique called an equivalent-input-disturbance (EID) approach is first proposed in She et al. (2008). It should be mentioned that the disturbances (matched and unmatched) can be effectively handled by using the EID approach without any prior information of the disturbance (Gao et al. 2016; Liu et al. 2014, 2014; Sakthivel et al. 2017). Moreover, one of the effective tools to remove time delay in the control system is Smith predictor. This tool stabilizes the time delay system by integrating the input/output delays (Smith 1959; Astrom et al. 1994; Lee et al. 1999). Very recently, in Gao et al. (2016) a controller that is combined with Smith predictor and the EID approach is proposed to improve the performance of disturbance rejection for a system subject to input time delay and external disturbances.

However, up to now, no work has been reported regarding the consensus for uncertain multi-agent systems via the controller design based on EID estimator and Smith predictor. Motivated by the above discussions, by using the Lyapunov stability approach, we aim to solve the robust consensus problem of a class of uncertain multi-agent systems in the presence of external disturbances via a feedback controller. Meanwhile, we also put effort to develop an EID based disturbance rejection method for multi-agent systems with input time delay. Specifically, in the EID-based control protocol, to reconstruct the state of multi-agent systems, a modified state observer is employed. Moreover, the Smith predictor is utilized in control protocol to deal with the input time delay. A set of LMI-based criteria is developed by choosing a suitable Lyapunov–Krasovskii functional which ensures the consensus of uncertain multi-agent systems in the presence of disturbances. Then, the LMIs are solved to further obtain the consensus controller gain.

The core contributions in this paper are outlined hereby

1. A robust feedback controller is proposed to guarantee the consensus of a class of input delay multi-agent systems with uncertainties and external disturbances, which is more applicable in the practical perspectives.
2. To effectively deal with input delay, a Smith predictor is utilized, which reduces the presence of the delay in the control system by shifting the delay outside the feedback loop.
3. An equivalent disturbance is incorporated into the proposed control protocol, which rejects the disturbance more accurately.
4. Sufficient criteria is developed in terms of LMIs to attain the desired consensus of the considered uncertain input delay multi-agent systems subject to external disturbances.

Preliminaries and model formulation

Initially, some graph theory concepts are provided and then the consensus problem of uncertain multi-agent systems through the feedback control protocol based on EID and Smith predictor approaches is discussed in this section.

As it is known, in graph theory, each agent can be considered as a node. Then the communication topology among the agents can be modelled as a dynamic graph. Let $\mathbb{G} = \{N, \mathbb{E}, \mathbb{A}\}$ be an undirected graph, where $N = \{1, 2, \ldots, \mathcal{N}\}$ is a set of nodes set, $\mathbb{E} \subseteq N \times N$ is a set of edges and $\mathbb{A} = [a_{ij}] \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}$ is an adjacency matrix with non-negative elements. (i, j) is an ordered edge of $\mathbb{G}$ if an agent i is able to communicate with agent j. The edges (i, j) is the same (j, i) in an undirected graph. In this paper, we have considered $\mathbb{G}$ to be undirected. The set $\mathcal{N}_i = \{j : j \in N, (j, i) \in \mathbb{E}\}$ denotes the set of neighbors which communicate with i, then $a_{ij} = a_{ji} > 0$, otherwise $a_{ij} = a_{ji} = 0$ with $a_{ii} = 0$. The Laplacian matrix denoted by $\mathcal{L}$ can be defined as

$$\mathcal{L} = [l_{ij}] = \begin{cases} -a_{ij}, & i \neq j, \\ \sum\limits_{j\in\mathcal{N}_i} a_{ij}, & i = j, \end{cases} \quad i,j = 1, 2, \ldots, \mathcal{N}.$$

Consider an input delay multi-agent system with external disturbances. Let the considered system be composed of $\mathcal{N}$ agents which are identical in nature. Subsequently the dynamics of each ith agent can be represented by

$$\begin{aligned} \dot{x}_i(t) &= \mathcal{A}x_i(t) + \mathcal{B}u_i(t-\tau) + \mathcal{E}d_i(t), \\ y_i(t) &= \mathcal{C}x_i(t), \quad i = 1, 2, \ldots, \mathcal{N}, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^q$ are the state, the control input; $d_i \in \mathbb{R}^q$ and $y_i \in \mathbb{R}^r$ are the external disturbance and the output of the ith agent, respectively; τ denotes the delay in the motion of the ith agent. Also $\mathcal{A}$, $\mathcal{B}$, $\mathcal{E}$ and $\mathcal{C}$ are real constant matrices with appropriate dimensions. The consensus problem of input delay multi-agent system (1) is solvable if $\forall i, j = 1, 2, \ldots, \mathcal{N}$ and as time approaches infinity, $x_i(t) \rightarrow x_j(t)$. To attain consensus, it is important to design the consensus such that input delay multi-agent system (1) is asymptotically stable about its zero fixed point. In particular, in this paper, we design a controller that rejects external disturbance and also compensates the input time delay in its design.

In order to reject the external disturbance, we incorporate an EID estimator block into the control protocol. The main importance of inserting an EID estimator in the control system is that it improves the disturbance rejection performance. Ultimately the EID estimator acts as a compensator for the disturbance rejection. Now, we introduce a state-dependent EID estimator, $d_{ie}(t)$. This $d_{ie}(t)$ when incorporated with the control input channel produces the same output $y_i(t)$. Then, (1) can be rewritten as

$$\begin{aligned} \dot{x}_i(t) &= \mathcal{A}x_i(t) + \mathcal{B}u_i(t-\tau) + \mathcal{B}d_{ie}(t), \\ y_i(t) &= \mathcal{C}x_i(t), \quad i = 1, 2, \ldots, \mathcal{N}. \end{aligned} \tag{2}$$

To estimate the EID, the local observer for each agent is designed by

$$\begin{aligned} \dot{\hat{x}}_i(t) &= \mathcal{A}\hat{x}_i(t) + \mathcal{B}u_{if}(t-\tau) + D[y_i(t) - \mathcal{C}\hat{x}_i(t)], \\ \hat{y}_i(t) &= \mathcal{C}\hat{x}_i(t), \quad i = 1, 2, \ldots, \mathcal{N}, \end{aligned} \tag{3}$$

where $\hat{x}_i$ and u_{if} are the observer state and control input vector; D and $\hat{y}_i$ are the observer gain and the output of each agent, respectively. According to Liu et al. (2014), if $\mathcal{B}^+$ is the pseudo inverse of $\mathcal{B}$, then we can represent an EID estimator as follows

$$\begin{aligned} \hat{d}_{ie}(t) &= \mathcal{B}^+ D(y_i(t) - \hat{y}_i(t)) - u_i(t-\tau) + u_{if}(t-\tau), \\ & i = 1, 2, \ldots, \mathcal{N}, \end{aligned} \tag{4}$$

where $\hat{d}_{ie}(t)$ is an estimate of the state-dependent EID $d_{ie}(t)$. It is noted that if (3) is stable asymptotically, then $\hat{d}_{ie}(t)$ converges to $d_{ie}(t)$ as $t \rightarrow \infty$. In particular, to establish the accuracy in estimation, we use a low-pass filter $F_i(t)$. This $F_i(t)$ is merged into the EID estimator. The low-pass filter $F_i(t)$ can be represented by the following state-space equation

$$F_i(t) : \begin{cases} \dot{x}_{iF}(t) = \mathcal{A}_F x_{iF}(t) + \mathcal{B}_F \hat{d}_{ie}(t), \\ \tilde{d}_{ie}(t) = \mathcal{C}_F x_{iF}(t), \quad i = 1, 2, \ldots, \mathcal{N}, \end{cases} \tag{5}$$

where $x_{iF}(t)$ is the state of the EID estimator with $\mathcal{A}_F$, $\mathcal{B}_F$ and $\mathcal{C}_F$ being constant matrices with suitable dimension. The output, $\tilde{d}_{ie}(t)$ is the filtered disturbance estimate from $F_i(t)$. This $\tilde{d}_{ie}(t)$ can be used to select the estimation frequency band. Now, we define a modified controller $u_i(t) = u_{if}(t) - \tilde{d}_{ie}(t)$ which contains the filtered estimate to suppress the disturbance effect in the system. It should be mentioned that choice of such a modified controller makes $y_i(t)$ to converge asymptotically to the origin. If $y_i(t) = 0$ then $x_i(t) = 0$ in (1), thereby solving the consensus problem of the multi-agent system (1).

In order to analyze the consensus issue, the system with $d_{ie}(t) = 0$ is considered. Further, by utilizing the concepts in Gao et al. (2016), the state-space form of the Smith Predictor can be written as

$$G_i(t) : \begin{cases} \dot{x}_{id}(t) = \mathcal{A}_d x_{id}(t) + \mathcal{B}_d[u_{if}(t) - u_{if}(t-\tau)], \\ y_{id}(t) = \mathcal{C}_d x_{id}(t), \quad i = 1, 2, \ldots, \mathcal{N}, \end{cases} \tag{6}$$

where x_{id} and y_{id} denote the state and output of the Smith Predictor for agent ith; $\mathcal{A}_d$ with $\mathcal{B}_d$ and $\mathcal{C}_d$ being suitably dimensioned real constant matrices. Now, let us present a state-space form of dynamic output feedback protocol $\mathcal{C}_i(t)$ based on the Smith Predictor as

$$\mathcal{C}_i(t) : \begin{cases} \dot{x}_{ic}(t) = \mathcal{A}_c x_{ic}(t) + \mathcal{B}_c[e_i(t) - y_{id}(t)], \\ u_{if}(t) = \mathcal{C}_c x_{ic}(t), \quad i = 1, 2, \ldots, \mathcal{N}, \end{cases} \tag{7}$$

where x_{ic} denotes the state of dynamic output feedback protocol; $\mathcal{A}_c$, $\mathcal{B}_c$ and $\mathcal{C}_c$ being suitably dimensioned real constant matrices; $e_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) - y_j(t))$ and the proposed consensus control protocol is $u_{if}(t)$.

Define $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ as the state estimation error. Consequently, if $\tilde{x}_i(t) = 0$ as $t \rightarrow \infty$ then, the multi-agent system will reach its consensus. Finally for (1) to reach consensus, we can observe that the five states $\dot{\tilde{x}}_i(t)$, $\dot{x}_i(t)$, $\dot{x}_{iF}(t)$, $\dot{x}_{ic}(t)$ and $\dot{x}_{id}(t)$ are involved in the stability analysis. This is clearly depicted in Fig. 1. In short, Fig. 1 can be described as a closed-loop system, $\dot{\phi}_i^T(t) = [\dot{\tilde{x}}_i^T(t)\ \dot{x}_i^T(t)\ \dot{x}_{iF}^T(t)\ \dot{x}_{ic}^T(t)\ \dot{x}_{id}^T(t)]$.

Using Eqs. (2)–(7), these five states can be alternately written in the following forms:

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= [\mathcal{A} - D\mathcal{C}]\tilde{x}_i(t) - \mathcal{B}\mathcal{C}_F x_{iF}(t-\tau), \\ \dot{x}_i(t) &= \mathcal{A}x_i(t) + \mathcal{B}\mathcal{C}_c x_{ic}(t-\tau) - \mathcal{B}\mathcal{C}_F x_{iF}(t-\tau), \\ \dot{x}_{iF}(t) &= \mathcal{A}_F x_{iF}(t) + \mathcal{B}_F\mathcal{C}_F x_{iF}(t-\tau) + \mathcal{B}_F\mathcal{B}^+ D\mathcal{C}\tilde{x}_i(t), \\ \dot{x}_{ic}(t) &= \mathcal{A}_c x_{ic}(t) + \mathcal{B}_c \sum_{j \in \mathcal{N}_i} a_{ij}\mathcal{C}(x_i(t) - x_j(t)) - \mathcal{B}_c\mathcal{C}_d x_{id}(t), \\ \dot{x}_{id}(t) &= \mathcal{A}_d x_{id}(t) + \mathcal{B}_d\mathcal{C}_c x_{ic}(t) - \mathcal{B}_d\mathcal{C}_c x_{ic}(t-\tau). \end{aligned}$$

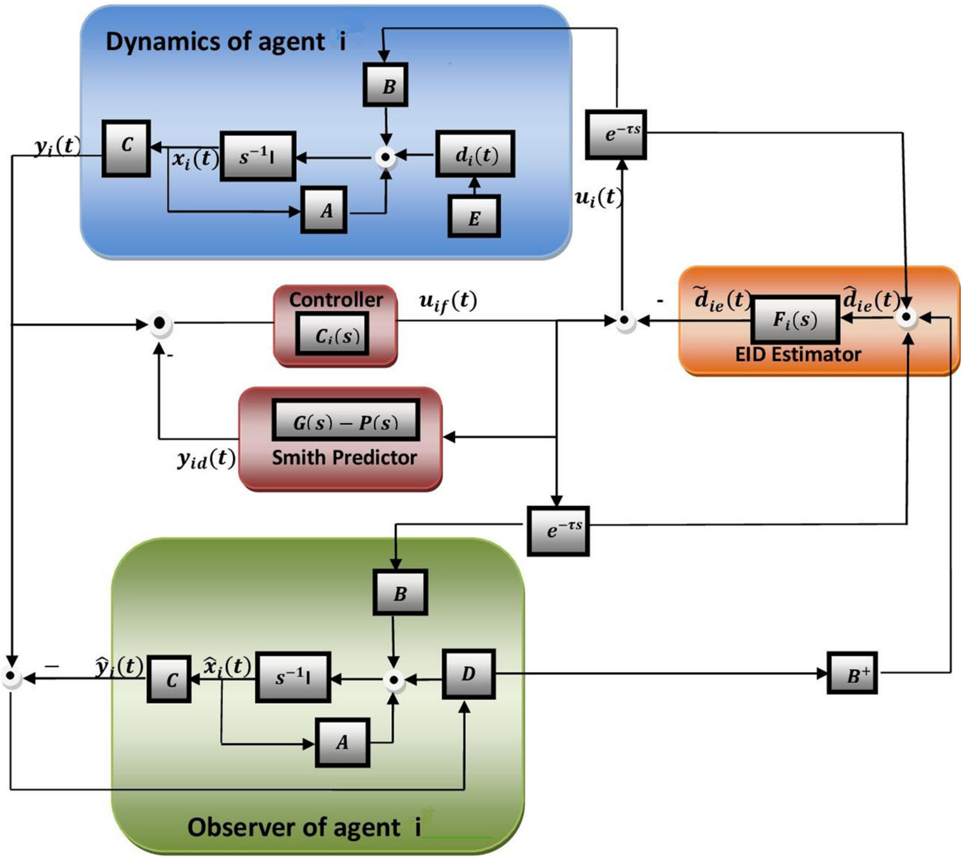


Fig. 1 Configuration of agent i with EID and Smith predictor-based dynamic output feedback controller

Now by utilizing the Kronecker product properties, the state-space representation of $\dot{\phi}(t)$ as described in Fig. 1 can be given in the form

$$\dot{\phi}(t) = A_1\phi(t) + A_2\phi(t-\tau), \tag{8}$$

where

$$A_1 = \begin{bmatrix} I_n\otimes(\mathcal{A}-D\mathcal{C}) & 0 & 0 & 0 & 0 \\ 0 & I_n\otimes\mathcal{A} & 0 & 0 & 0 \\ I_n\otimes\mathcal{B}_F\mathcal{B}^+D\mathcal{C} & 0 & I_n\otimes\mathcal{A}_F & 0 & 0 \\ 0 & \mathcal{L}\otimes\mathcal{B}_c\mathcal{C} & 0 & I_n\otimes\mathcal{A}_c & -(I_n\otimes\mathcal{B}_c\mathcal{C}_d) \\ 0 & 0 & 0 & I_n\otimes\mathcal{B}_d\mathcal{C}_c & I_n\otimes\mathcal{A}_d \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} 0 & 0 & -(I_n\otimes\mathcal{B}\mathcal{C}_F) & 0 & 0 \\ 0 & 0 & -(I_n\otimes\mathcal{B}\mathcal{C}_F) & (I_n\otimes\mathcal{B}\mathcal{C}_c) & 0 \\ 0 & 0 & (I_n\otimes\mathcal{B}_F\mathcal{C}_F) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(I_n\otimes\mathcal{B}_d\mathcal{C}_c) & 0 \end{bmatrix}.$$

Main results

In this section, we will establish a set of conditions which are sufficient to guarantee the consensus of input delay multi-agent system (1) via an EID and Smith predictor-based control design. These conditions are derived by utilizing the concepts of algebraic graph theory. Moreover by using Lyapunov stability theorem, these sufficient conditions are established in terms of LMIs to guarantee the consensus of (1).

Theorem 3.1 *Consider the input delay multi-agent system (1) under the assumption that $(\mathcal{A},\mathcal{B})$ is stabilizable and $(\mathcal{A},\mathcal{C})$ is detectable. For a given positive scalar τ, system (1) under the control protocol (7) achieves the consensus asymptotically, if there exist real constant symmetric positive definite matrices $\bar{P}$, $\bar{Q}$, $\bar{R}$ and any appropriate dimensioned matrix $\bar{X} = diag\{(I_n\otimes\bar{X}_1),(I_n\otimes\bar{X}_2),(I_n\otimes\bar{X}_3),(I_n\otimes\bar{X}_4),(I_n\otimes\bar{X}_5)\}$ such that the following LMI holds:*

$$\begin{bmatrix} \bar{Q}-\bar{R} & \bar{R} & \bar{X}A_1^T+\bar{P} \\ * & -\bar{Q}-\bar{R} & \bar{X}A_2^T \\ ** & \tau^2\bar{R}-2\bar{X} & \end{bmatrix} < 0. \tag{9}$$

Moreover, the control parameter can be obtained by $D = WUSX_{11}^{-1}S^{-1}U^T$, where $\bar{X}_1$ can be expressed by singular value decomposition of the form $\bar{X}_1 = [\,V_1 \quad V_2\,]\begin{bmatrix} X_{11} & 0 \\ 0 & X_{22}\end{bmatrix}\begin{bmatrix} V_1^T \\ V_2^T\end{bmatrix}$.

Proof To establish the consensus criterion for the considered system (1), it is sufficient to show that the augmented closed-loop system (8) is stable asymptotically. To establish these sufficient conditions we choose the Lyapunov–Krasovskii functional candidate for the system (8) as

$$V(t) = \phi^T(t)P\phi(t) + \int_{t-\tau}^{t} \phi^T(s)Q\phi(s)ds + \tau\int_{t-\tau}^{t}\int_{u}^{t} \dot{\phi}^T(s)R\dot{\phi}(s)dsdu,$$

where $P = diag\{(I_n \otimes P_1), (I_n \otimes P_2), (I_n \otimes P_3), (I_n \otimes P_4), (I_n \otimes P_5)\}$, $Q = diag\{(I_n \otimes Q_1), (I_n \otimes Q_2), (I_n \otimes Q_3), (I_n \otimes Q_4), (I_n \otimes Q_5)\}$ and $R = diag\{(I_n \otimes R_1), (I_n \otimes R_2), (I_n \otimes R_3), (I_n \otimes R_4), (I_n \otimes R_5)\}$. The first derivative of $V(t)$ along the solution of the closed-loop system (8) can be easily obtained as

$$\dot{V}(t) = 2\phi^T(t)P\dot{\phi}(t) + \phi^T(t)Q\phi^T(t) - \phi^T(t-\tau)Q\phi^T(t-\tau) + \tau^2\dot{\phi}^T(t)R\dot{\phi}(t) - \tau\int_{t-\tau}^{t} \dot{\phi}^T(s)R\dot{\phi}(s)ds. \tag{10}$$

By applying Jensen's inequality, we can rewrite the only integral term in (10) as

$$-\tau\int_{t-\tau}^{t} \dot{\phi}^T(s)R\dot{\phi}(s)ds \leq -[\phi(t) - \phi(t-\tau)]^T R[\phi(t) - \phi(t-\tau)].$$

In addition, for any appropriate matrix $X = diag\{(I_n \otimes X_1), (I_n \otimes X_2), (I_n \otimes X_3), (I_n \otimes X_4), (I_n \otimes X_5)\}$, the following equation holds:

$$2\dot{\phi}^T(t)X\{A_1\phi(t) + A_2\phi(t-\tau) - \dot{\phi}(t)\} = 0.$$

Now by adding the above zero equality to (10), we obtain

$$\begin{aligned}\dot{V}(t) \leq\; & 2\phi^T(t)P\dot{\phi}(t) + \phi^T(t)Q\phi^T(t) - \phi^T(t-\tau)Q\phi^T(t-\tau) \\ & + \tau^2\dot{\phi}^T(t)R\dot{\phi}(t) - \phi^T(t)R\phi(t) \\ & + \phi^T(t-\tau)R\phi(t) + \phi^T(t)R\phi(t-\tau) \\ & - \phi^T(t-\tau)R\phi(t-\tau) + 2\dot{\phi}^T(t)X\{A_1\phi(t) \\ & + A_2\phi(t-\tau) - \dot{\phi}(t)\}.\end{aligned} \tag{11}$$

By Defining $\xi(t) = [\phi^T(t), \phi^T(t-\tau), \dot{\phi}^T(t)]^T$, we can rewrite (11) in a simplified form as

$$\dot{V}(t) \leq \xi^T(t)\Gamma\xi(t), \tag{12}$$

where

$$\Gamma = \begin{bmatrix} Q-R & R & A_1^TX + P \\ * & -Q-R & A_2^TX \\ ** & \tau^2R - 2X & \end{bmatrix}.$$

It should be noted that if $\Gamma < 0$ in (12), then the augmented closed-loop system (8) can be guaranteed to have asymptotic stability. However, it is difficult to solve Γ since Γ contains some nonlinear terms. So, we convert it into an LMI format by applying the congruence transformation. For this purpose, let us define $\bar{X}_j = X_j^{-1}$, and represent $\bar{X} = diag\{(I_n \otimes \bar{X}_1), (I_n \otimes \bar{X}_2), (I_n \otimes \bar{X}_3), (I_n \otimes \bar{X}_4), (I_n \otimes \bar{X}_5)\}$. Further, let $\bar{P} = \bar{X}P\bar{X}$, $\bar{Q} = \bar{X}Q\bar{X}$ and $\bar{R} = \bar{X}R\bar{X}$. Now, we pre- and post-multiply Γ by $diag\{\bar{X}, \bar{X}, \bar{X}\}$. During the process of pre- and post-multiplication, the term $\mathcal{C}\bar{X}_1$ in $\Gamma_{1,3}$ can be equivalently written in the form $\hat{X}_1\mathcal{C}$ by utilizing the concept of singular value decomposition (Lemma 2 in Gao et al. (2016)), where $\hat{X}_1 = USX_{11}S^{-1}U^T$. Further, by defining $W = D\hat{X}_1$, the left-hand side of (9) can be easily acheived. Therefore, $\dot{V}(t) < 0$ if LMI (9) holds, which implies that (8) is stable asymptotically according to lyapunov stability theory. In other words, condition (9) guarantees that the input delay multi-agent system (1) reaches consensus. Thus the proof of Theorem 3.1 is complete. □

In the sequel, we will extend the LMIs-based sufficient conditions obtained in Theorem 3.1 to establish some conditions which guarantee the consensus of the uncertain input delay multi-agent system via the proposed control design method. Now, by considering uncertainty terms in the input delay multi-agent system (1), the corresponding motion of the agent i can be written by

$$\begin{aligned}\dot{x}_i(t) &= (\mathcal{A} + \triangle\mathcal{A}(t))x_i(t) + (\mathcal{B} + \triangle\mathcal{B}(t))u_i(t-\tau) + Ed_i(t), \\ y_i(t) &= \mathcal{C}x_i(t),\end{aligned} \tag{13}$$

where the parameter uncertainties $\triangle\mathcal{A}(t)$ and $\triangle\mathcal{B}(t)$ are time-varying matrices with appropriate dimensions and defined as

$$\triangle\mathcal{A}(t) = MJ(t)N_a \text{ and } \triangle\mathcal{B}(t) = MJ(t)N_b. \tag{14}$$

Then, according to system (13), the augmented closed-loop system (8) can be rewritten as

$$\dot{\phi}(t) = (A_1 + \triangle A_1(t))\phi(t) + (A_2 + \triangle A_2(t))\phi(t-\tau), \tag{15}$$

where $[\triangle A_1(t) \;\; \triangle A_2(t)] = \tilde{M}J(t)[N_1 \;\; N_2]$ with $\tilde{M} = \left[(I_n \otimes M)^T \;\; (I_n \otimes M)^T \;\; 0 \;\; 0 \;\; 0\right]^T$, $N_1 = [0 \;\; (I_n \otimes N_a) \;\; 000]$ and $N_2 = [0 \;\; 0 \;\; -(I_n \otimes N_b\mathcal{C}_F) \;\; (I_n \otimes N_b\mathcal{C}_c)0]$.

Now, by adopting the derivations in Theorem 3.1, we are going to obtain sufficient conditions ensuring robust consensus of input delay multi-agent system (13), which is derived in the following theorem.

Theorem 3.2 Under the assumptions in Theorem 3.1, consider the uncertain input delay multi-agent system (13). For a given scalar $\tau > 0$, if there exist positive scalars ϵ_1, ϵ_2, real constant symmetric positive definite matrices $\bar{P}$, $\bar{Q}$, $\bar{R}$ and any appropriate dimensioned matrix $\bar{X} = diag\{(I_n \otimes \bar{X}_1), (I_n \otimes \bar{X}_2), (I_n \otimes \bar{X}_3), (I_n \otimes \bar{X}_4), (I_n \otimes \bar{X}_5)\}$ such that the following inequality holds:

$$\begin{bmatrix} \bar{\Lambda} & \Lambda \\ 0 & \bar{\epsilon} \end{bmatrix} < 0, \tag{16}$$

where $\bar{\Lambda} = \begin{bmatrix} \bar{Q} - \bar{R} & \bar{R} & \bar{X}A_1^T + \bar{P} \\ * & -\bar{Q} - \bar{R} & \bar{X}A_2^T \\ * & * & \tau^2\bar{R} - 2\bar{X} \end{bmatrix}$, $\bar{\epsilon} = diag\{-\epsilon_1 I, -\epsilon_1 I, -\epsilon_2 I, -\epsilon_2 I\}$ and $\Lambda = \begin{bmatrix} \Lambda_1^T & \Lambda_2^T & \Lambda_3^T & \Lambda_4^T \end{bmatrix}$ with $\Lambda_1 = \begin{bmatrix} \epsilon_1 \tilde{M}^T & 0 & 0 \end{bmatrix}$, $\Lambda_2 = \begin{bmatrix} 0 & 0 & N_1\bar{X} \end{bmatrix}$, $\Lambda_3 = \begin{bmatrix} 0 & \epsilon_2 \tilde{M}^T & 0 \end{bmatrix}$ and $\Lambda_4 = \begin{bmatrix} 0 & 0 & N_2\bar{X} \end{bmatrix}$, then the system (13) achieves robust consensus under the proposed controller (7). Moreover, the control parameter is computed by $D = WUSX_{11}^{-1}S^{-1}U^T$.

Proof To establish the consensus criterion for the uncertain input delay multi-agent system (13), the Lyapunov–Krasovskii functional candidate for the system (15) can be chosen as the same in Theorem 3.1. Replacing the matrices A_1 and A_2 by $A_1 + \triangle A_1(t)$ and $A_2 + \triangle A_2(t)$, respectively and using the time-varying parametric uncertain matrices given in (14), we obtain $\bar{\Lambda} + \epsilon_1 \Lambda_1^T \Lambda_1 + \epsilon_1^{-1} \Lambda_2^T \Lambda_2 + \epsilon_2 \Lambda_3^T \Lambda_3 + \epsilon_2^{-1} \Lambda_4^T \Lambda_4$. By applying Schur complement, the expression given above could be equivalently rewritten as the left-hand side of (16). Therefore if (16) is valid, then the system (15) is stable both robustly and asymptotically. Thus, the consensus of system (13) is established via the EID and Smith Predictor-based controller. Thus the proof of this theorem is completed. □

Remark 3.3 Also as it is known, time delays usually appear during the processes of information exchange of agents. The existence of time delays often degrades the system performance and even jeopardizes consensus of system. In Zuo et al. (2016), Ou et al. (2012), simple Lyapunov–Krasovskii functional candidates are considered for multi-agent systems with input delay. In this paper, we incorporate the input delay in the Lyapunov–Krasovskii functional candidates which are taken as single and double integrals. In particular, the single and double integrals helps to reduce possible conservatism in the derivation of the main results. However, the introduction of these Lyapunov–Krasovskii functional candidates will make the consensus criterion more complex, due to the increasing number of variables in the LMI. But all the computations in the paper are done off-line. However, there should be a trade-off between the implementation of Lyapunov functions and computational burden.

Numerical simulations

In order to demonstrate the effectiveness of the theoretical results and to highlight the superiority of the developed control design technique, two numerical examples with its simulation results are presented here in this section.

Example 4.1 Let us consider an input delay multi-agent system as in (1) to consist of four agents. The parameters for the considered input delay multi-agent system (1) is chosen as follows:

$$\mathcal{A} = \begin{bmatrix} -0.1 & 0.5 \\ -0.5 & -5 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \mathcal{E} = \begin{bmatrix} -0.1 & -0.5 \\ 0.1 & 0.8 \end{bmatrix}$$

and $\mathcal{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Figure 2 depicts an undirected topology of the communication among the agents. In Fig. 2 the nodes denote the agents. Now, based on the communication topology we can obtain the Laplacian matrix as

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

Moreover, the input delay of the system (1) is considered as $\tau = 0.9$ s and the disturbance vector of each agent is assumed to be $d_i(t) = 0.1[\sin(-t/8) \quad \cos(t/8)]$ for all the four agents. If the transfer function of (1) is $\mathcal{C}(tI - \mathcal{A})^{-1}\mathcal{B}$, then the transfer function of Smith-predictor $G_i(t)$ is $\mathcal{C}(tI - \mathcal{A})^{-1}\mathcal{B} - e^{-\tau t}\mathcal{C}(tI - \mathcal{A})^{-1}\mathcal{B}$. Hence correspondingly the state space matrices of $G_i(t)$ can be obtained using MATLAB as $\mathcal{A}_d = \begin{bmatrix} 0 & 0.5 \\ -0.4 & 5.5 \end{bmatrix}$, $\mathcal{B}_d = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\mathcal{C}_d = \begin{bmatrix} 1 & 0 \\ 0 & 0.75 \end{bmatrix}$. Next, we design a strictly stable first-order low-pass filter as shown in (5) so that the EID could be estimated. Correspondingly, the parameters of $F_i(t)$ are taken as $\mathcal{A}_F = \begin{bmatrix} -51 & 0 \\ 0 & -51 \end{bmatrix}$, $\mathcal{B}_F = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$ and $\mathcal{C}_F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ such that the condition $\hat{d}_{ie}(t) - \tilde{d}_{ie}(t) = 0$ holds. A very small number was chosen for $\mathcal{C}_i(t)$, so that LMI (16) is feasible. Simulations showed that the choice $\mathcal{A}_c = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}$, $\mathcal{B}_c = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$ and $\mathcal{C}_c = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix}$ guarantees feasibility of the LMIs

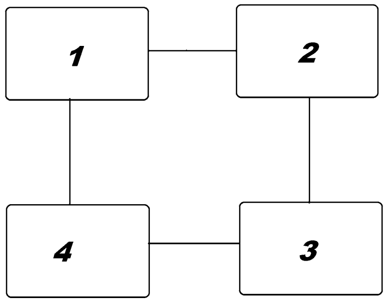


Fig. 2 Communication topology among four agents

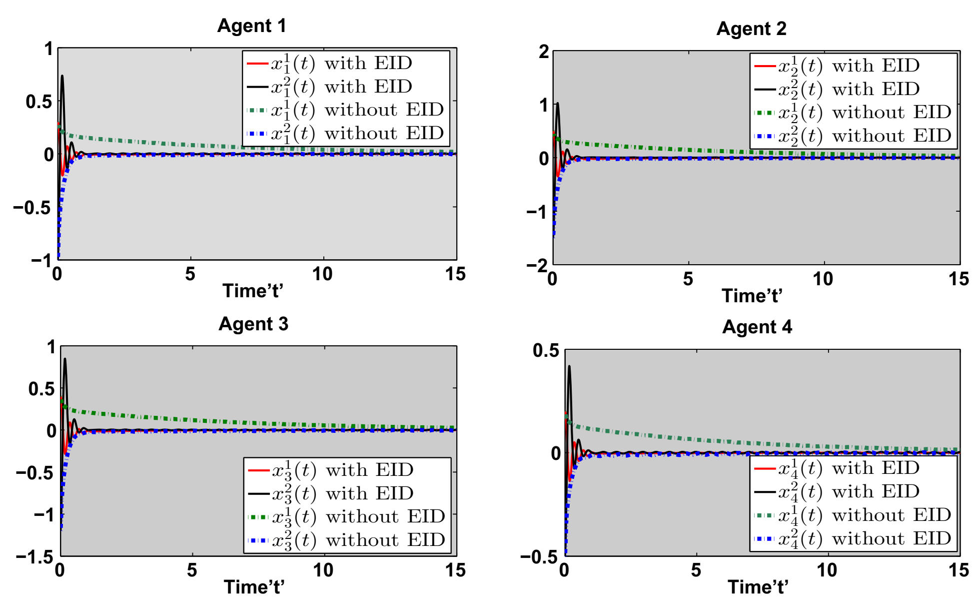
and also makes the closed-loop system (8) stable. By employing these values in the LMI of Theorem 3.1, we obtain a set of feasible solutions. Based on the obtained solutions of (9), the observer gain matrix is obtained as $D = \begin{bmatrix} 10.941 & 0 \\ 0 & 10.941 \end{bmatrix}$. By using this gain value and by selecting the initial conditions of the agents as $x_1(t) = [0.3 \;\; -1]^T$, $x_2(t) = [0.5 \;\; -1.5]^T$, $x_3(t) = [0.4 \;\; -1.2]^T$, $x_4(t) = [0.2 \;\; -0.5]^T$ and the initial conditions of their corresponding observers as $\hat{x}_i(t) = [0 \; 0.1]^T$, for $i = 1, 2, 3, 4$, we depict the state responses of the agents $x_i(t) = [x_i^1(t) \; x_i^2(t)]^T$ of the closed-loop system (8) in Fig. 3. Moreover in Fig. 3, the state responses of the agents without EID estimation is also shown. It can be observed from this Fig. 3 that the time taken for reaching consensus is effectively reduced by utilizing the EID estimator.

Further, to validate the robust consensus problem of system (13), we choose the time varying uncertainty parameters of (14) as $M = [\,0.2 \;\; 0.1\,]^T$, $F(t) = \sin(t)$, $N_a = [\,0 \;\; 0.02\,]$ and $N_b = [\,0 \;\; 0.01\,]$. By employing these values in the LMIs of Theorem 3.2, we obtain a set of feasible solutions. Based on the obtained solutions of (16), the observer gain matrix is estimated as $D = \begin{bmatrix} 11.334 & 0 \\ 0 & 11.334 \end{bmatrix}$. In Fig. 4 the state responses of the agents are represented by bold lines and the state responses of their estimates $\hat{x}_i(t) = [\hat{x}_i^1(t) \; \hat{x}_i^2(t)]^T$, are represented by dotted lines for $\imath = 1, 2, 3, 4$. From this figure, it is concluded that the state trajectories well coincides with the observer trajectories for all the four agents as time increases, by using the EID and smith predictor based controller. This implies that the robust consensus of the uncertain input delay multi-agent system (13) is attained within a very short time through the proposed controller (7).

Moreover, the estimation of disturbance $\tilde{d}_{ie}(t) = [\tilde{d}_{ie}^1(t) \;\; \tilde{d}_{ie}^2(t)]$ $(i = 1, 2, 3, 4)$ by using EID and Smith predictor-based approach, and the actual external disturbance $d_i(t) = [d_i^1(t) \;\; d_i^2(t)]$ $(i = 1, 2, 3, 4)$ acting on the dynamics (13) is shown in Fig. 5. The fact that the EID and Smith predictor based-estimator effectively rejects the external disturbances even when the system is subject to input time delay, can be observed in Fig. 5. In particular, the disturbance rejection realization, is determined with the aid of the frequency band of $F_i(t)$ in the EID and Smith predictor-based closed-loop system, which is estimated by the observer gain matrix D. To be precise, we may choose a higher frequency band for $F_i(t)$ to reject completely an external disturbance using the propsed EID and Smith predictor-based control protocol.

Figure 6 exhibits the achievement of consensus of the states of four agents. The region of attraction for various values of input time delay τ is shown in Fig. 7. Figure 7

Fig. 3 Evolution of $x_i(t)$ with and without EID



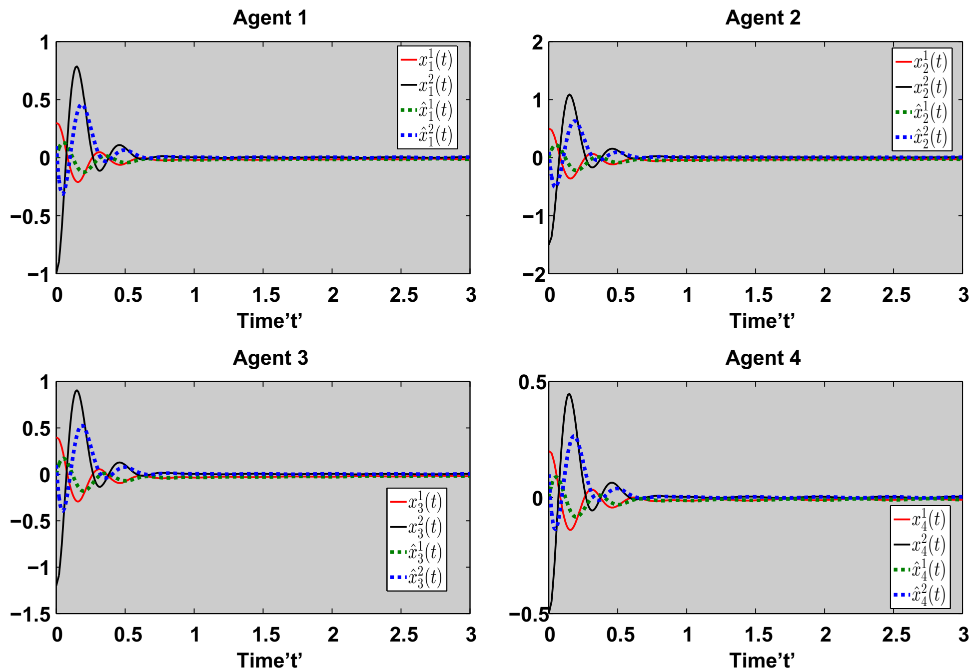


Fig. 4 Evolution of $x_i(t)$ and its estimation

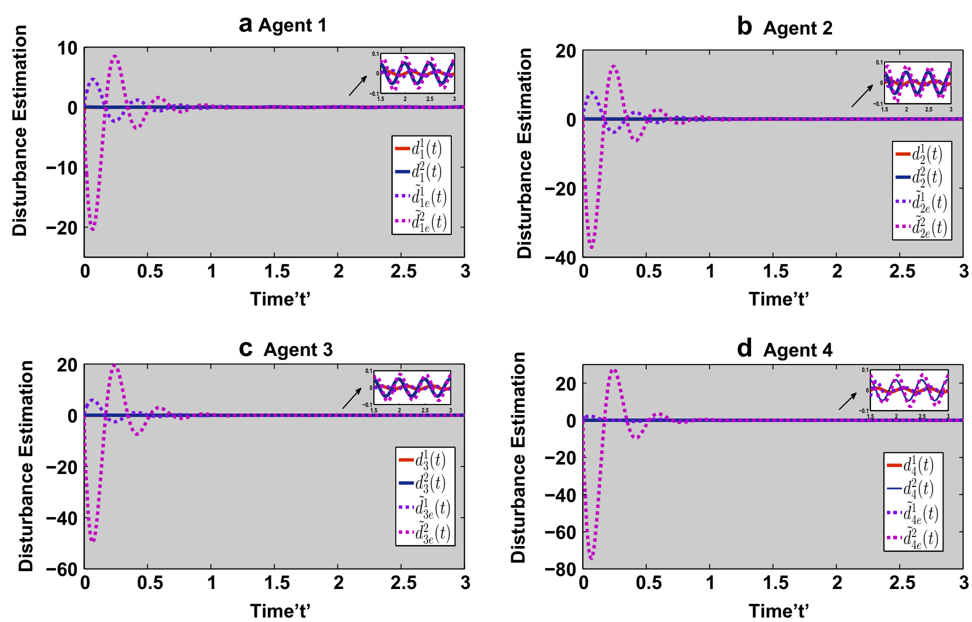


Fig. 5 Actual disturbance and the estimated EID for each agent

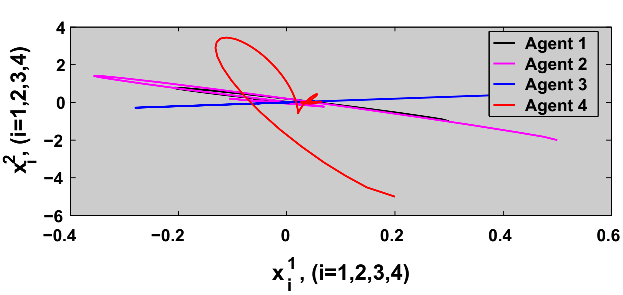


Fig. 6 Evolution of all four agents

reveals the fact that as delay bound steps up, the size of region of attraction steps down. In Fig. 8, we have depicted the region of attraction for $\tau = 0.4$ in the presence and absence of the Smith-Predictor block. It can be observed from Fig. 8 that the region of attraction increases as the effect time delay is reduced by the insertion of the Smith-Predictor block $G_i(t)$. The simulation graphs, reveal that the desired consensus of the considered multi-agent system (13) is achieved by not only considering time delay but also disturbances and uncertainties encountered by each agent,

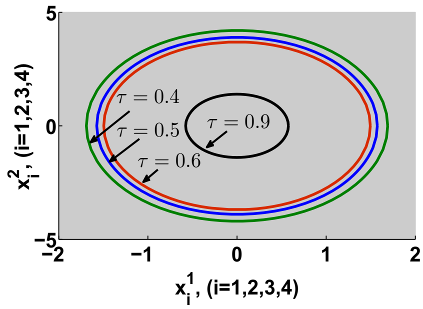


Fig. 7 Region of attraction

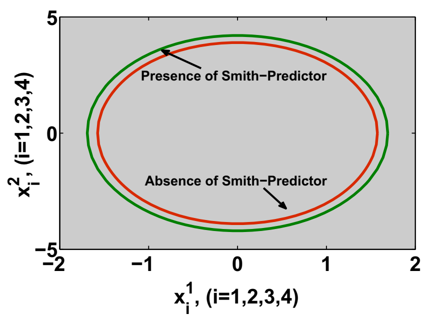


Fig. 8 Region of attraction when $\tau = 0.4$

which shows that the proposed controller (7) is more advantageous than those in the existing literature.

Example 4.2 To show the superiority of the method developed in this paper over the methods in Namerikawa and Yoshioka (2008) and Murray (2007), we provide this example along with its simulation results. Let us consider the agents as a collection of vehicles that are performing a shared task and that the vehicles are able to communicate with each other in carrying out the task. Under this scenario, we consider a four-vehicle formation problem and a two-wheel vehicle is treated as an agent. Borrowing the experimental data's from Namerikawa and Yoshioka (2008), the ith two-wheel vehicle of the multi-vehicle system takes the following form:

$$\begin{bmatrix} \dot{\tilde{r}}_{vi} \\ \dot{v}_{ei} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_{vr} & 0 \end{bmatrix} \begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t-\tau)$$

$$y_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix},$$

where $\tilde{r}_{vi}$ is the difference between the center of gravity of the virtual vehicle and the reference relative deviation for virtual vehicle system; v_{ei} is the difference between the velocity of the virtual vehicle and the reference velocity; $-k_{vr} = 0.23$ is the design parameter of the virtual vehicle. Further the maximum inter-vehicle communication delay τ was given as 0.3 s. Let us assume this multi-vehicle system is affected by external disturbances. Then the above system takes the following form:

$$\begin{bmatrix} \dot{\tilde{r}}_{vi} \\ \dot{v}_{ei} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_{vr} & 0 \end{bmatrix} \begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t-\tau) + \begin{bmatrix} 0.1 & -0.05 \\ -0.01 & -0.09 \end{bmatrix} \begin{bmatrix} sin(-t/6) \\ sin(-t/6) \end{bmatrix}$$

$$y_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{r}_{vi} \\ v_{ei} \end{bmatrix}.$$

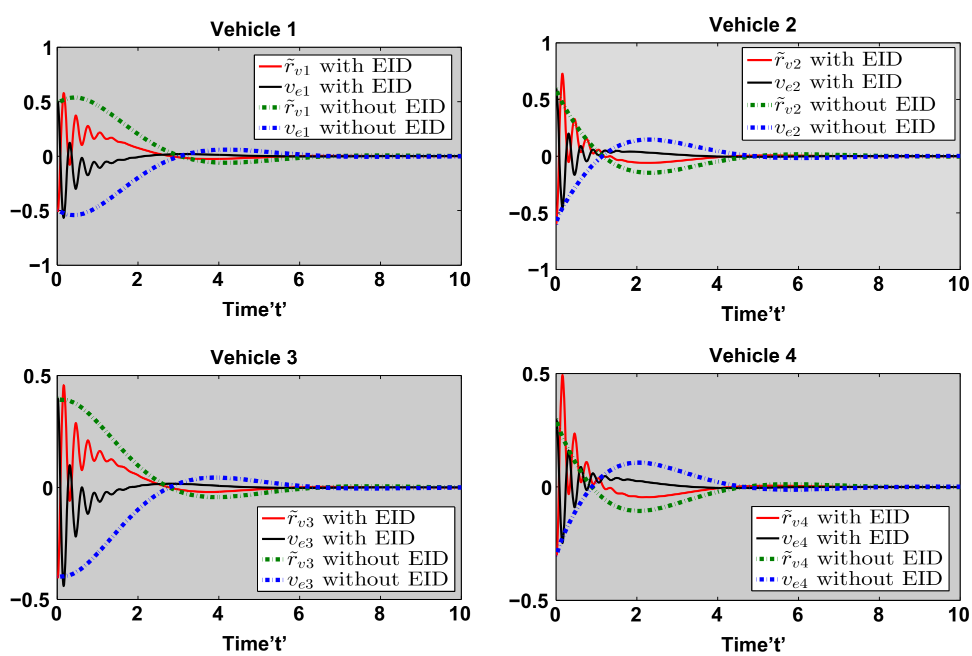


Fig. 9 Evolution of states with and without EID

By using the remaining parameters as in Example 4.1, we obtain a set of feasible solutions from the LMI of Theorem 3.1. Based on the obtained solutions of (9), the observer gain matrix is obtained as $D = \begin{bmatrix} 5.734 & 0 \\ 0 & 5.734 \end{bmatrix}$. By using this gain value and by selecting the initial conditions of the 4 vehicles as $[-0.5 \;\; -0.6]^T$, $[-0.4 \;\; -0.3]^T$, $[0.5 \; 0.6]^T$, $[0.4 \; 0.3]^T$ respectively, and the initial conditions of the observers for the 4 vehicles as $[0 \; 0.1]^T$, we depict the state responses of the vehicles with and without EID estimation in Fig. 9. It can be observed from Fig. 9 that the multi-vehicle system can reach consensus even in the presence of external disturbances by utilizing our proposed controller. It should be mentioned that the developed Smith predictor and EID-based controller not only eliminates the exogenous disturbances, but also helps to reach consensus with the proposed dynamic output feedback control strategy.

Remark 4.3 The presence of disturbances in real models is unavoidable. It is noted that a observer based control design technique has been utilized to reach consensus in Namerikawa and Yoshioka (2008), wherein the multi-agent system is not subject to uncertainties and external disturbances. However in our work, a multi-agent system subject to uncertainties and external disturbances has been considered, which is a more generalized case and hence it is more suitable for practical systems. Moreover, the robust dynamic output feedback control which is developed based on EID estimator and Smith-Predictor can actively reject the disturbances and the effect of time-delay to reach consensus of the system.

Conclusion

The robust consensus problem of input delay multi-agent systems via the controller design based on EID estimator and Smith predictor has been discussed in this paper. The proposed control protocols are developed based on the graph theory concept together with the EID based smith predictor disturbance rejection approach. The proposed protocols along with the properties of Kronecker product have stabilized the closed-loop system and thereby the problem of consensus for the uncertain multi agent system is solved. A delay-dependent criterion has been developed by using Lyapunov–Krasovskii approach to ensures the asymptotic stability of the control system which implies that the required consensus of the considered system is achieved. In particular, the EID and Smith predictor-based controller has been designed by solving the developed LMIs. At last, the presented numerical simulation demonstrate the effectiveness of the obtained theoretical results. In addition, our future research work considers the incorporation of packet losses over communication channels for stochastic nonlinear multi-agent systems with input time varying delay.

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