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Extensions of Intuitionistic Fuzzy Geometric Interaction Operators and Their Application to Cognitive Microcredit Origination

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Abstract
The intuitionistic fuzzy set (IFS), a popular tool to present decision makers' cognitive information, has received considerable attention from researchers. To extend the interaction operational laws in the computation of cognitive information, this paper focuses on investigating extensions of geometric interaction aggregation operators by means of the t-norm and the corresponding t-conorm under an intuitionistic fuzzy environment. We develop the extending intuitionistic fuzzy-weighted geometric interaction averaging (EIFWGIA) operator, the extending intuitionistic fuzzy-ordered weighted geometric interaction averaging (EIFOWGIA) operator, the intuitionistic fuzzy weighted geometric interaction quasi-arithmetic mean (IFWGIQAM), and the intuitionistic fuzzy-ordered weighted geometric interaction quasi-arithmetic mean (IFOWGIQAM). We investigate the properties of the proposed extensions and apply the extensions to the cognitive microcredit origination problem. For different generator functions h and ϕ, the proposed IFWGIQAM and IFOWGIQAM degenerate into existing intuitionistic fuzzy aggregation operators or extensions, some of which consider situations that in which no interactions exist between membership and non-membership functions, which can be used in more decision situations. The methods developed in this paper can be used to account for several decision situations. The numerical example demonstrates the validity of the proposed approaches by means of comparisons.

Keywords Intuitionistic fuzzy sets · The extending intuitionistic fuzzy-weighted geometric interaction averaging operator · The intuitionistic fuzzy-weighted geometric interaction quasi-arithmetic mean · Cognitive microcredit origination

Introduction

In the complex real world, decision makers rely on their cognition to present evaluation information. To model the fuzziness of the cognitive information of decision makers, many researchers [1, 2, 7, 12, 18, 44, 47, 49, 58, 63, 67, 68, 70] have investigated the fuzzy set (FS) and its extensions. Atanassov [4] and De et al. [13] investigated operational laws of intuitionistic fuzzy sets (IFSs). By means of entropy weight-based correlation coefficients, Ye [65] presented a decision-making approach with interval-valued IFSs. Li [29] proposed a multiple-attribute decision-making (MADM) method with the TOPSIS-based nonlinear programming technique. Zhu and Xu [71] proposed hesitant fuzzy linguistic preference relations with consistency measures. Beliakov et al. [5, 6] proposed alternative definitions of Atanassov's geometric operations on IFSs. Wang and Liu [56] proposed a MADM method with the intuitionistic fuzzy Einstein aggregation operators. Cognitive information has been widely used in management sciences [31, 57, 72], the design of decision support [53, 54], engineering, and other fields [22, 48, 55].

The suitable expression and effective aggregation of cognitive information [8, 9, 17, 21, 28, 32, 34, 36, 38–40, 43, 45, 64] are important in MADM. Li and Chen [30] aggregated cognitive information based on D-intuitionistic hesitant FSs. Liu and Zhang [37] proposed the Archimedean picture fuzzy linguistic-weighted arithmetic operator to fuse decision makers' cognitive information. Jiménez and Vargas discussed the future challenges in the field of cognitive MADM. Garg and Arora [20] proposed dual hesitant fuzzy soft operators to aggregate cognitive information from decision makers. Liu and Tang [35] addressed MADM with the interval neutrosophic uncertain linguistic Choquet geometric operator. Meng et al. [42] considered MADM problems with the linguistic interval hesitant fuzzy information. Liu and Li [33] developed new power Bonferroni aggregation operators with

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interval-valued IFSs, which provide creative methods to solve MADM problems.

Cognitive computation has received increased attention in MADM problems with uncertainty information. To improve the quality of decisions and take advantage of cognitive aspects, Carneiro et al. [10] included a cognitive analytic process in the MADM method. Jiménez and Vargas discussed future challenges in the field of cognitive MADM. Tao, Han, and Chen [51] developed a modified maximizing deviation decision procedure with cognitive information for MADM. Peng and Wang (2018) developed the outranking relations of Z-number cognitive information. Li and Wang [38] addressed hesitant MADM problems on the basis of possibility degrees. Zhang et al. [69] solved MADM with single-value neutrosophic information. Pires et al. [46] acquired biomedical signals wirelessly via an integrated e-healthcare system. Meng and Chen [41] solved clustering analysis problems with newly defined correlation coefficients. Tian et al. [52] addressed hesitant fuzzy linguistic MADM problems with a likelihood-based qualitative flexible approach.

To make effective decisions in the microcredit origination problem with cognitive information, considering the interactions between different IFSs in the cognitive computation process, this paper formulates the cognitive microcredit origination problem and proposes the extending intuitionistic fuzzy-weighted geometric interaction averaging (EIFWGIA) operator, the extending intuitionistic fuzzy-ordered weighted geometric interaction averaging (EIFOWGIA) operator, the intuitionistic fuzzy-weighted geometric interaction quasi-arithmetic mean (IFWGIQAM), and the intuitionistic fuzzy-ordered weighted geometric interaction quasi-arithmetic mean (IFOWGIQAM). The detailed motivations are as follows. (1) The existing intuitionistic fuzzy-weighted geometric interaction averaging (IFWGIA) operator in [23] is inconsistent with the operational laws of ordinary *FS*s ($u = 1-v$). To adapt the existing IFWGIA operator to more decision situations, this paper extends the geometric interactive operations in [23] by t-norm and t-conorm and develops the EIFWGIA operator and the EIFOWGIA operator, which include the consistent situation, as in Remark 1. (2) The existing GIFWGIA operator in [24] is also inconsistent with aggregation operations on ordinary *FS*s (when $u = 1-v$). To adapt the existing GIFWGIA operator to more decision situations, this paper extends the geometric interactive operations in [24] by the weighted quasi-arithmetic mean and develops the IFWGIQAM and the IFOWGIQAM. (3) the EIFGWIA operator, EIFOWGIA operator, IFWGIQAM, and IFOWGIQAM are applied to the rankings of cognitive microcredit origination to farmers, where the different generator functions h and ϕ are seen as the decision makers' different cognitive preferences for real situations. We discuss the detailed cases of the proposed extensions by considering various generator functions. In addition, an example is used to demonstrate the effectiveness of the proposed approaches.

The rest of this paper is structured as follows. Section 2 formulates the cognitive microcredit origination problem and reviews some related concepts. Section 3 develops the EIFWGIA and EIFOWGIA operators. Section 4 combines the proposed operators with the quasi-arithmetic means and develops the IFWGIQAM and IFOWGIQAM. We investigate special cases of the proposed extensions by considering certain generator functions. Section 5 applies the new extensions to the ordering of cognitive microcredit origination to farmers, and an illustrated example demonstrates the effectiveness of our approach. Finally, Section 6 provides some conclusions.

Preliminaries

Microcredit origination is a popular business activity around the world that involves cognitive decision-making [10, 26] and social cognition [19, 50]. For example, to expand the business of microcredit origination, the loan company will first form a professional and localized loan officer team. On the basis of their cognition and knowledge of microcredit origination, the loan officer team will define the rules for microcredit origination and define the evaluation criteria for applicants. When a set of farmers applies for microcredit from the loan company, the loan officer team will evaluate these farmers with respect to the evaluation criteria. Because of the fuzziness of complex environments, the loan officer team will rely on their cognition to evaluate the farmers. Cognitive information is expressed with IFSs in this paper. In the computation of cognitive information, the loan officer team will rely on cognition to aggregate the evaluation information. Finally, the loan officer team will rank the final aggregated values and farmers on the basis of their cognitive preferences.

Because the problem involves cognitive decision-making and social cognition, the microcredit origination problem formulated above is called the cognitive microcredit origination problem in this paper.

In the following, we briefly review the concepts of the IFS and some aggregation operators that will be used to make decisions in the cognitive microcredit origination problem under intuitionistic fuzzy environments.

Zadeh [66] first introduced the FS. Later, Atanassov [3] developed the IFS as follows.

Definition 2.1 [3]. An IFS A in X is defined as $A = \{\langle x, u_A(x), v_A(x), \pi_A(x)\rangle | x \in X\}$, where X is a fixed set and $u_A(x)$ and $v_A(x)$

are membership and non-membership functions that satisfy $0 \leq u_A(x) + v_A(x) \leq 1$ for all $x \in X$ and $\pi_A = 1 - u_A(x) - v_A(x)$.

Xu [61] denoted the intuitionistic fuzzy number (IFN) as $A = \langle u_A, v_A \rangle$. In addition, the set of all IFNs is denoted as IFNs(X) in [22].

Atanassov [4] and De et al. [13] investigated the operational laws of IFNs as follows.

Definition 2.2 [4]. Let $A = \langle u_A, v_A \rangle \in$ IFNs(X) and $B = \langle u_B, v_B \rangle \in$ IFNs(X). Then,

1) $A^C = \{\langle x, v_A(x), u_A(x) \rangle | x \in X\}$.
2) $A \subset B$ if $u_A(x) \leq u_B(x)$ and $v_A(x) \geq v_B(x)$.
3) $A \cap B = \{\langle x, \min\{u_A(x), u_B(x)\}, \max\{v_A(x), v_B(x)\} \rangle | x \in X\}$.
4) $A \cup B = \{\langle x, \max\{u_A(x), u_B(x)\}, \min\{v_A(x), v_B(x)\} \rangle | x \in X\}$.

Definition 2.3 [4, 13]. Let $A = \langle u_A, v_A \rangle \in$ IFNs(X) and $B = \langle u_B, v_B \rangle \in$ IFNs(X). Then,

$$1)\quad A \otimes B = \langle u_A u_B, v_A + v_B - v_A v_B \rangle \tag{1}$$

$$2)\quad A^\lambda = \left\langle {u_A}^\lambda, 1-(1-v_A)^\lambda \right\rangle, \lambda > 0 \tag{2}$$

Let w_i $(i = 1, 2, ..., n)$ be the weighting vector, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, with the operations in [4, 13], Xu and Yager [62] developed the intuitionistic fuzzy-weighted geometric averaging (IFWGA) and IF ordered WGA (IFOWGA) operators as follows.

Definition 2.4 [62]. Let $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, ..., n) \in$IFNs(X). The IFWGA operator is defined as

$$\text{IFWGA}_w(A_1, \cdots, A_n) = \left\langle \prod_{i=1}^{n} {u_{A_i}}^{w_i}, 1-\prod_{i=1}^{n} (1-v_{A_i})^{w_i} \right\rangle \tag{3}$$

Definition 2.5 [62]. Let $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, ..., n) \in$IFNs(X). The IFOWGA operator is defined as

$$\text{IFOWGA}_w(A_1, \cdots, A_n) = \left\langle \prod_{i=1}^{n} u_{A_{\sigma(i)}}^{w_i}, 1-\prod_{i=1}^{n} \left(1-v_{A_{\sigma(i)}}\right)^{w_i} \right\rangle \tag{4}$$

where$A_{\sigma(i)}$ is the ith largest value of A_i $(i = 1, ..., n)$.

As a complement to the above operations, He et al. [23] defined the interactive multiplication operation in Eq. (5).

$$A \hat{\otimes} B = \langle \text{PM}(u_A, u_B) - \text{PH}(u_A, v_B) - \text{PH}(u_B, v_A), \text{PN}(v_A, v_B) \rangle \tag{5}$$

where PM is the probability membership function, representing the probability of u_A and u_B occurring simultaneously; PH is the probability heterogeneous operator, representing the probability of v_B and u_A occurring simultaneously; and PN is the probability non-membership function, representing the probability of v_A and $\lambda A = \langle 1 - (1 - u_A)^\lambda, (1 - u_A)^\lambda - (1 - (u_A + v_A))^\lambda \rangle$, $\lambda > 0$ occurring simultaneously. For more detailed explanations, please refer to [23, 24].

By taking some special values for the above functions [23], such as $\text{PM}(u_A, u_B) = u_A + u_B - u_A \cdot u_B$, $\text{PH}(v_A, u_B) = u_B \cdot v_A$ and $\text{PN}(v_A, v_B) = v_A + v_B - v_A \cdot v_B$, we obtain Definition 2.6.

Definition 2.6 [23]. Let $A = \langle u_A, v_A \rangle \in$ IFNs(X) and $B = \langle u_B, v_B \rangle \in$ IFNs(X). Then,

$$1)\quad A \hat{\otimes} B = \langle u_A + u_B - u_A u_B - u_A v_B - v_A u_B, v_A + v_B - v_A v_B \rangle \tag{6}$$

$$2)\quad A^\lambda = \left\langle (1-v_A)^\lambda - (1-(u_A + v_A))^\lambda, 1-(1-v_A)^\lambda \right\rangle, \lambda > 0 \tag{7}$$

Based on the interactive operations in [23], the IFWGIA- and IF-ordered WGIA (IFOWGIA) operators were defined as Definitions 2.7 and 2.8.

Definition 2.7 [23]. Let $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, ..., n) \in$IFNs(X), and let w_i $(i = 1, 2, ..., n)$ be the weighting vector. The IFWGIA operator is defined as

$$\text{IFWGIA}_w(A_1, \cdots, A_n) = \left\langle \prod_{i=1}^{n} (1-v_{A_i})^{w_i} - \prod_{i=1}^{n} (1-(u_{A_i} + v_{A_i}))^{w_i}, 1-\prod_{i=1}^{n} (1-v_{A_i})^{w_i} \right\rangle \tag{8}$$

Definition 2.8 [23]. Let $A_i = \langle u_{A_i}, v_{A_i} \rangle$ $(i = 1, ..., n) \in$IFNs(X), and let w_i $(i = 1, 2, ..., n)$ be the weighting vector. The IFOWGIA is defined as

$$\text{IFOWGIA}_w(A_1, \cdots, A_n) = \left\langle \prod_{i=1}^{n} \left(1-v_{A_{\sigma(i)}}\right)^{w_i} - \prod_{i=1}^{n} \left(1-\left(u_{A_{\sigma(i)}} + v_{A_{\sigma(i)}}\right)\right)^{w_i}, 1-\prod_{i=1}^{n} \left(1-v_{A_{\sigma(i)}}\right)^{w_i} \right\rangle \tag{9}$$

where$A_{\sigma(i)}$ is the ith largest value of A_i $(i = 1, ..., n)$.

Chen and Tan [11] defined $S(A) = u_A - v_A$ as the score function, which is used to evaluate the degree of suitability from the decision-makers. Hong and Choi [25] defined $H(A) = u_A + v_A$ as the accuracy function, which is used to evaluate the accuracy degree of IFN.

Then, Xu and Yager [62] and Xu [58] defined the order of different IFNs as in Definition 2.9.

Definition 2.9 [61, 62]. Let $A=\langle u_A, v_A\rangle \in$ IFNs(X) and $B=\langle u_B, v_B\rangle \in$ IFNs(X). Then,

1) If $S(A)<S(B)$, then $A<B$.
2) If $S(A)>S(B)$, then $A>B$.
3) If $S(A)=S(B)$ and $H(A)<H(B)$, then $A<B$.
4) If $S(A)=S(B)$ and $H(A)>H(B)$, then $A>B$.

Alternative Definitions of *IFNs*

In this Section, the geometric interaction aggregation operators are investigated with the t-norm and t-conorm.

A. Alternative Interactive Multiplication Operation

Alternative operations on IFNs with the t-norm and t-conorm have received considerable attention [6, 8, 14–16].

Suppose that g is a strictly decreasing function $g:[0,1]\to[0,\infty]$ that satisfies $g(0)=1$ and that $g^{(-1)}(t)=\max\{0, g^{(-1)}(t)\}$ is the pseudoinverse of g. A continuous Archimedean t-norm is defined by additive generator g [27] in Eq. (10).

$$T(x,y)=g^{(-1)}(g(x)+g(y)) \tag{10}$$

Let $h(t)=g(1-t)$, and let $h^{(-1)}(t)=\min\{1, h^{(-1)}(t)\}$ be the pseudoinverse of h. The corresponding t-conorm [27] is

$$S(x,y)=h^{(-1)}(h(x)+h(y)) \tag{11}$$

Taking $g(t)=-\ln(t)$, we have $T(x,y)=xy$ and $S(x,y)=x+y-xy$.

With the above two equations, the interactive multiplication operation (Eq. (5)) is rewritten in [24] as

$$A\hat{\otimes}B=\langle S(u_A,u_B)-T(u_A,v_B)-T(v_A,u_B), S(v_A,v_B)\rangle \tag{12}$$

Eq. (5) can also be rewritten as

$$A\hat{\otimes}B=\langle S(u_A+v_A,u_B+v_B)-S(v_A,v_B), S(v_A,v_B)\rangle \tag{13}$$

We can prove that Eq. (12) is equal to Eq. (13) by the following Proposition 3.1.

Proposition 3.1. Suppose $T(x,y)$ is a continuous Archimedean t-norm and $S(x,y)$ is the corresponding dual t-conorm. Taking $g(t)=-\ln(t)$, we have

$$S(u_A,u_B)-T(u_A,v_B)-T(v_A,u_B)=S(u_A+v_A,u_B+v_B)-S(v_A,v_B) \tag{14}$$

Proof $S(u_A+v_A, u_B+v_B)-S(v_A,v_B)$

$$=(1-(1-(u_A+v_A))\cdot(1-(u_B+v_B)))-(1-(1-v_A)\cdot(1-v_B))$$

$$=(1-v_A)\cdot(1-v_B)-(1-(u_A+v_A))\cdot(1-(u_B+v_B))$$

$$=u_A+u_B-u_Au_B-u_Av_B-v_Au_B$$

$$=S(u_A,u_B)-T(u_A,v_B)-T(v_A,u_B).$$

Thus, Proposition 3.1 holds.

B. Alternative Geometric Interactive Aggregation Operators on IFNs

Let $C=\langle u_C,v_C\rangle=A\hat{\otimes}B$; by Eq. (12), we have

$$A\hat{\otimes}B=\left\langle \begin{matrix} h^{-1}(h(u_A)+h(u_B))- \\ g^{-1}(g(u_A)+g(v_B))-g^{-1}(g(v_A)+g(u_B)),\end{matrix} h^{-1}(h(v_A)+h(v_B))\right\rangle.$$

By Eq. (13), we have

$$\begin{cases} h(u_C)=h\left(h^{-1}(h(u_A+v_A)+h(u_B+v_B))-h^{-1}(h(v_A)+h(v_B))\right) \\ h(v_C)=h(v_A)+h(v_B)\end{cases} \tag{15}$$

Suppose $D=\langle u_D,v_D\rangle=A^{\lambda}$ $(\lambda>0)$; we have

$$\begin{cases} h(u_D)=h\left(h^{-1}(\lambda h(u_A+v_A))-h^{-1}(\lambda h(v_A))\right) \\ h(v_D)=\lambda h(v_A)\end{cases} \tag{16}$$

Proposition 3.2 proves Eq. (17) via mathematical induction and is used to prove Theorem 3.1.

Proposition 3.2. Let $A_i=\langle u_{A_i}, v_{A_i}\rangle$ $(i=1,\ldots,n)\in IFNs(X)$. If we denote $\langle h(u),h(v)\rangle$ as $\langle\overline{u},\overline{v}\rangle$ and deduce that $C=$ IFWGIA$(A_1,A_2,\ldots,A_n)$, then

$$\left\langle \overline{u}_C,\overline{u}_C\right\rangle=\left\langle h\left(h^{-1}\left(\sum_{i=1}^{n} w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right)-h^{-1}\left(\sum_{i=1}^{n} w_i\overline{v}_{A_i}\right)\right), \sum_{i=1}^{n} w_i\overline{v}_{A_i}\right\rangle \tag{17}$$

where $w=(w_1,w_2,\ldots,w_n)$ is the weighting vector, $w_i\in[0,1]$ and $\sum_{i=1}^{n} w_i=1$.

Proof. Eq. (17) is proved via mathematical induction.

1) When $n=1$ and $w_1=1$, by the *IFWGIA* operator in [23], we have

$$\langle u_C,v_C\rangle=IFWGIA_w(A_1)={A_1}^{w_1}=\langle w_1u_{A_1}, w_1v_{A_1}\rangle=\langle u_{A_1},v_{A_1}\rangle.$$

Then, $\begin{cases} h(u_C)=h\left(h^{-1}h(u_{A_1}+v_{A_1})-h^{-1}h(v_{A_1})\right) \\ h(v_C)=h(v_{A_1})\end{cases}$,

i.e., $\left\langle\overline{u}_C,\overline{u}_C\right\rangle=\left\langle h\left(h^{-1}\left(w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right)-h^{-1}\left(w_i\left(\overline{v}_{A_i}\right)\right)\right), w_i\overline{v}_{A_i}\right\rangle$

Thus, Eq. (17) holds for $n=1$.

2) If Eq. (17) holds for $n=k$, that is,

$$\left\langle \overline{u}_C, \overline{v}_C \right\rangle = \left\langle h\left(h^{-}\left(\sum_{i=1}^{k} w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right) - h^{-}\left(\sum_{i=1}^{k} w_i \overline{v}_{A_i}\right)\right), \sum_{i=1}^{k} w_i \overline{v}_{A_i} \right\rangle,$$

then, when $n = k+1$, let $B = w_{k+1}A_{k+1}$. By Eq. (16), we have

$$\begin{cases} h(u_B) = h\left(h^{-1}\left(w_{k+1}h\left(u_{A_{k+1}}+v_{A_{k+1}}\right)\right) - h^{-1}\left(w_{k+1}h\left(v_{A_{k+1}}\right)\right)\right) \\ h(v_B) = w_{k+1}h\left(v_{A_{k+1}}\right) \end{cases},$$

i.e.,
$$\begin{cases} \overline{u}_B = h\left(h^{-1}\left(w_{k+1}\left(\overline{u_{A_{k+1}}+v_{A_{k+1}}}\right)\right) - h^{-1}\left(w_{k+1}\overline{v}_{A_{k+1}}\right)\right) \\ \overline{v}_B = w_{k+1}\overline{v}_{A_{k+1}} \end{cases}.$$

By Eq. (15), we have

$$\begin{cases} \overline{u}_C = h\left(\begin{matrix} h^{-1}\left(\begin{matrix} h\left(h^{-}\left(\sum_{i=1}^{k} w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right) - h^{-}\left(\sum_{i=1}^{k} w_i\overline{v}_{A_i}\right) + h^{-}\left(\sum_{i=1}^{k} w_i\overline{v}_{A_i}\right)\right) \\ + h\left(h^{-1}\left(w_{k+1}\left(\overline{u_{A_{k+1}}+v_{A_{k+1}}}\right)\right) - h^{-1}\left(w_{k+1}\overline{v}_{A_{k+1}}\right) + h^{-}\left(w_{k+1}\overline{v}_{A_{k+1}}\right)\right) \end{matrix} \right) \\ -h^{-1}\left(\sum_{i=1}^{k} w_i\overline{v}_{A_i} + w_{k+1}\overline{v}_{A_{k+1}}\right) \end{matrix} \right) \\ \overline{v}_C = \sum_{i=1}^{k} w_i\overline{v}_{A_i} + w_{k+1}\overline{v}_{A_{k+1}} = \sum_{i=1}^{k+1} w_i\overline{v}_{A_i} \end{cases},$$

i.e.,
$$\begin{cases} \overline{u}_C = h\left(h^{-1}\left(h\left(h^{-}\left(\sum_{i=1}^{k} w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right)\right) + h\left(h^{-1}\left(w_{k+1}\left(\overline{u_{A_{k+1}}+v_{A_{k+1}}}\right)\right)\right)\right) - h^{-1}\left(\sum_{i=1}^{k+1} w_i\overline{v}_{A_i}\right)\right) \\ \overline{v}_C = \sum_{i=1}^{k} w_i\overline{v}_{A_i} + w_{k+1}\overline{v}_{A_{k+1}} = \sum_{i=1}^{k+1} w_i\overline{v}_{A_i} \end{cases},$$

i.e.,
$$\begin{cases} \overline{u}_C = h\left(h^{-1}\left(\sum_{i=1}^{k} w_i\left(\overline{u_{A_i}+v_{A_i}}\right) + w_{k+1}\left(\overline{u_{A_{k+1}}+v_{A_{k+1}}}\right)\right) - h^{-1}\left(\sum_{i=1}^{k+1} w_i\overline{v}_{A_i}\right)\right) \\ \overline{v}_C = \sum_{i=1}^{k+1} w_i\overline{v}_{A_i} \end{cases},$$

i.e.,
$$\begin{cases} \overline{u}_C = h\left(h^{-1}\left(\sum_{i=1}^{k+1} w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right) - h^{-1}\left(\sum_{i=1}^{k+1} w_i\overline{v}_{A_i}\right)\right) \\ \overline{v}_C = \sum_{i=1}^{k+1} w_i\overline{v}_{A_i} \end{cases}.$$

i.e., Eq. (17) holds for $n = k+1$.

Thus, Proposition 3.2 holds.

As extensions of the geometric interactive aggregation operators on IFNs in [23], we propose the EIFWGIA and EIFOWGIA operators in the following two theorems, where $w = (w_1, w_2, ..., w_n)$ is the weighting vector, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

Theorem 3.1. Let $A_i = \langle u_{A_i}, v_{A_i} \rangle \quad (i = 1, ..., n) \in \mathrm{IFNs}(X)$. The EIFWGIA operator is expressed as follows.

$$\mathrm{EIFWGIA}_w(A_1, A_2, \cdots, A_n) = \langle h^{-1}\left(\sum_{i=1}^{n} w_i h(u_{A_i}+v_{A_i})\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h(v_{A_i})\right), h^{-1}\left(\sum_{i=1}^{n} w_i h(v_{A_i})\right)\rangle \tag{18}$$

Proof. We may suppose $\mathrm{EIFWGIA}_w(A_1, A_2, ..., A_n) = \langle u_C, v_C \rangle$. By Proposition 3.2, we have $\langle u_C, \overline{u}_C \rangle = \langle h(h^{-1}\left(\sum_{i=1}^{n} w_i(\overline{u_{A_i}+v_{A_i}})\right) - h^{-1}\left(\sum_{i=1}^{n} w_i\overline{v}_{A_i}\right)), \sum_{i=1}^{n} w_i\overline{v}_{A_i} \rangle$.

Thus,

$$\begin{aligned} \langle u_C, v_C \rangle &= \left\langle h^{-1}\left(\overline{u}_C\right), h^{-1}\left(\overline{v}_C\right) \right\rangle \\ &= \left\langle h^{-1}\left(\sum_{i=1}^{n} w_i\left(\overline{u_{A_i}+v_{A_i}}\right)\right) - h^{-1}\left(\sum_{i=1}^{n} w_i\overline{v}_{A_i}\right), h^{-1}\left(\sum_{i=1}^{n} w_i\overline{v}_{A_i}\right) \right\rangle \\ &= \left\langle h^{-1}\left(\sum_{i=1}^{n} w_i h(u_{A_i}+v_{A_i})\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h(v_{A_i})\right), h^{-1}\left(\sum_{i=1}^{n} w_i h(v_{A_i})\right) \right\rangle \end{aligned}$$

Theorem 3.2. Let $A_i = \langle u_{A_i}, v_{A_i} \rangle \quad (i = 1, ..., n) \in IFNs(X)$. The EIFOWGIA operator is expressed as follows.

$$\mathrm{EIFOWGIA}_w(A_1, A_2, \cdots, A_n) = \langle h^{-1}\left(\sum_{i=1}^{n} w_i h\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h\left(v_{A_{\sigma(i)}}\right)\right), h^{-1}\left(\sum_{i=1}^{n} w_i h\left(v_{A_{\sigma(i)}}\right)\right)\rangle \tag{19}$$

Proof. Similar to Theorem 3.1 and omitted here.

Remark 1. If we take $h = h^{-1} = Id$, then

$$\begin{aligned}&\mathrm{EIFWGIA}_w(A_1, A_2, \cdots, A_n)\\ &= \left\langle h^{-1}\left(\sum_{i=1}^{n} w_i h(u_{A_i} + v_{A_i})\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h(v_{A_i})\right), h^{-1}\left(\sum_{i=1}^{n} w_i h(v_{A_i})\right)\right\rangle\\ &= \left\langle \sum_{i=1}^{n} w_i(u_{A_i} + v_{A_i}) - \sum_{i=1}^{n} w_i v_{A_i}, \sum_{i=1}^{n} w_i v_{A_i}\right\rangle = \left\langle \sum_{i=1}^{n} w_i u_{A_i}, \sum_{i=1}^{n} w_i v_{A_i}\right\rangle\end{aligned} \tag{20}$$

Remark 1 shows that, on ordinary FSs, the EIFWGIA operator includes a form that is consistent with the aggregation operations in the case where $u = 1–v$. For example, take $A = <0.4, 0.4>$ and $B = <0.3, 0.5>$ with weights $w = (0.3, 0.7)$. Then, as the weighted arithmetic means, we have $\mathrm{WAM}_w(0.4, 0.3) = 0.33$ and $\mathrm{WAM}_w(0.4, 0.5) = 0.47$. In addition, by Eq. (20), we have $\mathrm{EIFWGIA}_w(A_1, A_2, \cdots, A_n) = \langle 0.33, 0.47\rangle$. Thus, Eq. (20) is consistent with aggregation operations on FSs.

Remark 2. Similar to Remark 1, if we take $h = h^{-1} = Id$, we have

$$\begin{aligned}&\mathrm{EIFOWGIA}_w(A_1, A_2, \cdots, A_n)\\ &= \left\langle \sum_{i=1}^{n} w_i u_{A_{\sigma(i)}}, \sum_{i=1}^{n} w_i v_{A_{\sigma(i)}}\right\rangle\end{aligned} \tag{21}$$

Similar to Eq. (20), we have that Eq. (21) is consistent with ordered aggregation operations on FSs.

The IFWGIQAM and IFOWGIQAM

The IFWGIQAM and IFOWGIQAM are defined and investigated in this section.

Definition 4.1 [24]. Let $A_i = \langle u_{A_i}, v_{A_i}\rangle \quad (i = 1, \ldots, n) \in \mathrm{IFNs}(X)$ and $\lambda > 0$. $w = (w_1, w_2, \ldots, w_n)$ is the corresponding weighting vector, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. The generalized IFWGIA (GIFWGIA) operator is defined in Eq. (22).

$$\begin{aligned}\mathrm{GIFWGIA}_\lambda(A_1, \cdots A_n) = &\left\langle 1 - \begin{pmatrix} 1 - \prod_{i=1}^{n}\left(1-(1-u_{A_i})^\lambda + (1-(u_{A_i}+v_{A_i}))^\lambda\right)^{w_i} \\ + \prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\end{pmatrix}^{1/\lambda}, \right.\\ &\left. \begin{pmatrix} 1 - \prod_{i=1}^{n}\left(1-(1-u_{A_i})^\lambda + (1-(u_{A_i}+v_{A_i}))^\lambda\right)^{w_i} \\ + \prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda \cdot w_i}\end{pmatrix}^{1/\lambda} - \prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{w_i}\right\rangle\end{aligned} \tag{22}$$

Let ϕ be a strictly monotone continuous generating function, $\phi : [0, 1] \to [-\infty, \infty]$. The weighted quasi-arithmetic mean is expressed as

$$\mathrm{QAM}_w(x) = \phi^{-1}\left(\sum_{i=1}^{n} w_i \phi(x_i)\right) \tag{23}$$

If we view the GIFWGIA operator as a specific case of the IFWGIQAM for $\phi(t) = 1 - (1 - t)^\lambda$, $\lambda > 0$, then the IFWGIQAM is defined as follows.

Definition 4.2. Let $A_i = \langle u_{A_i}, v_{A_i}\rangle \quad (i = 1, \ldots, n) \in \mathrm{IFNs}(X)$, $w = (w_1, w_2, \ldots, w_n)$ be the corresponding weighting vector, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1. \lambda > 0$. The IFWGIQAM is defined as follows.

$$\begin{aligned}&\mathrm{IFWGIQAM}_w(A_1, A_2, \ldots, A_n)\\ &= \left\langle \phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}))\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}) - \phi(u_{A_i}))\right)\right)\right.\\ &\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}))\right)\right) - \phi^{-1}\\ &\left.\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}))\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}) - \phi(u_{A_i}))\right)\end{pmatrix}\right\rangle,\end{aligned} \tag{24}$$

where ϕ is a strictly monotone continuous generating function, $\phi : [0, 1] \to [-\infty, \infty]$, and h is a continuous Archimedean t-conorm.

Proposition 4.1. If $\phi(t) = 1 - (1 - t)^\lambda$, $\lambda > 0$ and $h(t) = -\ln(1 - t)$, then the IFWGIQAM is reduced to the GIFWGIA operator.

Proof $\mathrm{IFWGIQAM}_w(A_1, A_2, \ldots, A_n)$

$$\begin{aligned}&= \left\langle \phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}))\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}) - \phi(u_{A_i}))\right)\right),\right.\\ &\left.\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}))\right)\right) - \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}))\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h(\phi(u_{A_i}+v_{A_i}) - \phi(u_{A_i}))\right)\end{pmatrix}\right\rangle,\\ &= \left\langle \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left(1-(1-(u_{A_i}+v_{A_i}))^\lambda\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left((1-u_{A_i})^\lambda - (1-(u_{A_i}+v_{A_i}))^\lambda\right)\right)\end{pmatrix},\right.\\ &\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h\left(1-(1-(u_{A_i}+v_{A_i}))^\lambda\right)\right)\right)\\ &\left.-\phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left(1-(1-(u_{A_i}+v_{A_i}))^\lambda\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left((1-u_{A_i})^\lambda - (1-(u_{A_i}+v_{A_i}))^\lambda\right)\right)\end{pmatrix}\right\rangle\\ &= \left\langle \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i \ln\left((1-(u_{A_i}+v_{A_i}))^\lambda\right)^{-1}\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i \ln\left(1-(1-u_{A_i})^\lambda + (1-(u_{A_i}+v_{A_i}))^\lambda\right)^{-1}\right)\end{pmatrix},\right.\\ &\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i \ln\left((1-(u_{A_i}+v_{A_i}))^\lambda\right)^{-1}\right)\right)\\ &\left.-\phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i \ln\left((1-(u_{A_i}+v_{A_i}))^\lambda\right)^{-1}\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i \ln\left(1-(1-u_{A_i})^\lambda + (1-(u_{A_i}+v_{A_i}))^\lambda\right)^{-1}\right)\end{pmatrix}\right\rangle\end{aligned}$$

$$=\left\langle \phi^{-1}\begin{pmatrix} h^{-1}\left(\ln\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)^{-1}\right) \\ -h^{-1}\left(\ln\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right)^{-1}\right)\end{pmatrix},\right.$$

$$\phi^{-1}\left(h^{-1}\left(\ln\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)^{-1}\right)\right)$$

$$\left. -\phi^{-1}\begin{pmatrix} h^{-1}\left(\ln\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)^{-1}\right) \\ -h^{-1}\left(\ln\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right)^{-1}\right)\end{pmatrix}\right\rangle$$

$$=\left\langle \phi^{-1}\begin{pmatrix} \left(1-\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\right) \\ -\left(1-\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right)\right)\end{pmatrix},\right.$$

$$\phi^{-1}\left(1-\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\right)$$

$$\left. -\phi^{-1}\begin{pmatrix} \left(1-\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\right) \\ -\left(1-\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right)\right)\end{pmatrix}\right\rangle$$

$$=\left\langle \phi^{-1}\begin{pmatrix} \left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right) \\ -\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\end{pmatrix},\right.$$

$$\phi^{-1}\left(1-\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\right)$$

$$\left. -\phi^{-1}\begin{pmatrix} \left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right) \\ -\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\end{pmatrix}\right\rangle$$

$$=\left\langle 1-\begin{pmatrix} 1-\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right) \\ +\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\end{pmatrix}^{1/\lambda},\right.$$

$$1-\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)^{1/\lambda}$$

$$\left. -\left(1-\begin{pmatrix} 1-\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right) \\ +\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\end{pmatrix}^{1/\lambda}\right)\right\rangle$$

$$=\left\langle 1-\begin{pmatrix} 1-\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right) \\ +\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\end{pmatrix}^{1/\lambda},\right.$$

$$\left.\begin{pmatrix} 1-\left(\prod_{i=1}^{n}\left(1-(1-u_{A_i})^{\lambda}+(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{w_i}\right) \\ +\left(\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{\lambda w_i}\right)\end{pmatrix}^{1/\lambda} -\prod_{i=1}^{n}(1-(u_{A_i}+v_{A_i}))^{w_i}\right\rangle$$

$$=\mathrm{GIFWGIA}_{\lambda}(A_1,\ldots,A_n).$$

Thus, Proposition 4.1 holds.

Remark 3. For $\phi=\phi^{-1}=Id$, we have

$$\mathrm{IFWGIQAM}_w(A_1,A_2,\ldots,A_n)$$

$$=\left\langle \phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right)-h^{-1}\left(\sum_{i=1}^{n}w_i h(v_{A_i})\right)\right)\right.$$

$$\left.\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right)\right)-\phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right) \\ -h^{-1}\left(\sum_{i=1}^{n}w_i h(v_{A_i})\right)\end{pmatrix}\right\rangle$$

$$=\left\langle h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right)-h^{-1}\left(\sum_{i=1}^{n}w_i h(v_{A_i})\right),\right.$$

$$h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right)-h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right)$$

$$\left.+h^{-1}\left(\sum_{i=1}^{n}w_i h(v_{A_i})\right)\right\rangle$$

$$=\left\langle h^{-1}\left(\sum_{i=1}^{n}w_i h(u_{A_i}+v_{A_i})\right)-h^{-1}\left(\sum_{i=1}^{n}w_i h(v_{A_i})\right),h^{-1}\left(\sum_{i=1}^{n}w_i h(v_{A_i})\right)\right\rangle,$$

i.e., Eq. (24) is reduced to Eq. (18). Thus, the EIFWGIA operator is a special case of the IFWGIQAM.

Definition 4.3 [24]**.** Let $A_i=\langle u_{A_i},v_{A_i}\rangle\quad(i=1,\ldots,n)\in$IFNs$(X)$ and $\lambda>0$. $A_{\sigma(i)}$ is the ith largest value of A_i $(i=1,\ldots,n)$, and $w=(w_1,w_2,\ldots,w_n)$ is the corresponding weighting vector, with $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. The generalized IFOWGIA (GIFOWGIA) operator is defined as:

$$\mathrm{GIFOWGIA}_{\lambda}(A_1,\cdots A_n)=\left\langle 1-\begin{pmatrix} 1-\prod_{i=1}^{n}\left(1-\left(1-u_{A_{\sigma(i)}}\right)^{\lambda}+\left(1-\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right)^{\lambda}\right)^{w_i} \\ +\prod_{i=1}^{n}\left(1-\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right)^{\lambda w_i}\end{pmatrix}^{1/\lambda},\right.$$
$$\left.\begin{pmatrix} 1-\prod_{i=1}^{n}\left(1-\left(1-u_{A_{\sigma(i)}}\right)^{\lambda}+\left(1-\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right)^{\lambda}\right)^{w_i} \\ +\prod_{i=1}^{n}\left(1-\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right)^{\lambda w_i}\end{pmatrix}^{1/\lambda} -\prod_{i=1}^{n}\left(1-\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right)^{w_i}\right\rangle \quad (25)$$

Similar to Definition 4.2, we can regard the GIFOWGIA operator as a specific case of the IFOWGIQAM with $\phi(t)=1-(1-t)^{\lambda}$, $\lambda>0$. Then, the IFOWGIQAM can be defined as follows.

Definition 4.4. Let $A_i=\langle u_{A_i},v_{A_i}\rangle\quad(i=1,\ldots,n)\in$IFNs$(X)$, where $A_{\sigma(i)}$ is the ith largest value of A_i $(i=1,\ldots,n)$ and $w=(w_1,w_2,\ldots,w_n)$ is the corresponding weighting vector, with $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1.\lambda>0$. The IFOWGIQAM is defined as follows.

$$\begin{aligned}&\mathrm{IFOWGIQAM}_w(A_1, A_2, ..., A_n)\\ =&\ \langle \phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h\left(\phi\left(u_{A_{\sigma(i)}} + v_{A_{\sigma(i)}}\right)\right)\right) - h^{-1}\left(\sum_{i=1}^{n} w_i h\left(\phi\left(u_{A_{\sigma(i)}} + v_{A_{\sigma(i)}}\right) - \phi\left(u_{A_{\sigma(i)}}\right)\right)\right)\right)\\ &\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h\left(\phi\left(u_{A_{\sigma(i)}} + v_{A_{\sigma(i)}}\right)\right)\right)\right) - \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left(\phi\left(u_{A_{\sigma(i)}} + v_{A_{\sigma(i)}}\right)\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left(\phi\left(u_{A_{\sigma(i)}} + v_{A_{\sigma(i)}}\right) - \phi\left(u_{A_{\sigma(i)}}\right)\right)\right)\end{pmatrix}\rangle\end{aligned} \quad (26)$$

where ϕ is a strictly monotone continuous generating function, $\phi : [0, 1] \to [-\infty, \infty]$, and h is a continuous Archimedean t-conorm.

Similar to Proposition 4.1, we have Proposition 4.2.

Proposition 4.2. If $\phi(t) = 1 - (1 - t)^{\lambda}$, $\lambda > 0$ and $h(t) = -\ln(1 - t)$, then the IFOWGIQAM is reduced to the GIFOWGIA operator.

Remark 4. For $\phi = \phi^{-1} = Id$, similar to Remark 3, Eq. (26) is reduced to Eq. (19), which means the EIFOWGIA operator is a special case of the IFOWGIQAM.

Remark 5. For $h = h^{-1} = Id$ and $\phi(t) = 1 - (1 - t)^{\lambda}$, $\lambda > 0$, we have

$$\begin{aligned}&\mathrm{IFWGIQAM}_w(A_1, A_2, ..., A_n)\\ =&\ \langle \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left((1-u_{A_i})^{\lambda}-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\end{pmatrix}\\ &\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\right)\\ &-\phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left((1-u_{A_i})^{\lambda}-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\end{pmatrix}\rangle\\ =&\ \langle \phi^{-1}\begin{pmatrix} \left(\sum_{i=1}^{n} w_i\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right) \\ -\left(\sum_{i=1}^{n} w_i\left((1-u_{A_i})^{\lambda}-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\end{pmatrix}\\ &\phi^{-1}\left(\left(\sum_{i=1}^{n} w_i\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\right)\\ &-\phi^{-1}\begin{pmatrix} \left(\sum_{i=1}^{n} w_i\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right) \\ -\left(\sum_{i=1}^{n} w_i\left((1-u_{A_i})^{\lambda}-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\end{pmatrix}\rangle\\ =&\ \langle \phi^{-1}\left(\sum_{i=1}^{n} w_i\left(1-(1-u_{A_i})^{\lambda}\right)\right), \begin{matrix}\phi^{-1}\left(\left(\sum_{i=1}^{n} w_i\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)\right)\\ -\phi^{-1}\left(\sum_{i=1}^{n} w_i\left(1-(1-u_{A_i})^{\lambda}\right)\right)\end{matrix}\rangle\\ =&\ \langle 1-\left(1-\sum_{i=1}^{n} w_i\left(1-(1-u_{A_i})^{\lambda}\right)\right)^{1/\lambda}, \begin{matrix}\left(1-\left(1-\sum_{i=1}^{n} w_i\left(1-(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)\right)^{1/\lambda}\right)\\ -\left(1-\left(1-\sum_{i=1}^{n} w_i\left(1-(1-u_{A_i})^{\lambda}\right)\right)^{1/\lambda}\right)\end{matrix}\rangle\\ =&\ \langle 1-\left(\sum_{i=1}^{n} w_i(1-u_{A_i})^{\lambda}\right)^{1/\lambda}, \left(\sum_{i=1}^{n} w_i(1-u_{A_i})^{\lambda}\right)^{1/\lambda} - \left(\sum_{i=1}^{n} w_i(1-(u_{A_i}+v_{A_i}))^{\lambda}\right)^{1/\lambda}\rangle\end{aligned} \quad (27)$$

Similar to Remark 5, we obtain Remark 6.

Remark 6. For $h = h^{-1} = Id$ and $\phi(t) = 1 - (1 - t)^{\lambda}$, $\lambda > 0$, we have

$$\begin{aligned}\mathrm{IFOWIQAM}_w(A_1, A_2, ..., A_n) =&\ \langle 1-\left(\sum_{i=1}^{n} w_i\left(1-u_{A_{\sigma(i)}}\right)^{\lambda}\right)^{1/\lambda},\\ &\left(\sum_{i=1}^{n} w_i\left(1-u_{A_{\sigma(i)}}\right)^{\lambda}\right)^{1/\lambda} - \left(\sum_{i=1}^{n} w_i\left(1-\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)\right)^{\lambda}\right)^{1/\lambda}\rangle\end{aligned} \quad (28)$$

Remark 7. For $h = h^{-1} = Id$ and $\phi(t) = t^{\lambda}$, $\lambda > 0$, we have

$$\begin{aligned}&\mathrm{IFWGIQAM}_w(A_1, A_2, ..., A_n)\\ =&\ \langle \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left((u_{A_i}+v_{A_i})^{\lambda}-(u_{A_i})^{\lambda}\right)\right)\end{pmatrix}\\ &\phi^{-1}\left(h^{-1}\left(\sum_{i=1}^{n} w_i h\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right)\right)\\ &-\phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{i=1}^{n} w_i h\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right) \\ -h^{-1}\left(\sum_{i=1}^{n} w_i h\left((u_{A_i}+v_{A_i})^{\lambda}-(u_{A_i})^{\lambda}\right)\right)\end{pmatrix}\rangle\\ =&\ \langle \phi^{-1}\begin{pmatrix} \left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right) \\ -\left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}-(u_{A_i})^{\lambda}\right)\right)\end{pmatrix}, \begin{matrix}\phi^{-1}\left(\left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right)\right)\\ -\phi^{-1}\begin{pmatrix} \left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right) \\ -\left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}-(u_{A_i})^{\lambda}\right)\right)\end{pmatrix}\end{matrix}\rangle\\ =&\ \langle \phi^{-1}\left(\sum_{i=1}^{n} w_i(u_{A_i})^{\lambda}\right), \phi^{-1}\left(\left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right)\right) - \phi^{-1}\left(\sum_{i=1}^{n} w_i(u_{A_i})^{\lambda}\right)\rangle\\ =&\ \langle \left(\sum_{i=1}^{n} w_i(u_{A_i})^{\lambda}\right)^{1/\lambda}, \left(\sum_{i=1}^{n} w_i\left((u_{A_i}+v_{A_i})^{\lambda}\right)\right)^{1/\lambda} - \left(\sum_{i=1}^{n} w_i(u_{A_i})^{\lambda}\right)^{1/\lambda}\rangle\end{aligned} \quad (29)$$

Similar to Remark 7, we obtain Remark 8.

Remark 8. If taking $h = h^{-1} = Id$ and $\phi(t) = t^{\lambda}$, $\lambda > 0$. Then, the IFOWGIQAM (Eq. (26)) is reduced to the following formula.

$$\begin{aligned}&\mathrm{IFOWGIQAM}_w(A_1, A_2, ..., A_n)\\ =&\left\langle \left(\sum_{i=1}^{n} w_i\left(u_{A_{\sigma(i)}}\right)^{\lambda}\right)^{1/\lambda}, \left(\sum_{i=1}^{n} w_i\left(u_{A_{\sigma(i)}}+v_{A_{\sigma(i)}}\right)^{\lambda}\right)^{1/\lambda} - \left(\sum_{i=1}^{n} w_i\left(u_{A_{\sigma(i)}}\right)^{\lambda}\right)^{1/\lambda}\right\rangle\end{aligned} \quad (30)$$

Application of the Extensions to Cognitive Microcredit Origination

In this section, we use the proposed extensions of IFGIA operators to present approaches for the ranking of cognitive microcredit origination to farmers. Examples demonstrate the effectiveness of the new approaches.

A. Procedure of Cognitive Decision Making with the Extensions

In the cognitive microcredit origination problem, according to the loan officers' cognition and knowledge of microcredit origination, the company considers n attributes $g_i (i = 1, 2, ..., n)$ to evaluate the applicants. Assume that m farmers $x_i (i = 1, 2, ..., m)$ apply for microcredit; the company evaluates these famers based on n attributes with weights $w_i(i=1,2,...,n)$, where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The cognitive evaluation information is constructed by IFNs $A_{ij} = \langle u_{A_{ij}}, v_{A_{ij}} \rangle$. By Eq. (31) in [60], the evaluation information matrix $(A_{ij})_{m\times n}$ can be normalized as $R=(r_{ij})_{m\times n}$.

$$r_{ij} = \begin{cases} A_{ij}, & \text{for benefit attribute } g_j \\ (A_{ij})^c, & \text{for cost attribute } g_j \end{cases}, \; i=1,2,...,m; j=1,2,...,n \tag{31}$$

where $(A_{ij})^c$ is the complement of A_{ij} and $(A_{ij})^c = \langle v_{A_{ij}}, u_{A_{ij}} \rangle$.

Then, based on whether the weighting vectors are related to the IFNs or the locations of the input arguments, we present the detailed steps of the cognitive microcredit origination decision process with the extensions of the geometric interaction operators in two cases.

Method 1. The weights of the attributes are related to the IFNs.

Step 1. With Eq. (31), transform the given IF matrix into the normalized IF matrix.

Step 2. Aggregate all the cognitive preference IFNs r_{ij} $(j=1,2,...,n)$ into the collective *IFN*s r_i $(i=1,2,...,m)$ via the EIFWGIA operator, where

$$r_i = \mathrm{EIFWGIA}_w(r_{i1}, r_{i2}, ..., r_{in}) = \left\langle h^{-1}\left(\sum_{j=1}^{n} w_j h\left(u_{r_{ij}} + v_{r_{ij}}\right)\right) - h^{-1}\left(\sum_{j=1}^{n} w_j h\left(v_{r_{ij}}\right)\right), h^{-1}\left(\sum_{j=1}^{n} w_j h\left(v_{r_{ij}}\right)\right)\right\rangle \tag{32}$$

Alternatively, for the *IFWGIQAM*,

$$\begin{aligned} r_i &= \mathrm{IFWGIQAM}_w(r_{i1}, r_{i2}, ..., r_{in}) \\ &= \langle \phi^{-1}\left(h^{-1}\left(\sum_{j=1}^{n} w_j h(\phi(u_{r_{ij}} + v_{r_{ij}}))\right) - h^{-1}\left(\sum_{j=1}^{n} w_j h(\phi(u_{r_{ij}} + v_{r_{ij}}) - \phi(u_{r_{ij}}))\right)\right) \\ &\phi^{-1}\left(h^{-1}\left(\sum_{j=1}^{n} w_j h(\phi(u_{r_{ij}} + v_{r_{ij}}))\right)\right) - \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{j=1}^{n} w_j h(\phi(u_{r_{ij}} + v_{r_{ij}}))\right) \\ -h^{-1}\left(\sum_{j=1}^{n} w_j h(\phi(u_{r_{ij}} + v_{r_{ij}}) - \phi(u_{r_{ij}}))\right) \end{pmatrix} \rangle \end{aligned} \tag{33}$$

Step 3. Rank the candidate farmers x_i $(i=1,2,...,m)$ in descending order by score and accuracy degree of r_i $(i=1,2,...,m)$.

Step 4. Obtain the corresponding rakings for different functions h and ϕ.

Step 5. Analyze the results.

Step 6. End.

Method 2. The weights of the attributes are related to locations of the input IFNs.

Step 1. With Eq. (31), transform the given IF matrix into the normalized IF matrix.

Step 2. Aggregate all the preference IFNs r_{ij} $(j=1,2,...,n)$ into the collective *IFN*s r_i $(i=1,2,...,m)$ via the EIFOWGIA operator, where

$$r_i = \mathrm{EIFOWGIA}_w(r_{i1}, r_{i2}, ..., r_{in}) = \left\langle h^{-1}\left(\sum_{j=1}^{n} w_j h\left(u_{r_{\sigma(ij)}} + v_{r_{\sigma(ij)}}\right)\right) - h^{-1}\left(\sum_{j=1}^{n} w_j h\left(v_{r_{\sigma(ij)}}\right)\right), h^{-1}\left(\sum_{j=1}^{n} w_j h\left(v_{r_{\sigma(ij)}}\right)\right)\right\rangle \tag{34}$$

Alternatively, for the IFOWGIQAM,

$$\begin{aligned} r_i &= \mathrm{IFOWGIQAM}_w(r_{i1}, r_{i2}, ..., r_{in}) \\ &= \langle \phi^{-1}\left(h^{-1}\left(\sum_{j=1}^{n} w_j h\left(\phi\left(u_{r_{\sigma(ij)}} + v_{r_{\sigma(ij)}}\right)\right)\right) - h^{-1}\left(\sum_{j=1}^{n} w_j h\left(\phi\left(u_{r_{\sigma(ij)}} + v_{r_{\sigma(ij)}}\right) - \phi\left(u_{r_{\sigma(ij)}}\right)\right)\right)\right) \\ &\phi^{-1}\left(h^{-1}\left(\sum_{j=1}^{n} w_j h\left(\phi\left(u_{r_{\sigma(ij)}} + v_{r_{\sigma(ij)}}\right)\right)\right)\right) - \phi^{-1}\begin{pmatrix} h^{-1}\left(\sum_{j=1}^{n} w_j h\left(\phi\left(u_{r_{\sigma(ij)}} + v_{r_{\sigma(ij)}}\right)\right)\right) \\ -h^{-1}\left(\sum_{j=1}^{n} w_j h\left(\phi\left(u_{r_{\sigma(ij)}} + v_{r_{\sigma(ij)}}\right) - \phi\left(u_{r_{\sigma(ij)}}\right)\right)\right) \end{pmatrix} \rangle \end{aligned} \tag{35}$$

Step 3–Step 6 Refer to Method 1.

B. Numerical Example

An online, small loan limited company in Hainan province organized a loan officer team to expand their cognitive microcredit origination business to other provinces. After professional training, these loan officers were dispatched to different cities. Suppose three farmers $x_i(i=1,2,3)$ are applying for microcredit. Based on the loan officers' cognition and knowledge of microcredit origination, they define five important attributes $G_i(i=1,2,...,5)$ to evaluate the farmers' abilities. The cognitive evaluation information matrix is shown in Table 1.

- G_1: Household disposable income.
- G_2: Annual household income.
- G_3: Bank loan's deadline.
- G_4: Labor population of the family.
- G_5: Family population.

Step 1. The three candidate farmers are evaluated in terms of IFNs (Table 1), as follows.

Step 2. Suppose that the weights are directly related to the attributes and are determined on the basis of the normal distribution [59], i.e., $w=(0.112, 0.236, 0.304, 0.236, 0.112)$. Then, we use method 1 to

Table 1 IF matrix $(r_{ij})_{3\times 5}$

	G_1	G_2	G_3	G_4	G_5
x_1	⟨0.2,0.5⟩	⟨0.4,0.2⟩	⟨0.5,0.4⟩	⟨0.3,0.3⟩	⟨0.7,0.1⟩
x_2	⟨0.2,0.7⟩	⟨0.6,0.3⟩	⟨0.4,0.3⟩	⟨0.4,0.4⟩	⟨0.6,0.1⟩
x_3	⟨0.2,0.7⟩	⟨0.4,0.3⟩	⟨0.4,0.5⟩	⟨0.3,0.4⟩	⟨0.6,0.2⟩

solve the problem. From the IFWGIQAM (i.e., Eq. (24)) with $\phi(t)=1-(1-t)^2$ and $h=h^{-1}=Id$, we have

$$r_i = \text{IFWGIQAM}_w(r_{i1}, r_{i2}, \cdots, r_{in})\ (i=1,2,3)$$
$$=\left\langle 1-\left(\sum_{i=1}^{n} w_i(1-u_{A_i})^2\right)^{1/2}, \left(\sum_{i=1}^{n} w_i(1-u_{A_i})^2\right)^{1/2} -\left(\sum_{i=1}^{n} w_i(1-(u_{A_i}+v_{A_i}))^2\right)^{1/2}\right\rangle.$$

Inserting the data from Table 1 into the above equation yields.

$r_1=\langle 0.4014, 0.2935\rangle$, $r_2=\langle 0.4328, 0.3428\rangle$,$r_3=\langle 0.3678, 0.4061\rangle$.

Step 3. From Definition 2.9, we obtain $S(r_1)=0.1079$, $S(r_2)=0.0899$, and $S(r_3)=-0.0382$. Thus, $r_1>r_2>r_3$ and $x_1\succ x_2\succ x_3$.

Step 4. Taking $h=Id$ and $\phi=$1-(1-t)^3 and repeating Step 1 to Step 3, we obtain $S(r_1)=0.0994$, $S(r_2)=0.0757$, and $S(r_3)=-0.0386$; thus, $r_1>r_2>r_3$ and $x_1\succ x_2\succ x_3$. Taking $h=Id$ and $\phi=$1-(1-t)^5 and repeating Step 1 to Step 3 yields $S(r_1)=0.0755$, $S(r_2)=0.0420$, and $S(r_3)=-0.0477$; thus, $r_1>r_2>r_3$ and $x_1\succ x_2\succ x_3$.

The final candidate rankings are the same, which demonstrates the effectiveness of the proposed approaches.

C. Comparisons

The results of this paper and those of previous studies are compared in the following cases.

(1) If we use the method in [23], i.e., we use the IFWGIA operator to aggregate the individual cognitive decision information, we obtain $r_1=\langle 0.4477, 0.3172\rangle$, $r_2=\langle 0.4454, 0.3686\rangle$, and $r_3=\langle 0.3810, 0.4375\rangle$. Then, $S(r_1)=0.1305$, $S(r_2)=0.0768$, and $S(r_3)=-0.0564$; thus, we have $r_1>r_2>r_3$ and $x_1\succ x_2\succ x_3$. In fact, for $h=-\ln(1-t)$ and $\phi=Id$, we obtain the same results; therefore, the method in [23] is a special case of the method proposed in this paper.

(2) If the method in [24] is used, i.e., the GIFWGIA operator is used to aggregate the individual decision information, we obtain $r_1=\langle 0.4487, 0.3161\rangle$, $r_2=\langle 0.4486, 0.3654\rangle$, and $r_3=\langle 0.3814, 0.4371\rangle$. In this case, $S(r_1)=0.1326$, $S(r_2)=0.0832$, and $S(r_3)=-0.0557$; thus, we have $r_1>r_2>r_3$ and $x_1\succ x_2\succ x_3$. In fact, for $h=-ln(1-t)$ and $\phi=1-(1-t)^{0.8}$, we obtain the same results, i.e., the method in [24] is a special case of the method proposed in this paper.

(3) The proposed EIFWGIA operator includes a form that is consistent with aggregation operations on the ordinary FSs, which is explained in Remark 1. By contrast, the interaction aggregation operators on IFNs do not satisfy this property; for example, if we use the IFWGIA operator [23] to aggregate A and B in Remark 1, then

Table 2 Scores obtained by the IFWGIQAM and ranks of the alternatives for different h and ϕ

	$h=Id$, $\phi=Id$	$h=Id$, $\phi=t^2$	H=Id, $\phi=t^3$
x_1	0.1112	0.1450	0.1761
x_2	0.1012	0.1319	0.1574
x_3	−0.0416	−0.0191	0.0030
Rankings	$x_1\succ x_2\succ x_3$	$x_1\succ x_2\succ x_3$	$x_1\succ x_2\succ x_3$
	H=Id, $\phi=t^5$	$h=Id$, $\phi=1-(1-t)^3$	$h=Id$, $\phi=1-(1-t)^5$
x_1	0.2300	0.0994	0.0755
x_2	0.1968	0.0757	0.0420
x_3	0.0458	−0.0386	−0.0477
Rankings	$x_1\succ x_2\succ x_3$	$x_1\succ x_2\succ x_3$	$x_1\succ x_2\succ x_3$
	$h=-ln(1-t)$,$\phi=1-(1-t)^2$	$h=-ln(1-t)$,$\phi=1-(1-t)^5$	$h=-ln(1-t)$,$\phi=1-(1-t)^6$
x_1	0.0804	−0.0410	−0.0629
x_2	0.0440	−0.0408	−0.0625
x_3	−0.0774	−0.1345	−0.1463
Rankings	$x_1\succ x_2\succ x_3$	$x_2\succ x_1\succ x_3$	$x_2\succ x_1\succ x_3$

IFWGIA$_w(A, B) = \langle 0.3281, 0.4719\rangle$. Clearly, $\langle 0.3281, 0.4719\rangle \neq \langle 0.33, 0.47\rangle$.

(4) Similar to (3), the proposed EIFOWGIA operator includes a form that is consistent with ordered aggregation operations on ordinary FSs, whereas the ordered interaction aggregation operators on IFNs do not satisfy this property.

(5) Notably, Beliakov et al. [6] showed that the intuitionistic fuzzy-weighted averaging (IFWA) operator [58] is not consistent with aggregation operations on ordinary FSs. In fact, the IFWGA operator [62] also has the above weakness. However, for some special functions$h$ and ϕ, the extensions of the geometric interactive aggregation operators on IFNs can overcome the above weakness, which is also necessary for the proposed extensions.

(6) We combine the extended intuitionistic fuzzy geometric interaction aggregation operators with the quasi-arithmetic means and propose the IFWGIQAM and IFOWGIQAM. By taking different generator functions h and ϕ, the IFWGIQAM and the IFOWGIQAM degenerate into existing *IF* aggregation operators or extensions, some of which consider situations in which no interactions exist between membership and non-membership functions. In addition, for different functions h and ϕ, we obtain the rakings of the alternatives in the numerical example shown in Table 2, where the different generator functions h and ϕ can be viewed as the decision makers' cognitive preferences for real situations.

Table 2 shows that the overall scores of the alternatives vary for different generator functions h and ϕ. Thus, the management implications of different generator functions are that decision makers' different cognitive preferences affect the rankings of alternatives and lead to different results in real situations.

Conclusions

The extension of operational laws of IFNs has received considerable attention. This paper defines the extensions of geometric interaction aggregation operators by using the t-norm and the corresponding t-conorm under an IF environment. The EIFWGIA operator, EIFOWGIA operator, IFWGIQAM, and IFOWGIQAM are proposed. We explain some special cases of the proposed extensions and apply them to cognitive microcredit origination under the IF environment. Finally, an example demonstrates the feasibility of the developed approaches.

The limitations of this paper include the following. (1) The proposed operators are more complex than the previous ones, and the decision makers must spend more (but acceptable) time to obtain the final results. (2) If the decision makers involved have different cognitive preferences, then different rankings of the candidate alternatives will be obtained. Additionally, the decision makers need other techniques to assist the cognitive decision-making process.

Subsequent research will study the extensions in an interval-valued IF environment and propose interval-valued extensions of the interactive aggregation operators. These new operators will then be applied to cognitive microcredit origination, clustering and MADM.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

Informed Consent Informed consent was not required as no human or animals were involved.

Human and Animal Rights This article does not contain any studies with human or animal subjects performed by any of the authors.

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