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Bidirectional Cognitive Computing Model for Uncertain Concepts

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Abstract

Most intelligent computing models are inspired by various human/natural/social intelligence mechanisms during the past 60 years. Achievements of cognitive science could give much inspiration to artificial intelligence. Cognitive computing is one of the core fields of artificial intelligence. It aims to develop a coherent, unified, universal mechanism inspired by human mind's capabilities. It is one of the most critical tasks for artificial intelligence researchers to develop advanced cognitive computing models. The human cognition has been researched in many fields. Some uncertain theories are briefly analyzed from the perspective of cognition based on concepts. In classical intelligent information systems, original data are collected from environment at first; usually, useful information is extracted through analyzing the input data then, it is used to solve some problem at last. There is a common characteristic between traditional machine learning, data mining, and knowledge discovery models. That is, knowledge is always transformation from data. From the point of view of granular computing, it is a unidirectional transformation from finer granularity to coarser granularity. Inspired by human's granular thinking and the cognition law of "global precedence", the human cognition process is from coarser granularity to finer granularity. Generally speaking, concepts (information and knowledge) in a higher granularity layer would be more uncertain than the ones in a lower granularity layer. A concept in a higher granularity layer would be the abstraction of some objects (data or concepts in a lower granularity layer). Obviously, there is a contradiction between the unidirectional transformation mechanism "from finer granularity to coarser granularity" of traditional intelligent information systems with the global precedence law of human cognition. That is, the human cognition are different the computer cognition for uncertain concept. The human cognition for knowledge (or concept) is based on the intension of concept, while the computing of computer (or machine) is based on the extension. In order to integrate the human cognition of "from coarser to finer" and the computer's information processing of "from finer to coarser", a new cognitive computing model, bidirectional cognitive computing model between the intension and extension of uncertain concepts, is proposed. The purpose of the paper is to establish the relationship between the human brain computing mode (computing based on intension of concept) and the machine computing mode (computing based on extension of concept) through the way of computation. The cloud model theory as a new cognition model for uncertainty proposed by Li in 1995 based on probability theory and fuzzy set theory, which provides a way to realize the bidirectional cognitive transformation between qualitative concept and quantitative data—forward cloud transformation and backward cloud transformation. Inspired by the cloud model theory, the realization of the bidirectional cognitive computing process in the proposed method is that the forward cloud transformation algorithm can be used to realize the cognitive transformation from intension to extension of concept, while the backward cloud transformation algorithm is to realize the cognitive transformation from extension to intension. In other words, the forward cloud transformation is a converter "from coarser to finer", and the backward cloud transformation is a converter "from finer to coarser". Taking some uncertain concepts as cognitive unit of simulation, several simulation experiments of the bidirectional cognition computing process are implemented in order to simulate the human cognitive process, such as cognition computing process for an uncertain concept with fixed samples, cognition computing process of dynamically giving examples, and cognition computing process of passing a concept among people. These experiment results show the validity and efficiency of the bidirectional cognitive computing model for cognition study.

Keywords Bidirectional cognitive computing model · Cognitive computing · Cloud model

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Introduction

Cognition is a group of mental processes that humans recognize the external things. The human cognition is from "unknown" to "understand" and then to "know" in general [1, 2]. In the research of the cognitive process and the mind, cognitive science examines what the cognition is, and how the cognition works. The cognitive science also studies intelligence and behavior, especially focuses on how the information is represented, processed, and transformed within the nervous systems (human or other animal) and machines (e.g., computers). In the twenty-first century, cognitive computing is an iconic emerging subject. It is based on the scientific disciplines of artificial intelligence and signal processing. Inspired by some specific intelligence observation of brain/mind law, biological law, natural law, and social law, some artificial intelligence researchers have developed many intelligent computing models and machine learning models to address complex real-world problems. They are close to human' way of reasoning, use non-exact and non-complete knowledge, and could produce control actions in an adaptive way [3]. Accompanied by the development of computer science and inspired by the artificial intelligence technologies, the cognition computing has been developing rapidly. Meanwhile, artificial intelligence with uncertainty becomes a new research field in artificial intelligence(AI) [4], in which the representation and processing of uncertain knowledge are key problems [4–7]. The uncertainties of knowledge include randomness, fuzziness, incompleteness, and inconsistency. Wherein the randomness and the fuzziness are the two most important and fundamental ones [8–13]. Most obviously, language is the carrier of the human thinking and cognition and is also the carrier of knowledge. Furthermore, the basic unit of natural language is a series of concepts with uncertainty [8]. Thus, the cognition, expressing, and processing of uncertain concepts are a fundamental problem for both artificial intelligence and human intelligence [4].

Over the past few decades, there have been many theories about randomness and fuzziness research, for example, probability theory [9], fuzzy sets [10], rough sets [13, 14], formal concept analysis [15–17], and cloud model [4, 18, 19]. The probability theory uses the possibility to represent the random uncertainty of a concept [9]. Fuzzy set uses the membership function to express fuzziness of a concept [10, 20, 21], and the computing with words was used to realize the human cognitive computing process for words [22]. Rough set uses two certain sets, that is the lower approximation set and the upper approximation set, to define an uncertain concept based on an equivalence relation (indiscernibility relation). Formal concept analysis (FCA) is building a mathematical structure in order to depict the relationship between objects and attributes based on the information in the database. FCA, which is consisted of the extension and intension of concepts, models the concepts as the units of thought, where, the extension is all objects belonging to the concept and the intension is all attributes common to all those objects. The basic notions of FCA are concepts and concept lattices. The concept lattice is to establish the conceptual level model based on the dependent or causal relationship between intension and extension of concepts [15–17]. The cloud model theory as a new cognitive model for uncertainty based on the probability theory and fuzzy set theory [18], which provides a way to realize the bidirectional cognitive transformation between qualitative concept and quantitative data—forward cloud transformation and backward cloud transformation. The forward cloud transformation can be used to transform the intension of a fuzzy qualitative concept (e.g., "the Young") into a number of random quantitative unit with different membership degrees (e.g., some ages "25, 23, 18, $\cdots$ "), while the backward cloud transformation can be used to abstract a fuzzy qualitative concept from a group random data. The mutual transformation between intension and extension of the uncertain concept is implemented [5]. With the depth study of uncertainty, the uncertainty characteristics of the world have increasingly gained universal recognition. Although many people are still engaged in the research of the certainty, it has been difficult to propose substantive question for the nature of uncertainty of the world [4].

However, we are experiencing a revolution in the capacity to quickly collect and transport large amounts of data [23–26]. Not only has this revolution changed the means by which we store and access this data but also has caused a fundamental transformation in the methods and algorithms that we use to extract knowledge from data. The central goal of the from data to knowledge (FDK) is to develop new computational methods for the analysis of large and complicated data sets, i.e., to develop methods that help humans to extract knowledge from data [27]. The transformations from data to information, knowledge, and wisdom can be represented as part of a spectrum of cognition that characterizes human competence in dealing with life's events [28]. This spectrum is hierarchical. Each transformation (e.g., events to symbols, symbols to data, data to information,$\cdots$.) represents a step up [28, 29].

In classical intelligent information systems, the original data are usually collected from environment at first, and then, the useful information is extracted through analyzing the input data in order to solve some problem. There is a common characteristic between traditional machine learning, data mining, and knowledge discovery models [3]. That is, knowledge is always a transformation from data. From the point of view of granular computing [30–33], it is a unidirectional transformation from finer granularity to coarser granularity as shown in Fig. 1. In 1980s, the

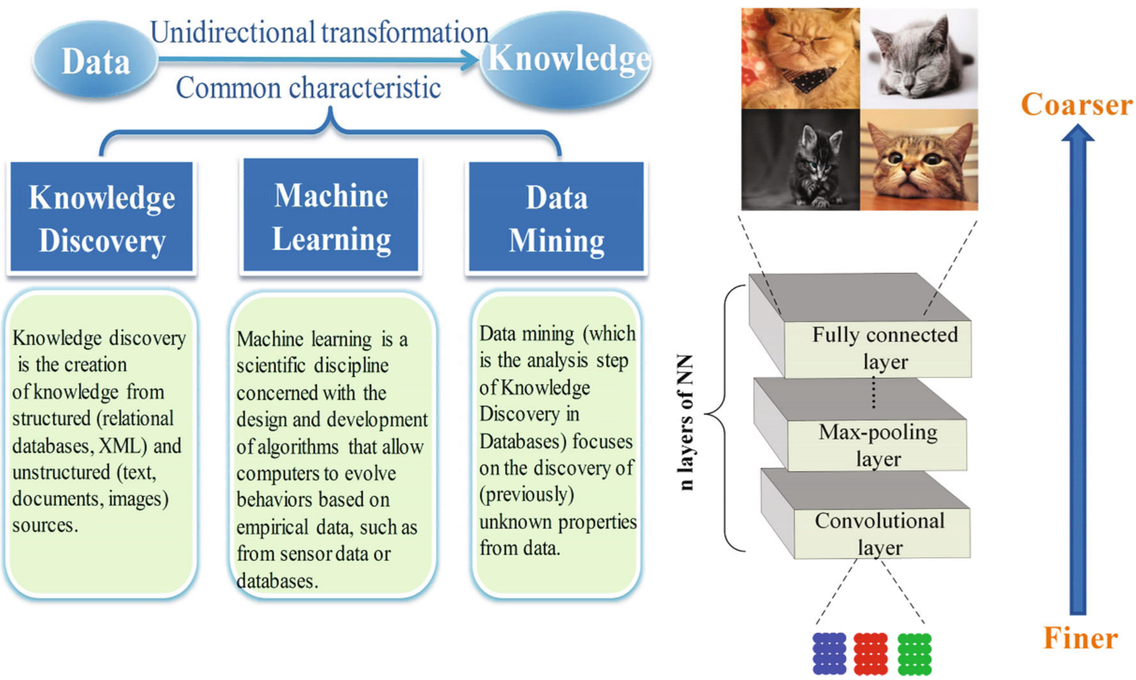


Fig. 1 Unidirectional transformation from finer granularity to coarser granularity [3]

cognitive scientist Chen developed a human cognition law called "global precedence" [34–36]. As shown in Fig. 2a [35], there are four large characters (the global level) made out of two small characters (the local level). People always recognize the large characters in the global level at first and then, the small characters in the local level. It is easy to draw and recognize a person, as shown in Fig. 2b, through his/her caricature, which has just a few lines, without analyzing detailed pixels. It is also easy for a person to recognize his/her old friend in a far distance at glance through its outline without detailed information. It shows the cognition law of the information processing in human visual perception [37]. It is a cognition process from coarser granularity to finer granularity. Generally speaking, concepts (information and knowledge) in a higher granularity layer would be more uncertain than the ones in a lower granularity layer. A concept in a higher granularity layer would be the abstraction of some objects (data or concepts in a lower granularity layer) [38, 39].

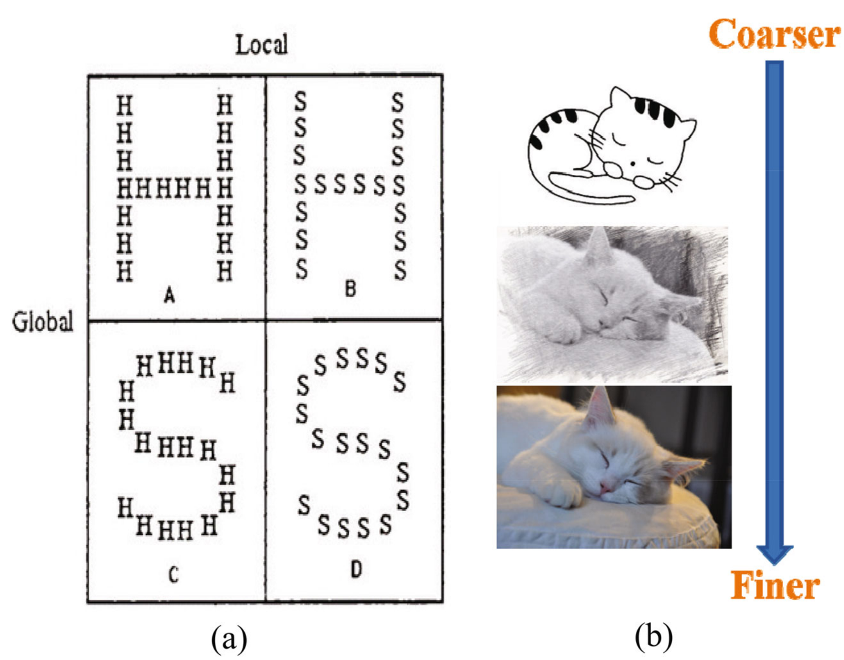


Fig. 2 Human cognition: from coarser granularity levels to finer levels [3]

Obviously, there is contradiction between the unidirectional transformation mechanism "from finer granularity to coarser granularity" of traditional intelligent information systems and the global precedence law of human cognition. That is, the human cognition for an uncertain concept is different from the computer cognition. The human cognition for knowledge (or concept) is based on the intension of concept, while the computing of computer (or machine) is based on the extension. The computer can not understand and process the intension of concept directly while the human can do. Additionally, it is impossible to store each individual (i.e., extension of concept) for human brain, while the computer can do. It would be a great challenge to integrate them into a machine learning model. For this purpose, Wang proposed the data-driven granular cognitive computing (DGCC) model [3], and he developed a general multiple granularity structure for DGCC [40]. Wang put forward many theoretical issues to be studied for implementing the DGCC model. One of the problems is to integrate human cognition of "from coarser to finer" and the information processing of "from finer to coarser" [3]. In DGCC, two kinds of transformation operators, namely upward operators and downward operators, are needed to simulate and implement such human intelligence mechanisms.

To this end, the paper proposes a bidirectional cognitive computing (BCC) model. The purpose of the current paper is twofold. First, we seek to establish the relationship between the human brain computing mode (computing based on intension of concept) and the machine computing mode (computing based on extension of concept) through the way of computation. The second objective is to illustrate the validity and efficiency of our method by the simulation realization of the BCC process. Wherein, taking some uncertain concepts as simulation unit, and using the forward cloud transformation (FCT) algorithm and the backward cloud transformation (BCT) algorithm to carry out some simulated experiments of the BCC processes, such as cognition process of a concept, cognition process of dynamically giving examples, and cognition process of passing a concept among people.

The remainder of the paper is organized as follows. A bidirectional cognitive computing model between concept extension and intension will be proposed in "Bidirectional Cognitive ComputingModel Between the Intension and Extension of a Concept". "Cloud Model and Cloud Transformation Algorithms" introduces the cloud model and the FCT and BCT algorithms. In "The Realization of Bidirectional Cognitive Process", some cognitive computing experiments will be given in detail. Final conclusion appears in "Conclusion".

Bidirectional Cognitive Computing Model Between the Intension and Extension of a Concept

Language and words are powerful tools for human thinking, but there is a fundamental difference between human intelligence and the other creatures' intelligence in the usage of the language and words [4]. Take cognition an uncertain concept "The Young" for example, there is a significant difference between the human brain and computer. The human brain is to understand "The Young" from the intension (although different people could have different understanding), while the computer is to know or understand "The Young" by extension. In other words, the human brain stores the concepts' intension, and the computer stores the concepts' extension. From the view of granular cognitive computing, the machine is always transformed (extracted) from finer granularity to coarser granularity, while in human visual perception, the cognition law of the information processing is a process from coarser to finer. Therefore, combining both the characteristics of human and computer processing of uncertain information, we propose a new model, BCC, in order to realize the mutual transformation between intension and extension of uncertain concepts in human brain and computer, namely, the bidirectional cognitive computing model of uncertain concepts (see Fig. 3). The formal definition are as follows.

Definition 1 Let $\mathcal{C}$ be an uncertain concept in a context, I and E are the intension and the extension of $\mathcal{C}$ respectively, and $\mathcal{T}$ be a cognitive transformation operator related to $\mathcal{C}$. Then the four tuple

$$(I, E, \mathcal{T}_{I\to E}, \mathcal{T}_{E\to I})$$

is called a BCC model. Where, $\mathcal{T}_{I\to E}$ is used to realize the cognitive transformation from intension to extension, while $\mathcal{T}_{E\to I}$ is from extension to intension.

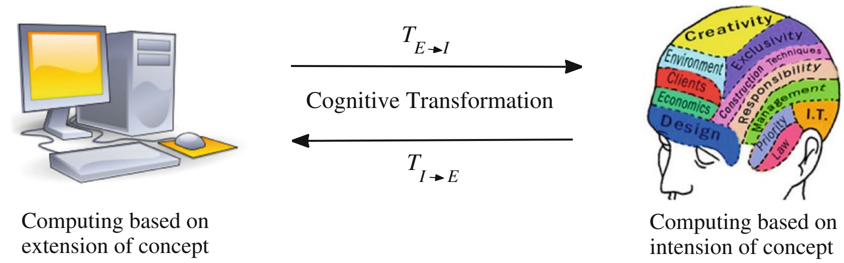


Fig. 3 Bidirectional cognitive computing model

From Definition 1, the two operators $\mathcal{T}_{I\to E}$ and $\mathcal{T}_{E\to I}$ can implement the mutual cognitive transformation between human and computers for uncertain concepts. In DGCC, two kinds of transformation operations, namely, upward operators and downward operators are needed to simulate and implement such human intelligence mechanisms. An upward operator transforms the data/ information/knowledge in a low granularity layer to a high granularity layer, while a downward operator transforms the data/information/knowledge in a high granularity layer to a low granularity layer [3]. Downward operators mimic the human cognition of "from coarser to finer", while upward operators mimic the information processing of "from finer to coarser". Therefore, the operators $\mathcal{T}_{I\to E}$ and $\mathcal{T}_{E\to I}$ could be viewed as the special downward operators and upward operators, respectively.

Cloud Model and Cloud Transformation Algorithms

In BCC, the key issue is to construct the specific implementation algorithms to realize the cognitive transformation processes. Based on the previous research achievements [5, 41–44], we found that the cloud transformation algorithms can be used to realize the functions of $\mathcal{T}_{I\to E}$ and $\mathcal{T}_{E\to I}$. So, the cloud model and the cloud transformation algorithms will be introduced in the this section. Firstly, the cloud model will be introduced specifically.

Cloud Model and Normal Cloud Model

Randomness and fuzziness are two main uncertainties in knowledge representation. Considering the fuzziness and the randomness, the cloud model, proposed by Li, interprets the randomness of membership degree based on probability statistics [18]. For example, some people think the membership degree of a 23 years old belonging to the "The Young" may be 0.8, while others think it could be 0.85 or other numerical values. In this sense, the membership degrees of uncertain concepts usually have some randomness. Although the randomness and the fuzziness are quite different in nature, people do not always care about what they express if it is randomness or fuzziness [19]. As another example, when we use the linguistic terms "about", "maybe", "probably", "sometimes," and "usually", they not only are random concepts but also embody the fuzziness due to the vague boundary of the extension. Compared with fuzzy sets, the cloud model doesn't concern a crisp membership degree, but produces random numbers automatically through computer algorithm. Different from type-2 fuzzy sets which focuses on the fuzziness of membership degrees, the cloud model discussed the randomness of them. The definition of the cloud model is as follows.

Definition 2 (**cloud**) [4]. Let U be a universal set described by precise numbers, and $\mathcal{C}$ be the qualitative concept related to U. If there is a number $x \in U$, which randomly realizes the concept $\mathcal{C}$, and the membership degree of x for $\mathcal{C}$, i.e., $\mu(x) \in [0, 1]$, is a random value with steady tendency: $\mu(x) : U \to [0, 1], \forall x \in U : x \to \mu(x)$, then, the distribution of x on U is defined as a *cloud*, and each x is defined as a *cloud drop*, noted $Drop(x, \mu)$.

In Definition 2, all the cloud drops constitute the extension of concept $\mathcal{C}$. The membership degree μ is a random number with steady tendency. It is used to express the randomness of an object x belonging to concept $\mathcal{C}$, namely, the membership degree has randomness.

In a cloud model, it uses three parameters, namely, expected value (Ex), entropy (En), and hyper entropy (He), to describe the intension of a concept, while a set of samples to describe its corresponding extension. The expected value Ex is the mathematical expectation of the cloud drops belonging to a concept $\mathcal{C}$, or it can be regarded as the core of a concept granular $\mathcal{C}$. The entropy En represents the uncertain measurement of a concept $\mathcal{C}$. It is determined by both the randomness and the fuzziness, and it is used to measure the size of a concept granular $\mathcal{C}$. The hyper entropy He is the uncertain degree of En, and it's used to depict the uncertainty of a concept granular $\mathcal{C}$.

In Definition 2, if the distribution of x on U is different, it will get the different cloud model, such as the uniform cloud based on the uniform distribution, the normal cloud based on the normal distribution, and the power-law cloud based on the power-law distribution. However, the normal cloud model, which is based on normal distribution and Gaussian membership function, is an extremely important model in the cloud model research. It has universality [45] and some successful applications in real life [46–48]. The definition of normal cloud model is as follows.

Definition 3 (**normal cloud**) [4]. Let U be a universal set described by precise numbers, and $\mathcal{C}$ be the qualitative concept containing three numerical characters (Ex,En,He) related to U. If there is a number $x \in U$, which is a random realization of the concept $\mathcal{C}$ and satisfies $x = R_N(\text{Ex}, y)$, where $y = R_N$ (En, He), and the Gaussian membership degree of x on U is

$$\mu(x) = \exp\left[-\frac{(x-\text{Ex})^2}{2y^2}\right],$$

then the distribution of x on U is a *second-order normal cloud*, simply referred to as *normal cloud*. Where, $y = R_N(\text{En}, \text{He})$ denoted a normally distributed random number with expectation En and variance He^2 ($x = R_N(\text{Ex}, y)$ has a similar meaning).

The key point in Definition 3 is the second-order relationship, i.e., within the two normal random numbers. If He=0, then, the distribution of x on U will become a normal distribution. If He=0, En=0, then, x will be a constant Ex and $\mu(x) \equiv 1$. In other words, certainty is the special case of the uncertainty.

Cloud Transformation Algorithms

The cloud model as a cognitive transformation model can realize the uncertain transformation between intension and extension of concepts through cloud transformation algorithms. For example, when we feel "hot" by perception, we usually use the quantitative data 38.1 °C, 39.5 °C, 40 °C, 41.3 °C, ..., to express it specifically; in other words, the uncertain concept "hot" (intension) are transformed into its extension (some numbers). Conversely, when the temperature reaches 38.1 °C, 39.5 °C, 40 °C, 41.3 °C,...., we feel "hot". The objective things are based on extension of concept, but human cognition is based on intension of concept, just like 35 °C and 35.1 °C are the same to our feeling. In fact, the transformation between extension and intension of concept happens around us all along. A concept is not meaning a set to people, and set operations do not happen in human thinking, the connotation of concept is more important than the set for people.

In normal cloud model, the forward cloud transformation(FCT) can realize the transformation from intension to extension, and the backward cloud transformation(BCT) realizes the transformation from extension to intension [4]. Thus, FCT and BCT could be used to realize the transformation between the computing of human brain and the computing of machine. They are introduced as follows.

(a) Forward cloud transformation algorithm

FCT algorithm transforms an uncertain concept with three numerical characters (Ex, En, He) into a number of cloud drops representing the quantitative description of the concept. According to Definition 3, the FCT algorithm is as follows.

Algorithm 1 [4] FCT algorithm.

Input: (Ex, En, He), the number of cloud drops n.
Output: n cloud drops and their membership degrees, i.e. $Drop(x_i, \mu(x_i)), i = 1, 2, \cdots, n$.
Step 1: Generate a normally distributed random number y_i with expectation En and variance He^2, i.e. $y_i = R_N(En, He)$.
Step 2: Generate a normally distributed random number x_i with expectation Ex and variance y_i^2, i.e. $x_i = R_N(Ex, y_i)$.
Step 3: Calculate Gaussian membership degree

$$\mu(x_i) = \exp\left[-\frac{(x_i - Ex)^2}{2y_i^2}\right].$$

So, x_i with certainty degree $\mu(x_i)$ is a cloud drop in the domain.
Step 4: Repeat the step 1 to step 3 until n cloud drops are generated.

For example, we use the FCT algorithm to describe human's cognition about the uncertain concept "The Young". Let the numerical characters $(25, 3, 0.3)$ express "The Young" (about 25 years old), then, "the Young" is transformed into some numbers (ages) by an FCT algorithm, shown in Fig. 4. The membership degree $\mu(x)$ is no longer a crisp membership degree. From Fig. 4, the same age belonging to "The Young" will have different memberships, so the normal cloud can better describe and express this cognitive uncertainty.

(b) Backward cloud transformation algorithm

The BCT is an algorithm based on probability statistics. Based on the cloud model, the concept granular's expression mainly uses the BCT algorithm to extract the intension of concept from sample data. Therefore, the BCT is a parameter estimation process in statistics. At present, there are four main BCT algorithms.

Liu proposed a BCT algorithm based on the sample variance and the first-order sample absolute central moment, and this method obtains the estimates $(\hat{E}x, \hat{E}n, \hat{H}e)$ from the sample data through the single step directly, so it is denoted as SBCT-1stM. The specific steps are as follows [49].

Algorithm 2 [52] SBCT-1stM.

Input: Drops(x_i), $i = 1, 2, \cdots, n$.
Output: The estimates $(\hat{E}x, \hat{E}n, \hat{H}e)$ of (Ex, En, He).
Step 1: Calculate the sample mean, the sample variance and the first-order sample absolute central moment from the random sample (drops (x_i)) $x_1, x_2, \cdots, x_n$, respectively, i.e.,

$$\hat{E}x = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i,\ S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2,$$

$$E|X - \hat{E}x| = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{X}|.$$

Step 2: According to the statistical properties of the normal cloud distribution, Liu got the equations:

$$S^2 = En^2 + He^2,\ E|X - Ex| = \sqrt{\frac{2}{\pi}}En.$$

Calculate the estimates of En and He, i.e.,

$$\hat{E}n = \sqrt{\frac{\pi}{2}} \times \frac{1}{n}\sum_{i=1}^{n}\left|x_i - \hat{E}x\right|,\quad \hat{H}e = \sqrt{S^2 - \hat{E}n^2}.$$

The SBCT-1stM algorithm can calculate the estimates $\hat{E}x, \hat{E}n, \hat{H}e$ from the sample directly, so this method is simple. But there are some problems, such as $S^2 - \hat{E}n^2 < 0$ when the sample size n is very little, that is, He will not be estimated sometimes (see the documents [43, 44]).

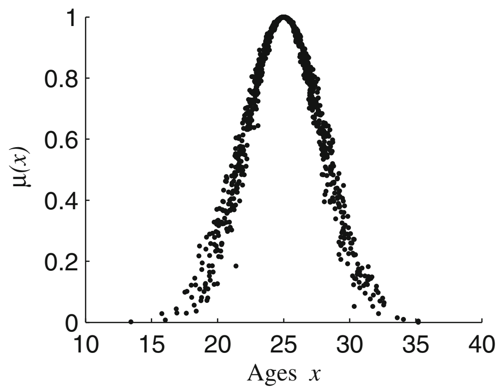


Fig. 4 "The Young" described by normal cloud model

This situation can be understood as—it will not obtain the cognitive concept when the information is very little.

In addition, Wang proposed another BCT algorithm to calculate the estimates $\hat{E}x, \hat{E}n, \hat{H}e$ directly according to the sample variance and the fourth-order sample central moment. It can be denoted as SBCT-4thM as the following [44, 50].

Algorithm 3 [53] SBCT-4thM.

Input: Drops(x_i), $i = 1, 2, \cdots, n$.
Output: The estimates $(\hat{E}x, \hat{E}n, \hat{H}e)$ of (Ex, En, He).
Step 1: Calculate the sample mean, the sample variance and the fourth-order sample central moment of cloud drops(x_i), respectively, i.e.,

$$\hat{E}x = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i,\ S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2,$$

$$\bar{\mu}_4 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^4.$$

Step 2: According to the statistical properties of the normal cloud distribution, Wang got the equations:

$S^2 = En^2 + He^2, \bar{\mu}_4 = 3(3He^4 + 6He^2En^2 + En^4)$.

Calculate the estimates of En and He, i.e.,

$$\hat{E}n = \sqrt[4]{\frac{9\left(S^2\right)^2 - \bar{\mu}_4}{6}},\ \hat{H}e = \sqrt{S^2 - \hat{E}n^2}.$$

Although the SBCT-4thM algorithm uses the fourth-order moment to calculate the estimates $\hat{E}x, \hat{E}n, \hat{H}e$, there are also the same problems with SBCT-1stM, that is, $\hat{E}n, \hat{H}e$ will not be estimated sometimes when the sample size n is little (see the document [43, 44]).

Then, Xu analyzes the shortages of the SBCT-1stM and SBCT-4thM algorithms. Two BCT algorithms (the two methods can be denoted as MBCT-SD and MBCT-SR [43, 44]) are proposed through the multi-step way based on sample division and sampling with replacement, and they are successfully applied into image segmentation. The details are as follows.

Algorithm 4 [47] MBCT-SD.

Input: Drops(x_i), $i = 1, 2, \cdots, n$.
Output: The estimates $(\hat{E}x, \hat{E}n, \hat{H}e)$ of (Ex, En, He).
Step 1: Calculate the sample mean $\hat{E}x = \frac{1}{n}\sum_{k=1}^{n} x_k$ from $x_1, x_2, \cdots, x_n$.
Step 2: Obtain the new sample $\hat{y_1^2}, \hat{y_2^2}, \cdots, \hat{y_m^2}$ from $x_1, x_2, \cdots, x_n$, that is, make the sample data $x_1, x_2, \cdots, x_n$ divide into m groups randomly, and each group will have r samples (i.e. $n = m \cdot r$ and n, m, r are positive integers). Calculate the sample variance

$$\hat{y_i^2} = \frac{1}{r-1}\sum_{j=1}^{r}(x_{ij} - \hat{E}x_i)^2$$

from each group, where, $\hat{E}x_i = \frac{1}{r}\sum_{j=1}^{r} x_{ij}$ $(i = 1, 2, \cdots, m)$. So, $\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_m$ are seen as a new random sample from a $N(En, He^2)$ distribution.
Step 3: Calculate the estimates of En^2 and He^2 from the new sample $\hat{y_1^2}, \hat{y_2^2}, \cdots, \hat{y_m^2}$. We have

$$\hat{E}n^2 = \frac{1}{2}\sqrt{4(\hat{E}Y^2)^2 - 2\hat{D}Y^2},\ \hat{H}e^2 = \hat{E}Y^2 - \hat{E}n^2.$$

Where, $\hat{E}Y^2 = \frac{1}{m}\sum_{i=1}^{m}\hat{y_i^2}$, $\hat{D}Y^2 = \frac{1}{m-1}\sum_{i=1}^{m}(\hat{y_i^2} - \hat{E}Y^2)^2$. The estimates $\hat{E}n$ and $\hat{H}e$ are obtained.

In step 2 of MBCT-SD, since the original sample $x_1, x_2, \cdots, x_n$ needs to be divided into m groups without replacement, this condition is too strong for sample size. It could not be satisfied sometimes. So the multi-step backward cloud transformation algorithm based on sampling with replacement (MBCT-SR) is proposed [43].

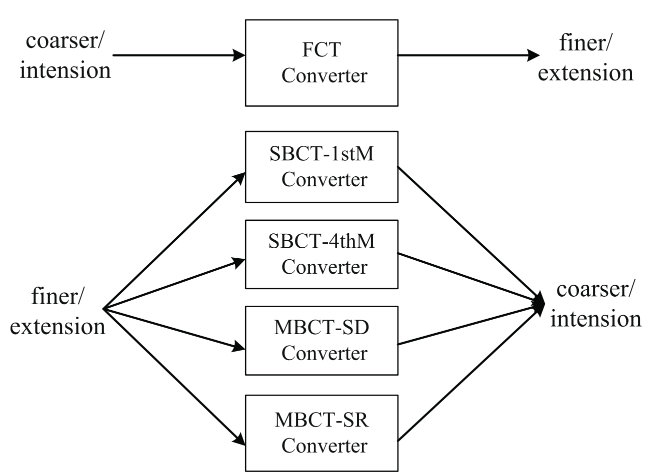


Fig. 5 FCT Converter and BCT Converter

Algorithm 5 [46] MBCT-SR.

Input: Drops(x_i), $i = 1, 2, \cdots, n$.
Output: The estimates $(\hat{E}x, \hat{E}n, \hat{H}e)$ of (Ex, En, He).

Step 1: Calculate the sample mean $\hat{E}x = \frac{1}{n}\sum_{k=1}^{n} x_k$ from $x_1, x_2, \cdots, x_n$.

Step 2: Obtain the new sample $\hat{y_1^2}, \hat{y_2^2}, \cdots, \hat{y_m^2}$ from $x_1, x_2, \cdots, x_n$, that is, drawing m groups sample with replacement from $x_1, x_2, \cdots, x_n$ randomly, and each group has r samples (n, m, r are positive integers), yielding m groups sample $x_{i1}, x_{i2}, \cdots, x_{ir}$ $(i = 1, 2, \cdots, m)$. Calculate each group sample variance

$$\hat{y_i^2} = \frac{1}{r-1}\sum_{j=1}^{r}(x_{ij} - \hat{E}x_i)^2,$$

where, $\hat{E}x_i = \frac{1}{r}\sum_{j=1}^{r} x_{ij}$ $(i = 1, 2, \cdots, m)$ is a sample mean of each group sample.

Step 3: Calculate the estimates $\hat{E}n^2$ and $\hat{H}e^2$ of En^2 and He^2 from the sample $\hat{y_1^2}, \hat{y_2^2}, \cdots, \hat{y_m^2}$ obtained by the step 2, respectively. We have

$$\hat{E}n^2 = \frac{1}{2}\sqrt{4(\hat{E}Y^2)^2 - 2\hat{D}Y^2},\ \hat{H}e^2 = \hat{E}Y^2 - \hat{E}n^2.$$

Where, $\hat{E}Y^2 = \frac{1}{m}\sum_{i=1}^{m}\hat{y_i^2}$, $\hat{D}Y^2 = \frac{1}{m-1}\sum_{i=1}^{m}(\hat{y_i^2} - \hat{E}Y^2)^2$ are the sample mean and sample variance from the sample $\hat{y_1^2}, \hat{y_2^2}, \cdots, \hat{y_m^2}$, respectively. Thus, $\hat{E}n$, $\hat{H}e$ can be obtained $(\hat{E}n > 0, \hat{H}e > 0)$.

The MBCT-SR can calculate the estimates $\hat{E}x$, $\hat{E}n$, $\hat{H}e$ for any sample size, so it has stronger adaptability.

Based on the moment estimation, the above four BCT algorithms are used to obtain the estimates $\hat{E}x$, $\hat{E}n$, and $\hat{H}e$ from different perspectives. If we think from the view of concept's cognition, the four BCT algorithms can be regarded as four different cognition algorithms. They could get the different intensions of concept for the same sample data. But the computing process of the different BCT algorithms could be considered as the different human brain' s cognitive computing process. That is, the four BCT algorithms cloud be used to simulate the four kinds of human brain's computation. In this sense, the four BCT algorithms cloud be considered as the four kinds of people with different thinking mode. The differences of BCT algorithms (defects and advantages) represent the differences of human cognition, just like different people have different viewpoints and understanding for the same thing. Thus, based on the four BCT algorithms, the different human cognition processes can be simulated.

The Realization of Bidirectional Cognitive Process

According to the introduction of the "Cloud Model and Cloud Transformation Algorithms", the FCT algorithm and the BCT algorithms can implement the function of two cognitive transformation operators $\mathcal{T}_{I\to E}$ and $\mathcal{T}_{E\to I}$ in the BCC process. So the FCT algorithm and the four BCT algorithms are used to realize several bidirectional cognition processes where, the forward cloud transformation is a converter "from coarser/intension to finer/extension", and the backward cloud transformation is a converter "from finer/extension to coarser/intension", shown as Fig. 5.

Initial Cognitive Concepts

In the cloud model, when He is small with respect to 3He ≤ En, the concept (Ex, En, He) is a relatively clear concept because the profile of cloud is relatively clear. When He is large with respect to 3He ≤ En, the concept (Ex, En, He) is a relatively uncertain concept because the profile of cloud is gradually away from the normal curve. When 3He ≥ En, the concept (Ex, En, He) becomes an atomized concept because the profile of cloud has become confusing. Thus, in order to facilitate the calculation of computer, the initial cognition concepts (ICC) are divided into three kinds of concepts roughly, that is, relatively clear concept $C(25, 3, 0.1)$, uncertain concept $C(25, 3, 0.55)$, and confusing concept $C(25, 1, 0.8)$. The cloud maps of the three concepts are shown in Fig. 6a, b, c respectively. In fact, there is no clear boundaries for the three concepts due to the

Fig. 6 Three kinds of initial cognitive concepts

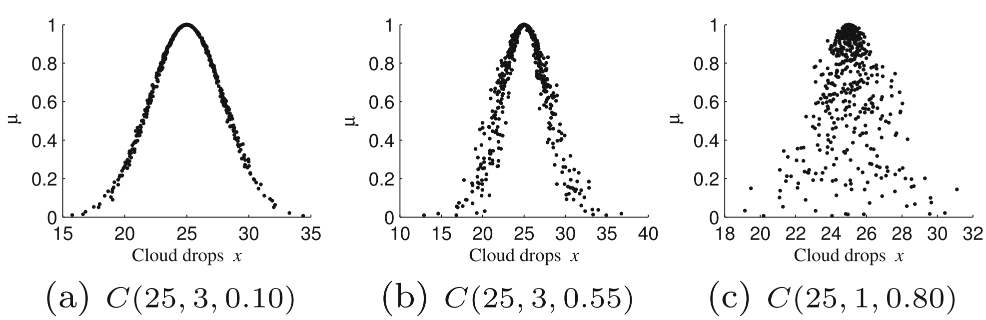


Table 1 One time cognition results of four BCTs

BCT \ ICC	$C(25, 3, 0.10)$	$C(25, 3, 0.55)$	$C(25, 1, 0.80)$
SBCT-1stM	(24.96, 2.98, 0.28)	(25.02, 3.01, 0.54)	(24.98, 1.25, 0.53)
SBCT-4thM	(24.96, 2.95, 0.21)	(25.02, 3.02, 0.52)	(24.98,1.04, 0.78)
MBCT-SD	(24.96, 3.01, 0.09)	(25.02, 3.01, 0.54)	(24.98, 0.98, 0.81)
MBCT-SR	(24.96, 2.97, 0.09)	(25.02, 2.99, 0.54)	(24.98, 1.04, 0.76)

uncertainty of concept. The different kinds of concepts are determined by the atomized feature of cloud concept [51].

If the above four BCT algorithms are regarded as the four kinds of people with different cognitive thinking, the four BCT algorithms and FCT algorithm can be used to study different bidirectional cognition processes for the given initial cognitive concepts. In the paper, we simulate three cognitive computing processes.

Cognition Process for an Uncertain Concept with Fixed Samples

Different people have different cognitive ability for the same things. For example, if there are some uncertain information about a scene, and these information is limited, it will be different cognitive results for different people based on these limited information. Abstractly, an uncertain concept with limited extension (i.e., the fixed sample quantity $n = 2000$) will have different cognitive results for different individuals. So, we firstly study the cognitive process for an uncertain concept with the fixed sample quantity from two parts: (a) One time cognition of different people for an uncertain concept; (b) Many times cognition of different people for an uncertain concept.

(a) One time cognition process for an uncertain concept

To the four different kinds of people (that is, four BCTs), one time cognition process for an uncertain concept is like this: firstly, generate the concept extension by FCT, and then, the four different kinds of people obtain a new concept intension from these extensions respectively; finally, using the cloud maps to depict the four new concepts' extensions. It is mainly to simulate what the one time cognitive results is for a given uncertain concept.

The one time cognition results are shown in Table 1. Their corresponding extensions (cloud maps) are shown in Figs. 7, 8 , and 9.

From Table 1, Figs. 7, 8, and 9, we can see that when the initial concept is quite clear ($C(25, 3, 0.10)$), the cognition results of SBCT-1stM and SBCT-4thM have some excursion, while MBCT-SD and MBCT-SR's cognition results are very similar to the initial concept $C(25, 3, 0.10)$. When the initial concept is uncertain in some degree ($C(25, 3, 0.55)$), the cognition results of SBCT-1stM, SBCT-4stM, MBCT-SD, and MBCT-SR are similar. When the initial concept is confusing ($C(25, 1, 0.80)$), SBCT-1stM's cognition result has much excursion, but the cognition results of SBCT-4thM, MBCT-SD, and MBCT-SR are similar to the initial concept $C(25, 1, 0.80)$. These results show that the cognitive difference exists indeed for different people.

(b) Many times cognition process for an uncertain concept

Although different people have different cognition for the same things, there may be some people with similar thinking mode. These people could obtain the similar cognition results for a concept. From this point of view, we want to simulate the cognition process with similar thinking mode. The one BCT algorithm is regarded as a family of people with similar thinking mode. If the same BCT algorithm are used repeatedly, the results may have some similarities.

In this section, many times cognition processes for a concept will be studied when the information quantity is fixed.

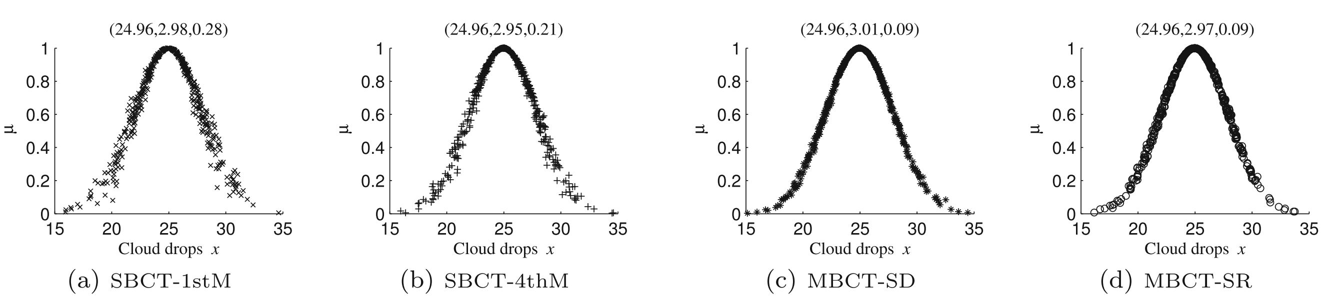
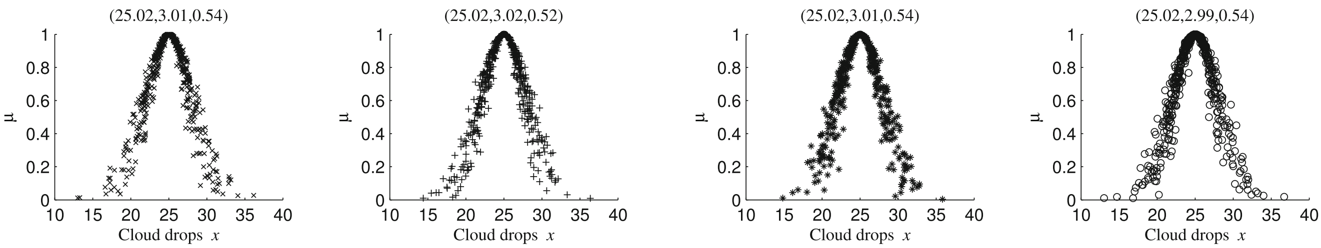
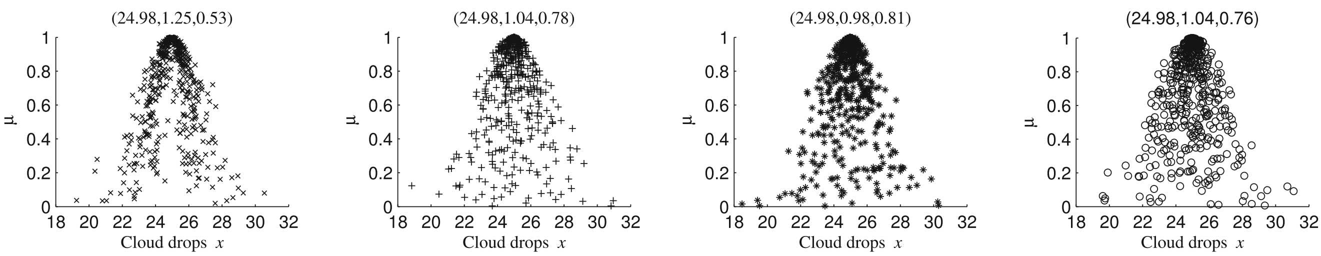


Fig. 7 One time cognition results of $C(25, 3, 0.10)$



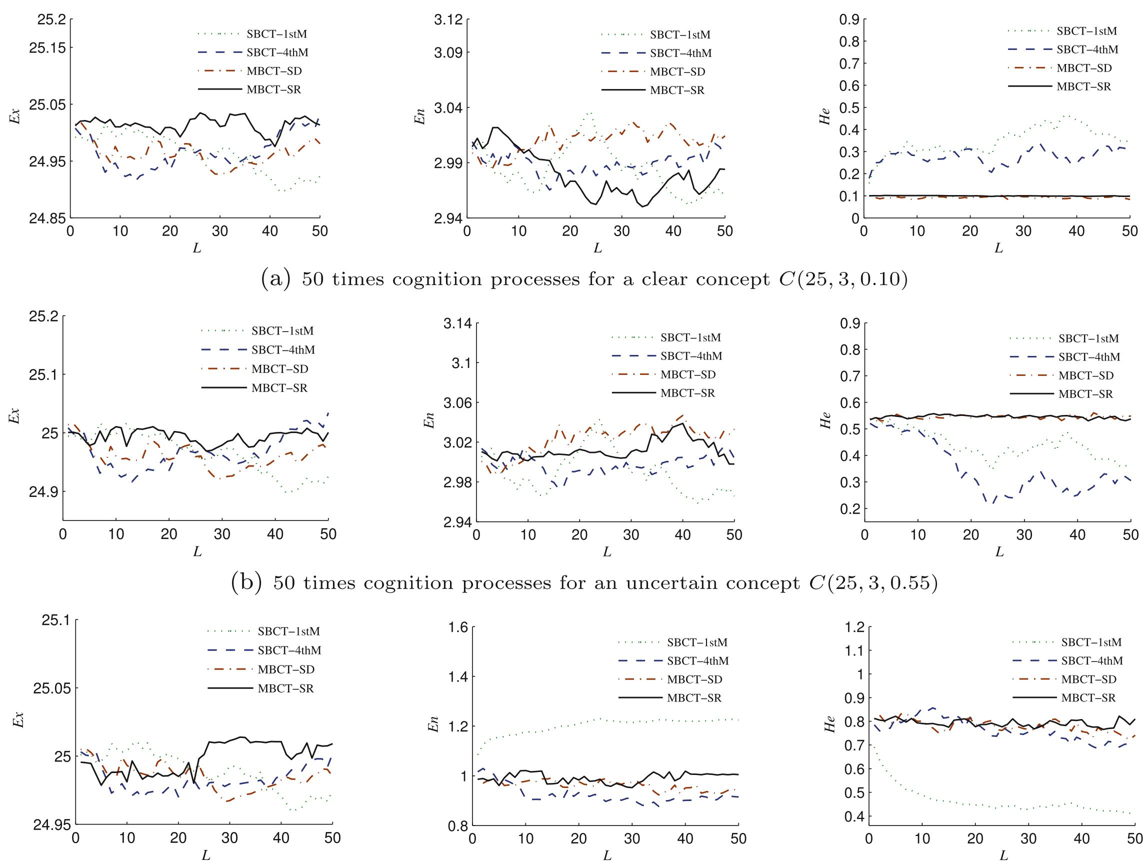
(a) SBCT-1stM (b) SBCT-4thM (c) MBCT-SD (d) MBCT-SR

Fig. 8 One time cognition results of $C(25, 3, 0.55)$



(a) SBCT-1stM (b) SBCT-4thM (c) MBCT-SD (d) MBCT-SR

Fig. 9 One time cognition results of $C(25, 1, 0.80)$



(a) 50 times cognition processes for a clear concept $C(25, 3, 0.10)$

(b) 50 times cognition processes for an uncertain concept $C(25, 3, 0.55)$

(c) 50 times cognition processes for a confusing concept $C(25, 1, 0.80)$

Fig. 10 Three different kinds of people 50 times cognition results for a concept

Firstly, as a group of 50 people, there are four groups of 200 individuals. In each group, they have similar cognition thinking mode. The result, which is obtained by using one BCT algorithm once, is regarded as one person's cognitive result. For each individual, the cognitive transformation from a concept's extension to its intension and then back to its extension is treated as a one-time cognitive process. Thus, 50 individuals' cognition for the same concept will have 50 times cognition results in each group of people where, using the FCT algorithm to realize the process from the concept's intension to its extension, and the process from the concept's extension to its intension is implemented by one BCT algorithm. The SBCT-1stM, SBCT-4thM, MBCT-SD, and MBCT-SR algorithms are used to represent the four group people's cognition transformation, respectively. So, the 50 times cognitive processes in each group can be summed up as

$$\text{a initial concept(intension)} \longrightarrow \underbrace{\xrightarrow{\text{FCT}} \text{extension} \xrightarrow{\text{BCT}} \text{intension}}_{\text{the 1st time cognitive process}} \longrightarrow \underbrace{\xrightarrow{\text{FCT}} \text{extension} \xrightarrow{BCT} \text{intension}}_{\text{the 2nd time cognitive process}} \longrightarrow \cdots \longrightarrow \underbrace{\xrightarrow{\text{FCT}} \text{extension} \xrightarrow{BCT} \text{intension}}_{\text{the 50th time cognitive process}}.$$

During the 50 times of cognition processes ($L = 1$ to 50), the change of the three numerical characters (Ex, En, He) is shown in Fig. 10a, b, c for the above three initial concepts. Some interesting phenomena can be seen from Fig. 10. If the initial concept is a quite clear concept $C(25, 3, 0.10)$ (Fig. 10a), the cognition results (Ex, En, He) have some excursion for the two groups of people (SBCT-1stM and SBCT-4thM). Especially for the hyper entropy He, the estimates $\hat{He}$ is gradually more than 0.1. While compared with the original concept $C(25, 3, 0.10)$, the cognition results (Ex, En, He) of MBCT-SD and MBCT-SR are almost no change. If the initial concept is an uncertain concept $C(25, 3, 0.55)$ in some degree (Fig. 10b), the cognition results of MBCT-SD and MBCT-SR are similar to the initial concept, while the other two groups of people (SBCT-1stM and SBCT-4thM) have a bit of excursion. Especially for the hyper entropy He, the estimate $\hat{He}$ is gradually less than 0.55. If the initial concept is a confusing concept $C(25, 1, 0.8)$ (Fig. 10c), the SBCT-1stM's cognition results have much excursion, especially for the entropy En and the hyper entropy He. But for the other three groups of people (SBCT-4tM, MBCT-SD, and MBCT-SR), there is almost no change. Despite that the entropy En and the hyper entropy He have some changes in SBCT-1stM and SBCT-4thM, the expected value Ex has almost no change in all BCT algorithms during the 50 times cognition processes. As a whole, the cognition results, which are obtained by the people with similar thinking mode, are similar.

Cognition Process of Dynamically Giving Examples

In the above two experiments, the information quantity (extension) of initial concept is fixed. If the number of cloud drops (extension) is dynamically changed from small to large, what kind of results will be generated for different people's cognition? Cognition computing process of dynamically giving examples will be simulated in this section.

Cognition of dynamically giving examples is an important source of human knowledge. It is also the main research object of machine learning. This is a process of knowledge accumulation, and also a process from "unknown" to "know". Cognition of dynamically giving examples summarizes the general rule from known example on a concept. Thus, it is a process from the particular to the general as well. Take for the cognition an uncertain concept "young people" example, at first, we don't know what is the young people. We only know people aged 24 years old is the "young people" if the experts tell us 24 years old people is

Table 2 Cognition results of dynamically giving examples for a clear concept

Initial cognition concept (Ex, En, He)—clear concept $C(25, 3, 0.10)$				
Sample size n	SBCT-1stM	SBCT-4thM	MBCT-SD	MBCT-SR
$n = 1$	(18, NaN, NaN)[a]	(18, NaN, NaN)	(18, NaN, NaN)	(18, 0, 0)
$n = 2$	(25.29, 1.23, NaN)	(25.29, 1.52, NaN)	(25.29, NaN, NaN)	(25.29, 0.77, 0.33)
$n = 10$	(24.07, 2.31, 1.02)	(24.07, 2.57, NaN)	(24.07, 2.31, 0.95)	(24.07, 2.34, 0.98)
$n = 50$	(24.92, 3.13, 0.89)	(24.92, 3.17, 0.73)	(24.92, 3.04, 0.68)	(24.92, 2.59, 0.63)
$n = 200$	(24.96, 2.85, 0.54)	(24.96, 2.85, 0.62)	(24.96, 2.87, 0.38)	(25.96, 2.91, 0.39)
$n = 400$	(25.01, 2.95, 0.48)	(25.01, 2.96, 0.49)	(25.01, 2.97, 0.21)	(25.01, 2.96, 0.22)
$n = 1000$	(25.02, 3.02, 0.31)	(25.02, 3.03, 0.22)	(25.02, 3.01, 0.09)	(25.02, 2.98, 0.09)

[a]"NaN" represents "not a number". It means that the algorithm fails to estimate the parameter

Table 3 Cognition results of dynamically giving examples for an uncertain concept

Initial cognition concept (Ex, En, He)—uncertain concept $C(25, 3, 0.55)$				
Sample size n	SBCT-1stM	SBCT-4thM	MBCT-SD	MBCT-SR
$n = 1$	(20, NaN, NaN)	(20, NaN, NaN)	(20, NaN, NaN)	(20, 0, 0)
$n = 2$	(25.87, 1.8, 0.94)	(25.87, 2.22, NaN)	(25.87, NaN, NaN)	(25.87, 1.92, 1.21)
$n = 10$	(24.37, 1.83, 0.98)	(24.37, 2.09, NaN)	(24.37, 2.05, 0.78)	(24.37, 2.14, 1.07)
$n = 50$	(24.74, 2.51, 0.83)	(24.74, 2.62, 0.91)	(24.74, 3.44, 0.83)	(24.74, 3.49, 0.88)
$n = 200$	(24.86, 2.83, 0.71)	(24.86, 2.89, 0.75)	(24.86, 3.17, 0.64)	(24.86, 3.12, 0.68)
$n = 400$	(24.91, 3.07, 0.64)	(24.91, 3.10, 0.67)	(24.91, 3.08, 0.52)	(24.91, 3.03, 0.53)
$n = 1000$	(25.02, 3.04, 0.53)	(25.02, 2.96, 0.52)	(25.02, 3.02, 0.54)	(25.02, 3.01, 0.54)

the young people. Of course, the experts could tell us that people aged 18 years old, 19 years old, 20 years old, ..., 28 years old, ..., are also the young people. So a concept about "young people" is gradually formed with the increase of information. In other words, the greater extension of concept, the smaller intension, then, the concept is more clearly. It is a learning and cognition process based on dynamically giving examples.

The experimental method: for the different initial concepts, generate different numbers of extension through FCT firstly, and then obtain the intension by four BCTs. The clear concept $C(25, 3, 0.1)$, the uncertain concept $C(25, 3, 0.55)$, and the confusing concept $C(25, 1, 0.8)$ are taken as the initial cognitive concepts. Let the numbers of the concept's extension be $n = 1$, $n = 2$, $n = 10$, $n = 50$, $n = 200$, $n = 400$, and $n = 1000$. The intensions (numerical characters) obtained by four BCTs are shown in Tables 2, 3, and 4 with the increasing of the sample size n.

Based on the results of the Tables 2, 3, and 4, the cognition processes of the four BCTs have some differences when the sample size n is increasing. When the initial concept is a clear concept $C(25, 3, 0.1)$ and the number of extension (sample) is increasing, the numerical characters obtained by MBCT-SD and MBCT-SR are gradually approaching the initial concept's numerical characters, that is, the greater extension, the smaller intension. These show that the MBCT-SD and the MBCT-SR have good cognition results, while the cognition results of SBCT-1stM and SBCT-4thM have some excursion. For instance, the third numerical character He has some excursion with respect to the initial concept (see Table 2). If the initial concepts are the uncertain concept $C(25, 3, 0.55)$ and the confusing concept $C(25, 1, 0.8)$, SBCT-1stM, SBCT-4thM, MBCT-SD, and MBCT-SR have good cognition results. This is because the uncertainty of concept is increased. However, if there are only few samples, SBCT-1stM, SBCT-4thM, and MBCT-SD may fail to obtain a concept's intension. In other words, the smaller extension, the greater intension, and even it is difficult to form the intension of the concept. This reveals that it will not reach a consensus if the sample size of the concept's extension is few, so that the intension of the concept can not be formed. With this, just the opposite is, the obtained concepts' intension are becoming more and more close to the initial concepts' intension with the increasing of the sample size n.

In order to intuitively understand the cognitive processes with the increasing of extension, the cloud maps of the clear concept obtained by SBCT-1stM, SBCT-4thM, MBCT-SD, and MBCT-SR are given in Figs. 11, 12, 13, and 14, respectively. Some cloud maps are "Null" in SBCT-1stM,

Table 4 Cognition results of dynamically giving examples for a confusing concept

Initial cognition concept (Ex, En, He)—confusing concept $C(25, 1, 0.80)$				
Sample size n	SBCT-1stM	SBCT-4thM	MBCT-SD	MBCT-SR
$n = 1$	(23, NaN, NaN)	(23, NaN, NaN)	(23, NaN, NaN)	(23, 0, 0)
$n = 2$	(26.34, 2.35, 1.23)	(26.34, 2.89, NaN)	(26.34, NaN, NaN)	(26.34, 2.13, 1.58)
$n = 10$	(25.84, 1.53, 1.06)	(25.84, 1.60, 1.02)	(25.84, 1.43, 1.08)	(25.84, 1.47, 1.12)
$n = 50$	(25.49, 1.41, 0.62)	(25.49, 1.47, 0.64)	(25.49, 1.32, 0.67)	(25.49, 1.36, 0.65)
$n = 200$	(25.09, 1.22, 0.68)	(25.09, 1.11, 0.73)	(25.09, 1.12, 0.75)	(25.09, 1.10, 0.77)
$n = 400$	(24.93, 1.15, 0.72)	(24.93, 0.89, 0.86)	(24.93, 0.97, 0.78)	(24.93, 0.95, 0.81)
$n = 1000$	(24.97, 1.09, 0.75)	(24.97, 1.09, 0.77)	(24.97, 1.04, 0.81)	(24.97, 0.98, 0.79)

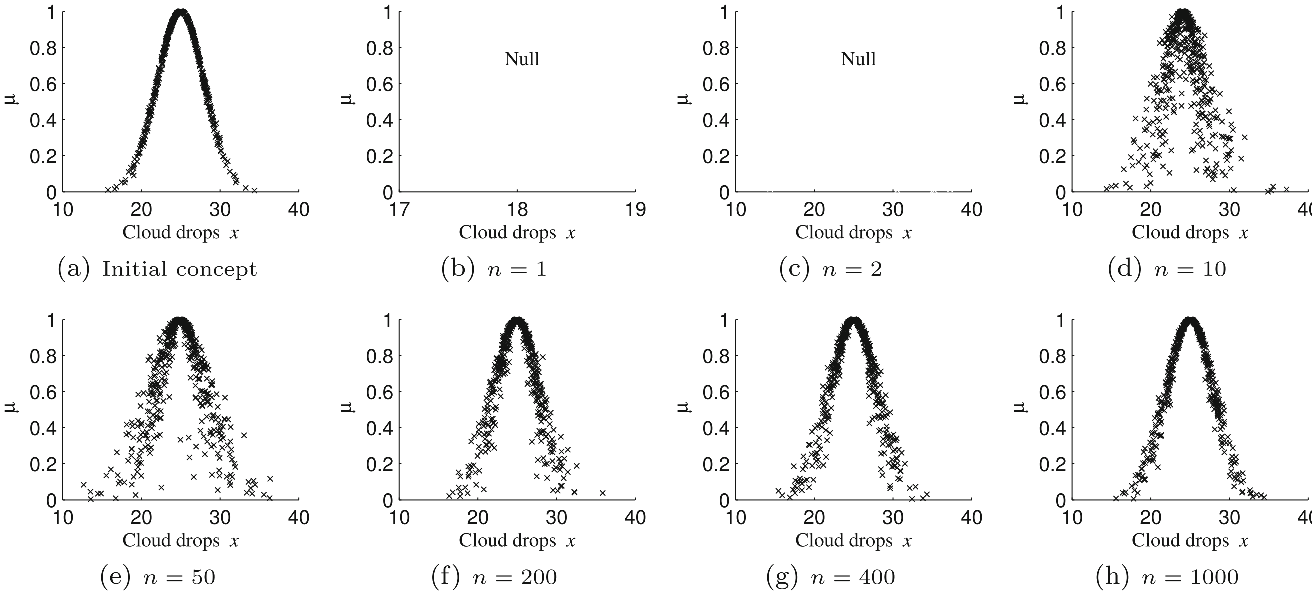


Fig. 11 SBCT-1stM's cognitive process with the increasing of sample size n for a clear concept $C(25, 3, 0.10)$

SBCT-4thM, and MBCT-SD. This is because they may fail to obtain the concept's intension when the number of extension is very few. Furthermore, the concept obtained by MBCT-SR gradually approaches the initial concept $C(25, 3, 0.1)$ with the increase of concept's extension n. The cloud maps of other concepts are omitted.

Cognition of Passing a Concept Among People

In our life, we often play passing on the message game, that is, a game of language that as many players as possible will line up such that they can whisper to their immediate neighbors but not hear any players further away. A phrase or a sentence will be told by the judges and the first player whispers it as quietly as possible to his or her neighbor. The neighbor then passes on the message to the next player to the best of his or her ability. The passing continues in this fashion until it reaches the player at the end of the line, who says to the judges the message he or she received. In this game, one person whispering a message to another and another people getting a message is a cognition process, and the messages obtained by different players may be different

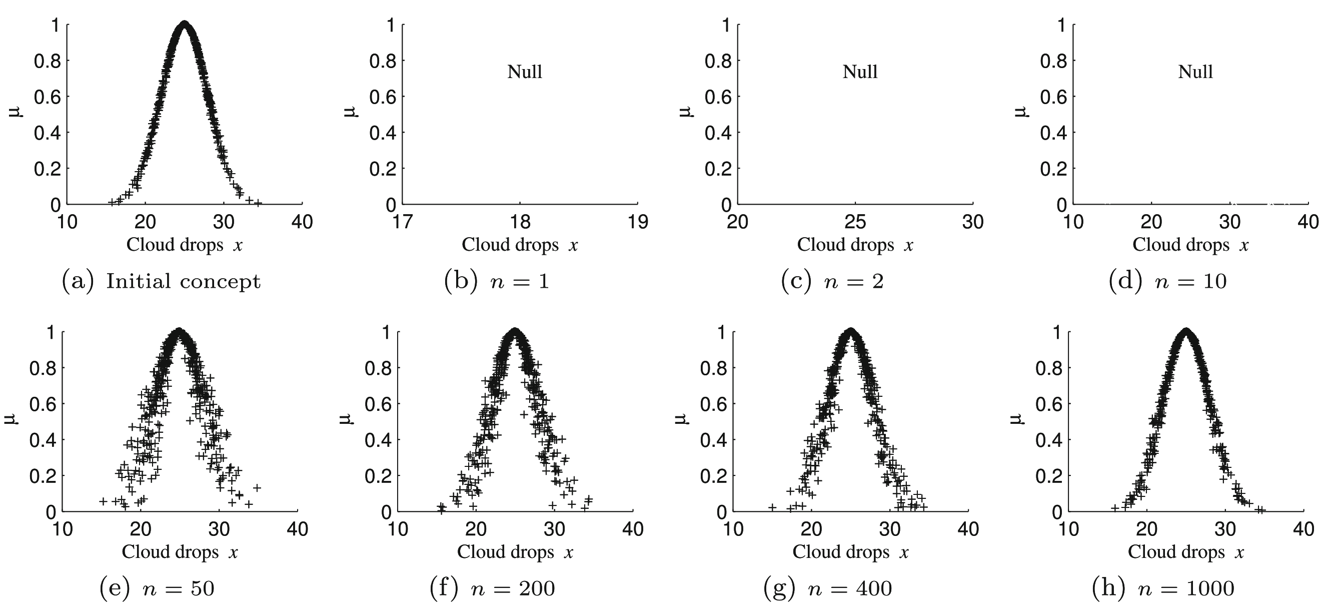


Fig. 12 SBCT-4thM's cognitive process with the increasing of sample size n for a clear concept $C(25, 3, 0.10)$

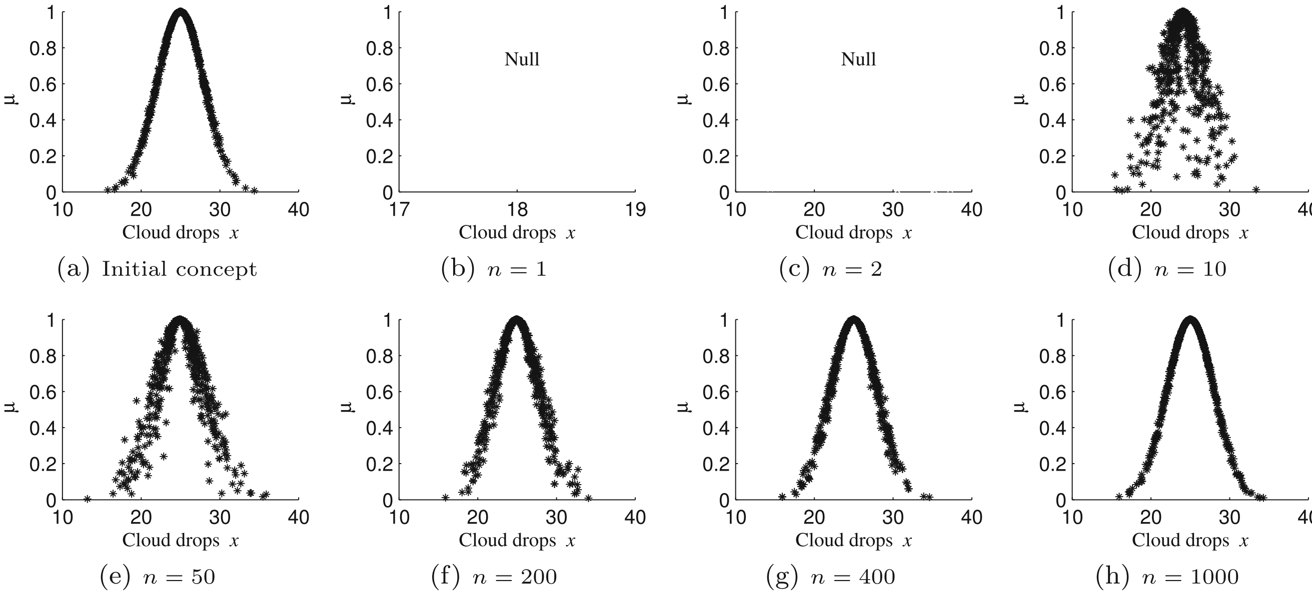


Fig. 13 MBCT-SD's cognitive process with the increasing of sample size n for a clear concept $C(25, 3, 0.10)$

due to the different players having different expression or cognition ability. In this section, we use the BCC model to imitate this process.

The experiment method: a phrase or a sentence told by the judges is expressed with different initial concepts with numerical characters (Ex, En, He), and these initial concepts are passed to players as the way of denotation, so they need to be transformed into extension by FCT, and then one player (one BCT) obtains the intension from the extension. The passing continues in this way until it reaches the last player (BCT). They are described briefly below: initial cognition concept (Ex, En, He) → BCTi → BCTj → BCTk → BCTl, $i, j, k, l = 1, 2, 3, 4$, and $i \neq j \neq k \neq l$, wherein BCT1, BCT2, BCT3, and BCT4 represent SBCT-1stM, SBCT-4thM, MBCT-SD, and MBCT-SR respectively.

If the clear concept $C(25, 3, 0.1)$, the uncertain concept $C(25, 3, 0.55)$, and the confusing concept $C(25, 1, 0.8)$ are taken as the the initial concepts, the cognition results are shown in Table 5, wherein the concepts C_1, C_2, C_3, and C_4 are obtained by SBCT-1stM, SBCT-4thM, MBCT-SD, and MBCT-SR respectively. There will be 24 different results

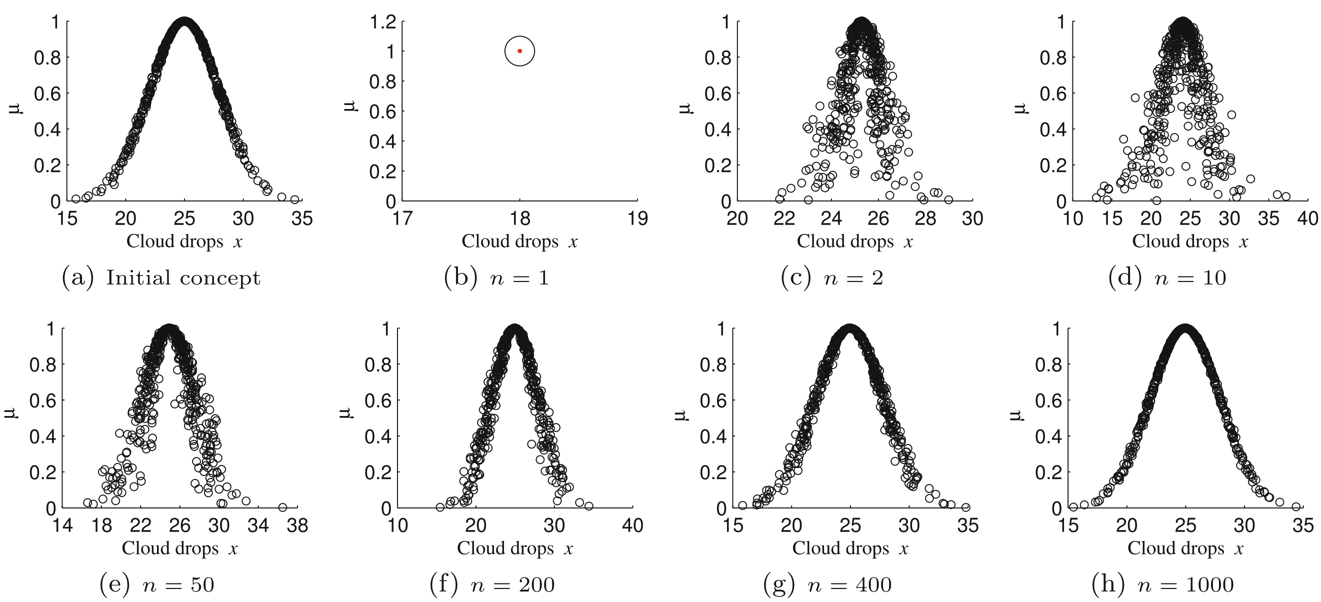
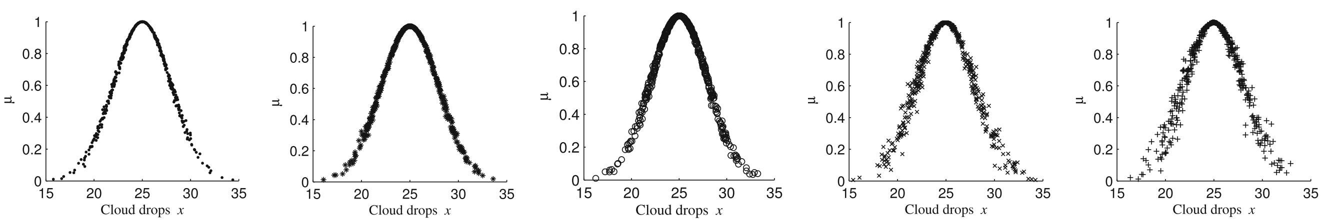


Fig. 14 MBCT-SR's cognitive process with the increasing of sample size n for a clear concept $C(25, 3, 0.10)$. Note: "Null" represents the cloud map can be formed because the corresponding concept can not be obtained

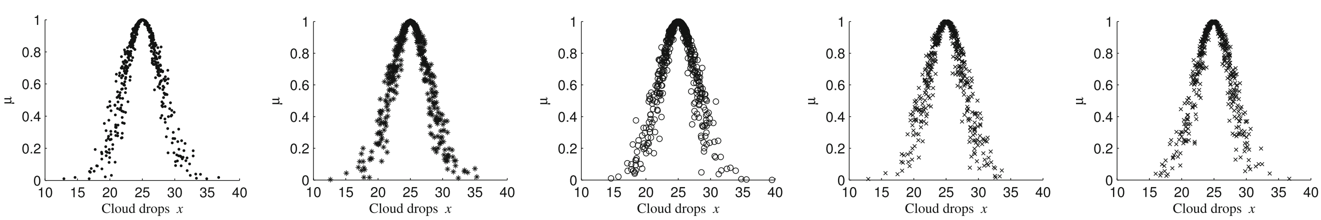
Table 5 Passing cognition results among different BCTs for different concepts

Initial cognition concept $(Ex, En, He) \rightarrow$ BCTi $\rightarrow$ BCTj $\rightarrow$ BCTk $\rightarrow$ BCTl, $i, j, k, l = 1, 2, 3, 4$, and $i \neq j \neq k \neq l$				
	$\rightarrow C1(24.99, 2.96, 0.25)$	$\rightarrow C2(25.01, 3.03, 0.27)$	$\rightarrow C3(25.02, 2.97, 0.28)$	$\rightarrow C4(24.97, 3.02, 0.27)$
	$\rightarrow C2(25.02, 2.98, 0.23)$	$\rightarrow C3(24.99, 2.97, 0.22)$	$\rightarrow C4(25.03, 3.01, 0.24)$	$\rightarrow C1(24.98, 2.97, 0.32)$
$C(25, 3, 0.10)$	$\rightarrow C3(25.01, 3.01, 0.09)$	$\rightarrow C4(25.03, 2.98, 0.11)$	$\rightarrow C1(24.97, 3.06, 0.28)$	$\rightarrow C2(24.98, 3.11, 0.35)$
	$\rightarrow C4(24.99, 2.98, 0.09)$	$\rightarrow C1(25.03, 3.03, 0.25)$	$\rightarrow C2(24.98, 3.01, 0.32)$	$\rightarrow C3(25.03, 2.97, 0.33)$
	$\rightarrow \cdots\cdots$			
	$\rightarrow C1(25.02, 2.97, 0.53)$	$\rightarrow C2(24.97, 3.03, 0.54)$	$\rightarrow C3(24.98, 2.98, 2.56)$	$\rightarrow C4(25.01, 2.99, 0.54)$
	$\rightarrow C2(24.99, 2.96, 0.56)$	$\rightarrow C3(25.03, 3.02, 0.55)$	$\rightarrow C4(24.97, 2.95, 0.54)$	$\rightarrow C1(25.03, 3.04, 0.53)$
$C(25, 3, 0.55)$	$\rightarrow C3(25.01, 2.99, 0.54)$	$\rightarrow C4(24.99, 2.98, 0.55)$	$\rightarrow C1(25.03, 3.02, 0.57)$	$\rightarrow C2(29.96, 2.97, 0.52)$
	$\rightarrow C4(25.01, 2.98, 0.56)$	$\rightarrow C1(24.97, 3.04, 0.53)$	$\rightarrow C2(25.04, 2.95, 0.52)$	$\rightarrow C3(25.03, 2.97, 0.53)$
	$\rightarrow \cdots\cdots$			
	$\rightarrow C1(24.98, 1.22, 0.75)$	$\rightarrow C2(25.06, 1.19, 0.77)$	$\rightarrow C3(25.04, 1.20, 0.78)$	$\rightarrow C4(25.02, 1.18, 0.76)$
	$\rightarrow C2(25.02, 1.03, 0.78)$	$\rightarrow C3(24.98, 0.99, 0.79)$	$\rightarrow C4(25.01, 1.02, 0.81)$	$\rightarrow C1(24.97, 1.23, 0.76)$
$C(25, 1, 0.80)$	$\rightarrow C3(24.98, 0.99, 0.81)$	$\rightarrow C4(25.03, 1.02, 0.79)$	$\rightarrow C1(25.02, 1.24, 0.75)$	$\rightarrow C2(25.04, 1.22, 0.77)$
	$\rightarrow C4(25.01, 0.98, 0.79)$	$\rightarrow C1(24.97, 1.19, 0.74)$	$\rightarrow C2(25.03, 1.16, 0.76)$	$\rightarrow C3(25.05, 1.15, 0.78)$
	$\rightarrow \cdots\cdots$			



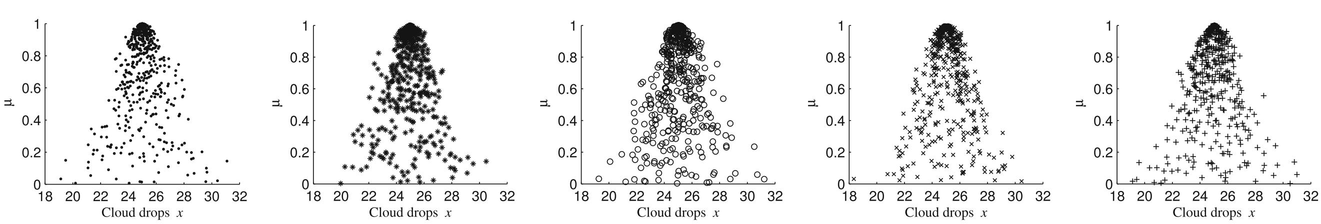
(a) Initial concept → (b) MBCT-SD → (c) MBCT-SR → (d) SBCT-1stM → (e) SBCT-4thM

Fig. 15 Passing cognition results of MBCT-SD→MBCT-SR→SBCT-1stM→SBCT-4thM for a clear concept $C(25, 3, 0.10)$



(a) Initial concept → (b) MBCT-SD → (c) MBCT-SR → (d) SBCT-1stM → (e) SBCT-4thM

Fig. 16 Passing cognition results of MBCT-SD→MBCT-SR→SBCT-1stM→SBCT-4thM for an uncertain concept $C(25,3,0.55)$



(a) Initial concept → (b) MBCT-SD → (c) MBCT-SR → (d) SBCT-1stM → (e) SBCT-4thM

Fig. 17 Passing cognition results of MBCT-SD→MBCT-SR→SBCT-1stM→SBCT-4thM for a confusing concept $C(25,1, 0.8)$

for each initial concept owing to the four BCTs (four players) having 4! = 24 kinds of permutations. Four group experiment results are only presented in Table 5 for each initial concept, and other results can be taken as well. The cloud maps from initial concept → MBCT-SD → MBCT-SR → SBCT-1stM → SBCT-4thM are shown in Figs. 15, 16, and 17 for each initial concept. Other cloud maps are omitted.

From Figs. 15 to 17, we can see that the clear concept and the uncertain concept passed among different kinds of people (BCTs) have some excursion, and there is almost no change for the confusing concept due to the initial concept can be an atomized concept (concept which can not be reached consensus), that is, the judge gives a very unclear concept to the first player so that other players obtain an atomized concept as well.

The above three experiments carry out some cognitive processes from different perspectives according to the BCC model. The experiment results are basically consistent with the human cognition.

Conclusion

It is a natural way to develop intelligent computing models with inspiration of natural/brain/social cognition laws. There are many disciplines in research about human cognition. Different disciplines have different viewpoints to study it. In this work, we introduce the cognition from artificial intelligence, as well as the cognition based on concept. And then based on the characteristics of human and computer processing information, we propose the BCC model and study the human cognition as the way of computing. As a new cognition model, cloud model can realize the uncertain transformation between intension and extension of concept. The cloud transformations are used to carry out the BCC process. At last, the different ways of human cognition are imitated through some experiments. The results show the validity and efficiency of the cloud model for the BCC model. However, we just study the human cognition from single granularity. Thus, there are some new research directions to be studied in the future: (1) multi granularity bidirectional cognitive computing might be a way to simulate the multi granularity cognition process of human being; (2) how to measure the excursion between the initial concept and the concept which it cognized; (3) for efficient knowledge discovery from big data, the data-driven granular cognitive computing can be used to study and develop big data intelligent computing algorithms and cognitive systems.

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflicts of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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