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Group Decision-Making with Linguistic Cognition from a Reliability Perspective

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Abstract

To deal reliably with the cognitive uncertainty experienced by decision-makers when facing problems involving linguistic group decision-making, we investigate a new research perspective: cognitive familiarity is regarded as a measure of cognitive reliability. The linguistic variables examined in this work are quantified with the use of several granulation optimization models that include consideration of cognitive reliability. Three types of linguistic variables are used for describing the alternative grades, attribute weights, and levels of cognitive familiarity associated with the experts involved. Three interrelated optimization models are built to quantify these linguistic variables. An information entropy model first determines cognitive familiarity, which is applied as a measure of cognitive reliability. Using group consistency and cognitive reliability, two proposed optimization models then successively establish the attribute weights and alternative grades. The final element is a proposed new selection method based on the aggregated values for the alternative grades and cognitive reliability. An illustrative example clarifies the steps in the proposed method, which produces a ranking of three alternatives as a final decision. The validity and advantages of the proposed method are verified through a comparison with existing approaches. The proposed method can be employed for effectively resolving decision-making uncertainty through the improvement of the group consistency and cognitive reliability of the experts. Sensitivity analysis also reveals that cognitive reliability has a strong impact on decision-making and should thus be considered during fusion processes.

Keywords Decision analysis · Cognitive reliability · Group consistency · Cognitive familiarity

Introduction

In real-world decision-makings, researchers often advocate for granting full credence to collective wisdom in order to improve decision-making accuracy [1, 2]. Moreover, decision-makers would rather express their cognitions in a natural and appropriate way instead of specifying a crisp value [3, 4]. So, linguistic multi-attribute group decision-making (LMAGDM) is preferred by administrators over individual decision-making [5, 6]. The literature reveals that although multi-view learning methods [7, 8] entail the collection of information from multiple sources, they serve as machine learning tools that involve the use of large-scale data and cannot use small-scale information for solving decision-making problems. LMAGDM methods, which have been successfully examined in numerous studies [10–12], aggregate information from different decision-makers as a means of managing both large- and small-scale linguistic information related to the problems under study [9].

The primary goal in the quantitative solution of LMAGDM problems is to quantify the linguistic cognitions of decision-makers (or so-called experts) [6]. Many researchers have investigated methods for quantifying linguistic information using such techniques as fuzzy membership functions [12], symbolic transferring scales [13], 2-tuple linguistics [14], cloud models [15], and discrete fuzzy numbers [16]. However, to some extent, these numerical methods either weaken the flexible superiority of information or lead to information loss. To make up for these limitations, some scholars have adopted granulation

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methods as a means of setting an appropriate numerical scale for linguistic terms [17, 18]. Such methods offer two advantages: (1) information granules are uneven in each interval, and (2) basic linguistic information is not lost during the granulation process. With respect to group decision-making, group conflicts often occur due to social and cultural differences of decision-makers [19]. To enhance the acceptability of decision-making, group consistency [20] is frequently employed as a means of overcoming inadequacies associated with the conflicts and biases of decision-makers' opinions. Especially, the granulation method to quantify the linguistic information by maximizing group consistency is applied to solve LMAGDM problems. For example, Fu et al. [21] proposed an optimization method to determine the utilities of linguistic terms by maximizing group consistency; Lin et al. [22] proposed a multi-granulation method for rough sets.

However, in the above reported studies, in spite of its essential role in helping administrators make reliable decisions, no consideration was given to the cognitive reliability of the experts either during the granulation or throughout the decision-making process. Although group consistency can improve information reliability to some extent [23], it does not reflect the intrinsic properties of experts who are information providers. In many practical decision-making cases, decision-makers must make a decision despite inherent uncertainty in the information they are using. This factor can enhance any risk related to the problems under review [24] but can be compensated for when decision-makers take into account the cognitive reliability of the decision-making process. Consideration of cognitive reliability is an effective way to mitigate uncertainty in LMAGDM problems [25], and ensuring greater cognitive reliability means that less uncertainty is associated with experts' cognitions. Cognitive familiarity, as proposed by Zhu et al. [26], denotes the cumulative experience and knowledge of the experts (i.e., the intrinsic properties of the experts). Experts with a higher degree of cognitive familiarity offer more extensive experience and knowledge in a given field [27, 28]. In general, with respect to making reliable and convincing decisions, greater persuasive power is attributed to experts who have more experience and knowledge. Similarly, experts whose levels of cognitive familiarity are higher are more likely to provide more reliable information. Based on these considerations, we employed cognitive familiarity as a measure of cognitive reliability and took into account both group consistency and cognitive reliability in the quantification and fusion process. The proposed method is useful for improving the acceptability of decisions and for differentiating between expert information and any intrinsic aspects. This feature allows the maintenance of expert information as well as its full and reliable use.

Paper Contributions

This study is the first to extend classical LMAGDM methods using an ELMAGDM (extended LMAGDM) approach that incorporates consideration of the cognitive familiarity of the experts. Cognitive familiarity is employed for measuring the cognitive reliability of the experts using a granulation optimization model, and two other linguistic granulation optimization models have been developed based on a reliability perspective. These elements constitute the primary contribution of the work presented in this paper. It should be noted that information about cognitive familiarity is generally given by experts because only experts truly comprehend the levels of their cognitive familiarity. The ELMAGDM approach entails the use of three linguistic granulation optimization models for the successive quantification of three linguistic variables. The first model measures the levels of cognitive familiarity and the corresponding cognitive reliability of the experts. Based on consideration of group consistency and cognitive reliability, the second and third models are used for determining attribute weights and alternative grades, respectively. The weighted values assigned to the experts are then determined from the levels of cognitive reliability and group consistency and from prior subjective information. A ranking method is also proposed as a means of managing the aggregated results related to alternative grades and cognitive reliability. The final step is the application of the proposed method for solving the selection issues of small- and medium-sized enterprises (SMEs) associated with a specific illustrative case.

The remainder of the paper is organized as follows. A "Preliminaries" section first introduces concepts and descriptions related to classical LMAGDMs and ELMAGDMs. The "ELMAGDM Method" section explains the techniques employed for establishing the three linguistic optimization models and for generating a ranked order of alternatives. A "Case Study" section provides a case whereby the proposed approach is applied to the SME selection and is compared to several existing methods. The final "Conclusion" section presents the findings from our study as well as suggested avenues for future research.

Preliminaries

This section introduces some basic concepts related to our research and describes the problem addressed in this work.

Basic LMAGDM Concepts

Some basic notations and operational laws related to linguistic variables [29] are introduced in this section. Suppose that $G = \{g_{i_G} | i_G = 1, \ldots, M\}$ is a finite and ordered discrete linguistic set, where g_{i_G} denotes a possible linguistic label in G, where M (an odd number) indicates its cardinality, and where $g_1 \prec g_2 \prec \cdots \prec g_M$. G has two properties: (1) if $i_G \le j_G$ ($i_G, j_G = 1, \ldots, M$), then $g_{i_G} \le g_{j_G}$; (2) if $j_G = M - i_G$, then $Neg(g_{i_G}) = g_{j_G}$ is a negation operator.

A classical LMAGDM problem often involves multiple attributes, experts, and alternatives [30]. Suppose that $E = \{e^1, e^2, \ldots, e^l\}(e^k \in E, k = 1, 2, \ldots, l, l \ge 2)$ is a set of experts and that $\lambda = (\lambda^1, \lambda^2, \ldots, \lambda^l)^T$ denotes an expert weighting vector where $\sum_{k=1}^{l} \lambda^k = 1$ and $\lambda^k \in [0, 1]$. Let $A = \{a_1, a_2, \ldots, a_m\}(a_i \in A, i = 1, \ldots, m, m \ge 2)$ be a finite set of alternatives, and let $G = \{g_1, g_2, \ldots, g_M\}(g_{i_G} \in G, i_G = 1, \ldots, M)$ denote a set of linguistic terms representing their grades. Suppose that $F = \{f_1, \ldots, f_n\}(f_j \in F, j = 1, \ldots, n, n \ge 2)$ denotes a set of attributes. Linguistic information for attribute weights must then be provided by experts [31], and the set of corresponding linguistic terms is $H = \{h_1, h_2, \ldots, h_N\}(h_{i_H} \in H, i_H = 1, \ldots, N)$. Then, let $g_{ij}^k(g_{ij}^k \in G)$ be a linguistic variable given by expert e^k for the grade of alternative a_i with respect to attribute f_j, and let $h_j^k(h_j^k \in H)$ indicate a linguistic variable for the weight of attribute f_j given by expert e^k.

We introduce two definitions that are useful for the proposed method.

Definition 1 Zhao [32] supposing that $0 \le \alpha^D \le \alpha^U$, $\alpha = [\alpha^D, \alpha^U]$ is defined as a non-negative interval number if and only if $\alpha^D = \alpha^U$, then α is a non-negative real number.

Definition 2 De Almeida et al. [33] supposing that $\alpha = [\alpha^D, \alpha^U]$ is a non-negative interval number, then $\hat{\alpha} = \alpha^D + (\alpha^U - \alpha^D)\pi$ ($\pi \in [0, 1]$) is defined as a binary connection number, which is the whitening form of α.

Problem Description

With the LMAGDM framework as the basis, cognitive familiarity is further considered and a new ELMAGDM problem is thereby generated. Supposing that $C = \{c_1, c_2, \ldots, c_L\}$ ($c_{i_C} \in C, i_C = 1, \ldots, L$) is a set of the linguistic variables representing the cognitive familiarity of experts and that $c_{ij}^k(c_{ij}^k \in C)$ is a linguistic variable denoting the cognitive familiarity of expert e^k with alternative a_i with respect to attribute f_j, then the information included in the ELMAGDM framework is shown in Table 1. Figure 1 provides a further illustration that enables a comparison between new problems that can be solved only with the use of the proposed method and those solved using the LMAGDM method.

To measure the scales of the aforementioned linguistic terms, we apply information granulation methods based on a reliability perspective. The key purpose is to determine the optimal granular segmentation points of an appropriate range [6]. Given that an obvious linear order exists between two adjacent linguistic terms of one set, linguistic granulation is implemented in terms of intervals. In set $G = \{g_1, \ldots, g_M\}$, the value of g_M is the maximum. Based on the principles of the symbol transferring method, the value of g_M is equal to M, so the terms of G are set to be located within the range of $[0, M]$. Then, according to the granulation method described in [6], we have $g_1 = [0, \alpha_1)$, $g_2 = [\alpha_1, \alpha_2)$, …, $g_M = [\alpha_{M-1}, M]$, where $\alpha_1, \ldots, \alpha_{M-1}$ denotes the segmentation points of the set. For linguistic sets $H = \{h_1, \ldots, h_N\}$ (attribute weights) and $C = \{c_1, \ldots, c_L\}$ (cognitive familiarity), it is obvious that they should both belong to range [0, 1]. Thus, we have $h_1 = [0, \beta_1)$, $h_2 = [\beta_1, \beta_2)$, …, $h_N = [\beta_{N-1}, 1]$, and $c_1 = [0, \gamma_1)$, $c_2 = [\gamma_1, \gamma_2)$, …, $c_L = [\gamma_{L-1}, 1]$. Following completion of the information granulation, the next task is to determine the whitening form of each interval because it is difficult to optimize variables in terms of intervals. We therefore apply Definition 2 in order to transform each interval number into a binary connection number, so that we then have $\hat{g}_{i_G} = \alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G$ ($i_G = 1, \ldots, M, \alpha_0 = 0, \alpha_M = M, \pi^G \in [0, 1]$), $\hat{h}_{i_H} = \beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H$ ($i_H = 1, \ldots, N, \beta_0 = 0, \beta_N = 1, \pi^H \in [0, 1]$), $\hat{c}_{i_C} = \gamma_{i_C-1} + (\gamma_{i_C} - \gamma_{i_C-1})\pi^C$ ($i_C = 1, \ldots, L, \gamma_0 = 0, \gamma_L = 1, \pi^C \in [0, 1]$).

The ELMAGDM method has two significant features: (1) Each expert can choose an appropriate variable from specific linguistic labels according to his or her cognition, which is comparatively convenient and practical for experts. (2) More expert information can be excavated to help

Table 1 Linguistic information considered under the ELMAGDM framework

	e^1			$\cdots$	e^l		
	f_1	$\cdots$	f_n	$\cdots$	f_1	$\cdots$	f_n
a_1	$(g_{11}^1, c_{11}^1, h_1^1)$	$\cdots$	$(g_{1n}^1, c_{1n}^1, h_n^1)$	$\cdots$	$(g_{11}^l, c_{11}^l, h_1^l)$	$\cdots$	$(g_{1n}^l, c_{1n}^l, h_n^l)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_m	$(g_{m1}^1, c_{m1}^1, h_1^1)$	$\cdots$	$(g_{mn}^1, c_{mn}^1, h_n^1)$	$\cdots$	$(g_{m1}^l, c_{m1}^l, h_1^l)$	$\cdots$	$(g_{mn}^l, c_{mn}^l, h_n^l)$

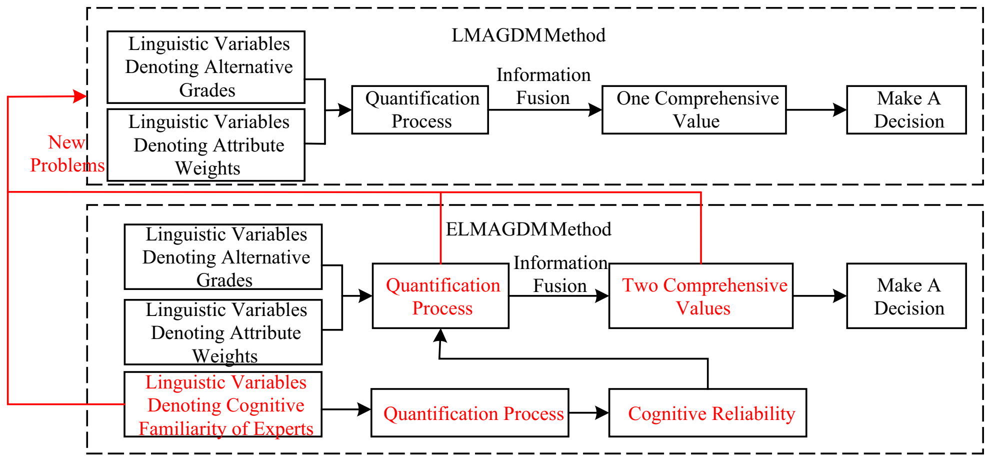


Fig. 1 Illustration of the proposed problem solution

produce a reasonable decision. When some experts hold a full range of cognitive familiarity, they cannot necessarily provide this kind of information, which renders this situation an exception to the usual functioning of the method.

ELMAGDM Method

In this section, we illustrate the building of the three linguistic granulation optimization models, as well as the selection from alternatives based on a reliability perspective. Cognitive familiarity is employed mainly to describe the experience and knowledge of an expert that are independent from those of other experts. Attribute weights and alternative grades describe attributes and alternatives, respectively, and it is preferable for experts to provide similar information with respect to the same attribute or alternative. Because optimized variables for cognitive familiarity, attribute weights, and alternative grades also differ, three interrelated optimization models are proposed for their determination. The first model is designed to optimize linguistic variables that represent cognitive familiarity and thereby to determine the corresponding cognitive reliability levels; the second model optimizes linguistic variables that denote attribute weights based on the group consistency and cognitive reliability of the experts; the third model optimizes linguistic variables that designate alternative grades in order to obtain the aggregated values of the alternatives derived from the group consistency and cognitive reliability of the experts. Figure 2 presents a diagram of the ELMAGDM method.

Linguistic Granulation Optimization Model for Quantifying Cognitive Familiarity

Cognitive familiarity denotes the depth and scope of the information held by an expert in a relevant area of

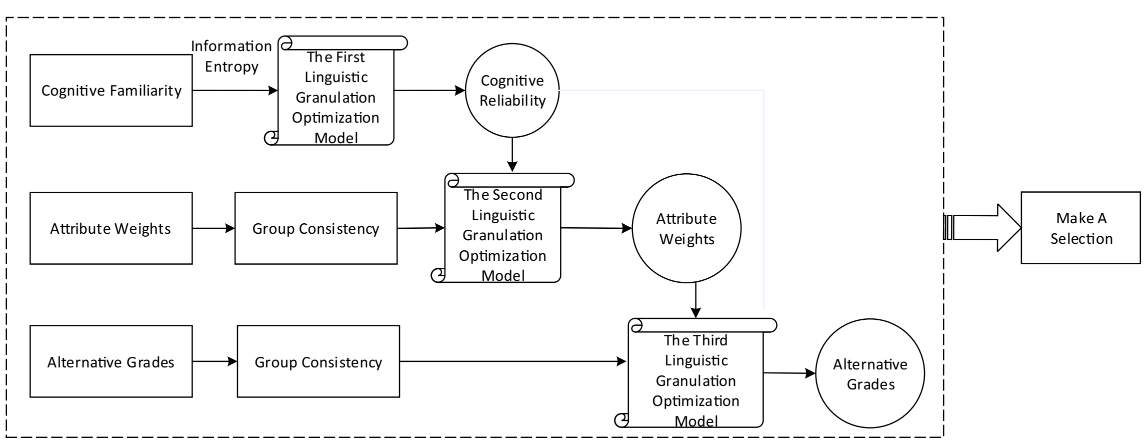


Fig. 2 Diagram of the ELMAGDM method

expertise. Experts who are comparatively more familiar with their disciplines are considered to have greater experience and knowledge in a relevant field, thus enabling them to make more significant contributions in terms of reliable decision-making [27]. Accordingly, if one expert offers full knowledge (i.e., 100% cognitive familiarity), he or she would be able to make a fully accurate judgment. Clearly, in many decision-making scenarios, experts with greater experience and knowledge are more likely to be selected to participate in decision-making processes, since these experts are more familiar with the field in question and are thus more helpful with respect to making reliable assessments. Cognitive familiarity can be used for identifying essential differences among experts, with different experts offering different degrees of cognitive familiarity due to differences in their knowledge, the methods used to collect information, etc. In many applications, an expert who offers more cognitive familiarity is often considered to have a stronger impact on decision-making and is thus more highly valued. When cognitive familiarity is not included in consideration, decision-making processes may involve greater risk [26]. Cognitive familiarity can thus be used as a means of objectively judging the experience and influence of the experts. The familiarity of the experts can also be easily and directly expressed using linguistic variables, thus facilitating quantification. Treating the cognitive familiarity of the experts as a measure of information reliability is therefore reasonable.

Definition 3 Suppose that $\hat{c}_{ij}^k(\hat{c}_{ij}^k \in \{\hat{c}_1, \ldots, \hat{c}_L\})$ denotes the degree of cognitive familiarity of expert e^k with alternative a_i with respect to attribute f_j, cognitive reliability can be defined as:

$$\delta_{ij}^k = \hat{c}_{ij}^k$$

This definition of cognitive reliability reflects the cognitive familiarity of the experts. In general, a positive relationship exists between these two concepts: a higher degree of cognitive familiarity denotes a higher level of cognitive reliability, and vice versa.

Measuring cognitive reliability involves identifying segmentation points $\gamma_1, \ldots, \gamma_{L-1}$ and π^C, and obtaining its constraints should be the first step in the building of a linguistic optimization model. To prevent the distribution of the lengths of all intervals ($\gamma_{i_C} - \gamma_{i_C-1}, i_C = 1, \ldots, L, \gamma_0 = 0, \gamma_L = 1$) from being too uneven and to avoid having two adjacent segmentation points (γ_{i_C} and γ_{i_C-1}) positioned too closely together, $\gamma_{i_C} - \gamma_{i_C-1}$ is set to be limited to an approximate range. Let τ_U^C and τ_D^C ($0 < \tau_D^C < \tau_U^C < 1$) be the respective upper and lower limits of $\gamma_{i_C} - \gamma_{i_C-1}$, with values provided by experts according to real conditions. When τ_U^C and τ_D^C are not provided, they can be approximately set around the upper and lower limits of the intervals, which are uniformly distributed. We then have $\tau_D^C \le \gamma_{i_C} - \gamma_{i_C-1} \le \tau_U^C$. In addition, because linguistic variable $c_{(L+1)/2}$ of C denotes the average level, its value $\hat{c}_{(L+1)/2}$ should be 0.5, i.e.,

$$\hat{c}_{(L+1)/2} = \gamma_{(L-1)/2} + (\gamma_{(L+1)/2} - \gamma_{(L-1)/2})\pi^C = 0.5.$$

Since the cognitive familiarity of the experts denotes the uncertainty associated with their cognitions, a higher level of cognitive familiarity indicates a lower degree of uncertainty, thus enabling a higher level of cognitive reliability to be obtained. Therefore, when segmentation points are being determined, the linguistic cognitions of the experts should be fully applied in order to limit the experts' cognitive uncertainty. However, with the exception of their ranges, the specific probability distribution of the segmentation points is unknown so that the cognitive uncertainty based on the information given should be minimized. According to the literature, entropy [34] can be used as a measure of the uncertainty of a system: the objective information of the experts is taken into account in order to determine the probability distribution of the information through the maximization of negative entropy, which is expressed as follows:

$$\max \ S(q) = -\sum_{i_C=1}^{L} q_{i_C} \ln q_{i_C}$$
$$s.t. \sum_{i_C=1}^{L} q_{i_C} = 1. \tag{1}$$

As defined in [34], we use the convention that $0 \ln 0 = 0$, which is easily proven by continuity when $q_{i_C} \to 0$, $q_{i_C} \ln q_{i_C} \to 0$. The entropy method can thus be used for establishing segmentation points. According to the definition of information entropy, the first step is to generate probabilities with respect to segmentation points, which are expressed as

$$q_{i_C} = \frac{\gamma_{i_C-1} + (\gamma_{i_C} - \gamma_{i_C-1})\pi^C}{\sum_{i_C=1}^{L}[\gamma_{i_C-1} + (\gamma_{i_C} - \gamma_{i_C-1})\pi^C]}, \tag{2}$$

where $q_{i_C}(i_C = 1, \ldots, L)$ is the corresponding probability, $0 < q_1 < \ldots < q_{L-1} < 1$, and where $\sum_{i_C=1}^{L} q_{i_C} = 1$.

The information entropy model (expressed by Eq. M1) can therefore be constructed as follows:

$$\max S(q) = -\sum_{i_C=1}^{L}\left[\frac{\gamma_{i_C-1}+(\gamma_{i_C}-\gamma_{i_C-1})\pi^C}{\sum_{i_C=1}^{L}[\gamma_{i_C-1}+(\gamma_{i_C}-\gamma_{i_C-1})\pi^C]}\ln\left(\frac{\gamma_{i_C-1}+(\gamma_{i_C}-\gamma_{i_C-1})\pi^C}{\sum_{i_C=1}^{L}[\gamma_{i_C-1}+(\gamma_{i_C}-\gamma_{i_C-1})\pi^C]}\right)\right]$$
$$s.t.\begin{cases}\tau_D^C \le \gamma_{i_C}-\gamma_{i_C-1}\le \tau_U^C,\\ \gamma_{(L-1)/2}+(\gamma_{(L+1)/2}-\gamma_{(L-1)/2})\pi^C=0.5,\\ 0\le\pi^C\le 1,\\ 0\le\gamma_{i_C}\le 1,\\ \gamma_0=0,\gamma_L=1, i_c=1,\ldots,L.\end{cases} \tag{M1}$$

In Eq. M1, the first equality measures the range of the interval length, and the next one denotes the value of $c_{(L+1)/2}$. From Eq. M1, the degree of cognitive familiarity $\hat{c}_{ij}^k$ can be obtained, as can the cognitive reliability measure δ_{ij}^k.

Theorem 1 *An optimal solution is given by model* (M1).

Proof Let the feasible region of model (M1) be denoted as $\Omega_1 = \{\gamma = (\gamma_0,\ldots,\gamma_L,\pi^C)|\tau_D^C \le \gamma_1-\gamma_0 \le \tau_U^C,\ldots,\tau_D^C \le \gamma_{(L+1)/2}-\gamma_{(L-1)/2} \le \tau_U^C,\ldots,\tau_D^C \le \gamma_L-\gamma_{L-1} \le \tau_U^C, \gamma_0 = 0, \gamma_L = 1, \pi^C = (0.5-\gamma_{(L-1)/2})/(\gamma_{(L+1)/2}-\gamma_{(L-1)/2}), 0\le\pi^C\le 1, 0\le\gamma_{i_C}\le 1\}$. Since τ_D^C and τ_U^C are given parameters, and since $0 \le \tau_D^C < \tau_U^C$, π^C exists and is bounded, it is not difficult to find that other constraints in Ω_1 are bounded and that Ω_1 is non-empty. Ω_1 is thus non-empty and occupies a closed bounded region.

Based on Ω_1, $\sum_{i_C=1}^{L}[\gamma_{i_C-1}+(\gamma_{i_C}-\gamma_{i_C-1})\pi^C] > 0$, $\gamma_{i_C-1}+(\gamma_{i_C}-\gamma_{i_C-1})\pi^C \ge 0$, then $q_{i_C} \ge 0$. In agreement with the definition of information entropy [34], $S(q)$ is a continuous function when $q_{i_C} \ge 0$. According to the extreme value theorem of multivariate functions, the objective function must thus obtain a maximum [35], and an optimal solution therefore exists for model (M1). □

A Linguistic Granulation Optimization Model for Quantifying Attribute Weights

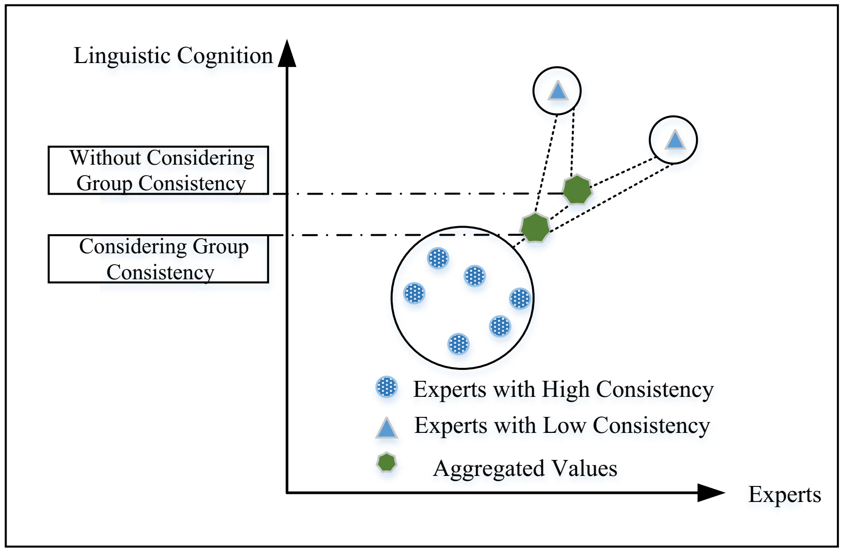
Unlike cognitive familiarity, which is the term used for describing experts, linguistic terms for attribute weights and alternative grades are used mainly for describing attributes and alternatives. This distinction means that the method for quantifying them should be established based on a different perspective. With respect to the examination of an LGDM problem, group consistency significantly enhances the effectiveness and acceptability of decision-making [36]. For example, as shown in Fig. 3, for a given case, while the linguistic information about alternative grades given by most experts is similar, that given by several individual experts is not. When consistency among experts is neglected, the results may deviate from the opinions of most experts because experts with significantly different opinions can have a considerable effect on the aggregated result. Group consistency can also weaken the influence of individual experts, rendering results more consistent with those of most experts (Fig. 3). An additional factor is that the cognitive reliability of the experts significantly shapes linguistic cognitions: a higher level of cognitive reliability reflects the use of more effective information. Incorporating consideration of cognitive reliability into the linguistic quantification process can help mitigate the risk associated with decision-making. Two linguistic granulation optimization methods are therefore proposed as a means of quantifying the linguistic variables that represent attribute weights and alternative grades based on both group consistency and cognitive reliability. In this section, we introduce an optimization model for determining attribute weights.

During the modeling process, specific constraints on variables included in the model are established from a determination of the segmentation points of C (the range of β_{i_H} is [0, 1]). Let τ_U^H and τ_D^H $(0 < \tau_D^H < \tau_U^H < 1)$ be the respective upper and lower limits of $\beta_{i_H}-\beta_{i_H-1}$; we then have $\tau_D^H \le \beta_{i_H}-\beta_{i_H-1} \le \tau_U^H$ $(i_H = 1,\ldots,N, \beta_0 = 0, \beta_N = 1)$. As well, because linguistic variable $\beta_{(N+1)/2}$ in H denotes the average level, we have

$$\hat{h}_{(N+1)/2} = \beta_{(N-1)/2}+(\beta_{(N+1)/2}-\beta_{(N-1)/2})\pi^H = 0.5.$$

Let $\hat{h}_j^k$ represent the specific value of h_j^k, which signifies the weight of attribute f_j with respect to expert e^k, and let h_j^k indicate the information initially given by the experts,

Fig. 3 Role of group consistency



where $h_j^k = \{h_{i_H}\}$ $(i_H = 1, \ldots, N; \hat{h}_j^k = \hat{h}_{i_H})$. For directly expressing the relationship between $\hat{h}_j^k$ and the segmentation points (β_{i_H}) of H, the following formula is used:

$$\hat{h}_j^k = \sum_{i_H=1}^{N} \varepsilon_{j,i_H}^{Hk} \hat{h}_{i_H} = \sum_{i_H=1}^{N} \varepsilon_{j,i_H}^{Hk} [\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]. \tag{3}$$

In Eq. 3, $\varepsilon_{j,i_H}^{Hk} = 0$ or 1, and $\sum_{i_H=1}^{N} \varepsilon_{j,i_H}^{Hk} = 1$, meaning that only one linguistic variable is assigned for describing h_j^k and that the specific value of ε_{j,i_H}^{Hk} is determined by h_j^k. Equation 3 does not increase the calculation complexity, because ε_{j,i_H}^{Hk} is known for a specific case and since (3) is the same as the expression $h_j^k = \{h_{i_H}\}$ in this case. Then, if $h_j^k = \{h_{i_H}\}$, we have $\varepsilon_{j,i_H}^{Hk} = 1$ and $\varepsilon_{j,j_H}^{Hk} = 0$ $(j_H = 1, \ldots, N$, and $j_H \neq i_H)$. For example, if $h_j^k = \{h_{(N+1)/2}\}$, we have $\hat{h}_j^k = \hat{h}_{(N+1)/2} = \beta_{(N-1)/2} + (\beta_{(N+1)/2} - \beta_{(N-1)/2})\pi^H$ directly; when Eq. 3 is used, we have $\varepsilon_{j,(N+1)/2}^{Hk} = 1$ and $\varepsilon_{j,i_H}^{Hk} = 0$ $(i_H = 1, \ldots, N$, and $i_H \neq (N+1)/2)$, and then we also have $\hat{h}_j^k = \beta_{(N-1)/2} + (\beta_{(N+1)/2} - \beta_{(N-1)/2})\pi^H$. To ensure that the sum of all attribute weights is equal to 1, a normalization method is applied in order to generate the normalized weight w_j^k, as follows:

$$w_j^k = \frac{\hat{h}_j^k}{\sum_{j=1}^{n} \hat{h}_j^k} = \frac{\sum_{i_H=1}^{N} \varepsilon_{j,i_H}^{Hk} [\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]}{\sum_{j=1}^{k} \sum_{i_H=1}^{N} \varepsilon_{j,i_H}^{Hk} [\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]}, \tag{4}$$

where $\sum_{j=1}^{n} w_j^k = 1$, $j = 1, \ldots, n$ and $k = 1, \ldots, l$.

As noted above, we consider an optimization model based on group consistency and cognitive reliability.

Definition 4 Suppose that w_j^k is the normalized weight of attribute f_j with respect to expert e^k, that δ_{ij}^k is the cognitive reliability of expert e^k for alternative a_i with respect to attribute f_j, and thus that attribute reliability with respect to expert e^k for alternative a_i is defined as:

$$\delta_i^k = \sum_{j=1}^{n} w_j^k \delta_{ij}^k = \frac{\sum_{j=1}^{n} \sum_{i_H=1}^{N} \delta_{ij}^k \varepsilon_{j,i_H}^{Hk} [\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]}{\sum_{j=1}^{n} \sum_{i_H=1}^{N} \varepsilon_{j,i_H}^{Hk} [\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]}, \tag{5}$$

where $i = 1, \ldots, m$, and $k = 1, \ldots, l$.

To improve the degree of cognitive reliability and thus to mitigate decision-making risks that can arise from attribute information, the process includes a minimum level of attribute reliability (denoted by $\bar{\delta}^H$) required by the experts, written as $\delta_i^k = \sum_{j=1}^{n} w_j^k \delta_{ij}^k \geq \bar{\delta}^H$, where δ_{ij}^k is obtained using Eq. M1.

Definition 5 Suppose that $\hat{h}_j^k$ and $\hat{h}_j^o$ respectively denote the weights of attribute f_j for experts e^k and e^o $(k = 1, \ldots, l, o = 1, \ldots, l$, and $k \neq o)$; the degree of group consistency for expert e^k with respect to the attribute weights is then defined as:

$$\theta^{Hk} = \frac{1}{n(l-1)} \sum_{o=1, o\neq k}^{l} \sum_{j=1}^{n} \left[1 - \left|\hat{h}_j^k - \hat{h}_j^o\right|\right]. \tag{6}$$

Based on Eqs. 3 and 6, the comprehensive degree of group consistency for all experts with respect to attribute weights can then be written as:

$$\theta^{H}=\frac{1}{nl(l-1)}\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{j=1}^{n}\left[1-\left|\sum_{i_H=1}^{N}\left(\varepsilon_{j,i_H}^{Hk}-\varepsilon_{j,i_H}^{Ho}\right)[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^{H}]\right|\right]. \tag{7}$$

The key goal of the consistency-driven method of linguistic granulation is to obtain the maximum degree of comprehensive group consistency [36], which is helpful for reducing bias among experts and thus for improving the acceptability of the decision-making. A linguistic granulation optimization model is then applied for determining segmentation points $\beta_1,\ldots,\beta_N$ and π^H. The purpose of the model is to maximize group consistency while ensuring that $\delta_i^k=\sum_{j=1}^{n}w_j^k\delta_{ij}^k\geq\bar{\delta}^H$, thus improving the results produced by the model. To conclude, a mathematical model (designated (M2)) is established to maximize the comprehensive group consistency θ^H of the attribute weights.

$$\max\theta^{H}=\frac{1}{nl(l-1)}\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{j=1}^{n}\left[1-\left|\sum_{i_H=1}^{N}\left(\varepsilon_{j,i_H}^{Hk}-\varepsilon_{j,i_H}^{Ho}\right)[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^{H}]\right|\right]$$
$$s.t.\begin{cases}\frac{\sum_{j=1}^{n}\sum_{i_H=1}^{N}\delta_{ij}^k\varepsilon_{j,i_H}^{Hk}[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]}{\sum_{j=1}^{n}\sum_{i_H=1}^{N}\varepsilon_{j,i_H}^{Hk}[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]}\geq\bar{\delta}^H,\\ \beta_{(N-1)/2}+(\beta_{(N+1)/2}-\beta_{(N-1)/2})\pi^H=0.5,\\ \tau_D^H\leq\beta_{i_H}-\beta_{i_H-1}\leq\tau_U^H,\\ 0\leq\pi^H\leq1,0\leq\beta_{i_H}\leq1,\\ \beta_0=0,\beta_N=1,i_H=1,\ldots,N.\end{cases} \tag{M2}$$

In model (M2), the first equality indicates that attribute reliability with respect to expert e^k on alternative a_i is not less than the level of reliability required. The next one denotes the value of $\hat{h}_{(N+1)/2}$ and the third one indicates the range of each interval length. It should be noted that ε_{j,i_H}^{Hk} and ε_{j,i_H}^{Ho} are known model parameters. If the first constraint cannot be obtained from the region of the other constraints, the level of reliability of the information about the attribute weights is too low to satisfy the requirement, so the information should be adjusted so that the first constraint is met. The first constraint is intended to be derived without violation of the other constraints involved in the model. With the use of Eq. M2, w_j^k, δ_i^k, and θ^{Hk} can then be acquired successively from Eqs. 4, 5, and 6.

Theorem 2 *An optimal solution exists in model (M2).*

Proof Let the feasible region of model (M2) be denoted as Ω_2. In Ω_2, $\bar{\delta}^H\leq\delta_i^k\leq1$ should be obtained from the region occupied by the other constraints. Based on the constraints on β_{i_H}, π^H, and $0\leq\delta_{ij}^k\leq1$, inequality is transformed into

$$0\leq\sum_{j=1}^{n}\sum_{i_H=1}^{N}\varepsilon_{j,i_H}^{Hk}[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]-\sum_{j=1}^{n}\sum_{i_H=1}^{N}\delta_{ij}^k\varepsilon_{j,i_H}^{Hk}[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]$$
$$=\sum_{j=1}^{n}\sum_{i_H=1}^{N}\left(\varepsilon_{j,i_H}^{Hk}-\delta_{ij}^k\varepsilon_{j,i_H}^{Hk}\right)[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]\leq\sum_{j=1}^{n}\left(1-\delta_{ij}^k\right),$$
$$0\leq\sum_{j=1}^{n}\sum_{i_H=1}^{N}\delta_{ij}^k\varepsilon_{j,i_H}^{Hk}[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]-\sum_{j=1}^{n}\sum_{i_H=1}^{N}\bar{\delta}^H\varepsilon_{j,i_H}^{Hk}[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]$$
$$=\sum_{j=1}^{n}\sum_{i_H=1}^{N}\left(\delta_{ij}^k\varepsilon_{j,i_H}^{Hk}-\bar{\delta}^H\varepsilon_{j,i_H}^{Hk}\right)[\beta_{i_H-1}+(\beta_{i_H}-\beta_{i_H-1})\pi^H]\leq\sum_{j=1}^{n}|\delta_{ij}^k-\bar{\delta}^H|.$$

The above two inequalities are clearly bounded. The other constraints are similar to those of model (M1) and have been proven to be bounded and feasible in the proof for Theorem 1. Ω_2 is thus non-empty and occupies a closed bounded region. □

In model (M2), the absolute value sign is determined from the values of ε_{j,i_H}^{Hk} and ε_{j,i_H}^{Ho}. Supposing that $\varepsilon_{j,i_{H1}}^{Hk} = 1$, $\varepsilon_{j,i_{H2}}^{Ho} = 1$, if $i_{H1} \geq i_{H2}$, we have

$$\left|\sum_{i_H=1}^{N}\left(\varepsilon_{j,i_H}^{Hk} - \varepsilon_{j,i_H}^{Ho}\right)[\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]\right|$$
$$= [\beta_{i_{H1}-1} + (\beta_{i_{H1}} - \beta_{i_{H1}-1})\pi^H]$$
$$-[\beta_{i_{H2}-1} + (\beta_{i_{H2}} - \beta_{i_{H2}-1})\pi^H].$$

If $i_{H1} < i_{H2}$, then

$$\left|\sum_{i_H=1}^{N}\left(\varepsilon_{j,i_H}^{Hk} - \varepsilon_{j,i_H}^{Ho}\right)[\beta_{i_H-1} + (\beta_{i_H} - \beta_{i_H-1})\pi^H]\right|$$
$$= [\beta_{i_{H2}-1} + (\beta_{i_{H2}} - \beta_{i_{H2}-1})\pi^H]$$
$$-[\beta_{i_{H1}-1} + (\beta_{i_{H1}} - \beta_{i_{H1}-1})\pi^H].$$

This calculation means that the absolute value sign is independent of the values of β_{i_H} and π^H. The objective function is thus a continuous function of region Ω_2 and must obtain a maximum value in Ω_2 according to the extreme value theorem of multivariate functions [35]. An optimal solution therefore exists in model (M2).

A Linguistic Granulation Optimization Model for Quantifying Alternative Grades

In this section, we introduce the third optimization model used for quantifying segment points α_{i_G} $(i_G = 1, \ldots, M)$ in order to obtain the value of each alternative grade. To prevent the distribution of the length of the intervals $(\alpha_{i_G} - \alpha_{i_G-1}, \alpha_0 = 0, \alpha_M = M)$ from being too uneven and $\alpha_{i_G} - \alpha_{i_G-1}$ from being too small, the length of each interval is limited to an approximate range. Let τ_U^G and τ_D^G $(0 < \tau_D^G < \tau_U^G < M)$ be the respective upper and lower limits of $\alpha_{i_G} - \alpha_{i_G-1}$. These values are determined in the same way that τ_U^C and τ_D^C are established. We then have $\tau_D^G \leq \alpha_{i_G} - \alpha_{i_G-1} \leq \tau_U^G$. In addition, as $\alpha_{(M+1)/2}$ in G denotes the average level, its value $\hat{g}_{(M+1)/2}$ should be $M/2$, i.e.,

$$\hat{g}_{(M+1)/2} = \alpha_{(M-1)/2} + (\alpha_{(M+1)/2} - \alpha_{(M-1)/2})\pi^G = M/2.$$

Suppose that $\hat{g}_{ij}^k$ denotes the value of g_{ij}^k, which represents the linguistic variable for the grade of alternative a_i given by expert e^k with respect to attribute f_j . Based on the premise expressed in Eq. 3, the relationship between r_{ij}^k and the segmentation points (α_{i_G}) of G can then be presented as follows:

$$\hat{g}_{ij}^k = \sum_{i_G=1}^{M} \varepsilon_{ij,i_G}^{Gk}\hat{g}_{i_G} = \sum_{i_G=1}^{M} \varepsilon_{ij,i_G}^{Gk}[\alpha_{i_G-1}+(\alpha_{i_G}-\alpha_{i_G-1})\pi^G], \tag{8}$$

where $\varepsilon_{ij,i_G}^{Gk} = 0$ or 1, and where $\sum_{i_G=1}^{M}\varepsilon_{ij,i_G}^{Gk} = 1$. The specific value of $\varepsilon_{ij,i_G}^{Gk}$ is known according to the initial information g_{ij}^k.

Let $\hat{g}_{ij}^k$ and $\hat{g}_{ij}^o$ denote the values of g_{ij}^k and g_{ij}^o, respectively. The degree of group consistency of expert e^k with respect to alternative grades can then be expressed as

$$\theta^{Gk} = \frac{1}{mn(l-1)}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}[M - |\hat{g}_{ij}^k - \hat{g}_{ij}^0|]$$
$$= \frac{1}{mn(l-1)}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]. \tag{9}$$

The comprehensive degree of group consistency for all experts with respect to alternative grades can then be written as:

$$\theta^G = \frac{1}{mnl(l-1)}\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]. \tag{10}$$

In group decision-making settings, the weights assigned to the experts are crucial factors in the final decisions produced. Experts differ from one another due to individual dissimilarities with respect to a variety of areas (e.g., their knowledge and experience). For a comprehensive evaluation of the alternatives, the impact of group consistency and the effect of cognitive reliability should be considered as two measures of expert weighting that also avoid the subjective randomness of information combinations that subjective information can create. Because group consistency is a criterion of the validity of group decision-making, problems associated with improving the group consistency of the experts require further study [36]. In general, a positive correlation exists between group consistency and expert weights: experts with greater group consistency exhibit less resistance to achieving group consistency and thus contribute a larger weight to the information aggregation, and vice versa. Group consistency can therefore be considered a measure of expert weighting.

Definition 6 Let θ^{Hk} and θ^{Gk} denote the group consistency of expert e^k $(k = 1, \ldots, l)$ with respect to attribute weights and alternative grades, respectively. The consistency weight of expert e^k is then defined as:

$$\lambda^{\theta k} = \frac{1}{2}\left(\frac{\theta^{Hk}}{\sum_{k=1}^{l}\theta^{Hk}} + \frac{\theta^{Gk}}{\sum_{k=1}^{l}\theta^{Gk}}\right) \tag{11}$$

In this study, the level of cognitive reliability is established based on cognitive familiarity. Experts who have more cognitive familiarity also have more extensive experience with and knowledge about the decision-making elements, so the information they provide is more readily adopted by administrators in many practical cases. This means that an expert who has more cognitive familiarity becomes more important to the decision-making process and is thus assigned a higher weight. Since cognitive reliability can be considered the quantification form of cognitive familiarity (Definition 3), we use it as a measure of expert weighting.

Definition 7 Let δ^k denote the cognitive reliability of expert e^k; the reliability weight of expert e^k is then defined as:

$$\lambda^{\delta k} = \frac{\delta^k}{\sum_{k=1}^{l}\delta^k} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\delta_{ij}^k}{\sum_{k=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\delta_{ij}^k}. \tag{12}$$

In addition to objective factors, a priori subjective experience also affects expert weighting. Suppose that λ^{sk} is the priori subjective weight of expert e^k. Three types of weights then denote different decision-making preferences, meaning that group consistency, cognitive reliability, and a priori subjective experience might be emphasized differently. To highlight differences in their importance, we apply two preference coefficients in terms of $\hbar$ ($\hbar \in [0, 1]$) and ℓ ($\ell \in [0, 1]$) in order to emphasize the weights for consistency and reliability, respectively. The two coefficients can then be adjusted to reflect the different decision-making conditions. Thus, the comprehensive weight λ^k of expert e^k is set as:

$$\lambda^k = \hbar\lambda^{\theta k} + \ell\lambda^{\delta k} + (1 - \hbar - \ell)\lambda^{sk}. \tag{13}$$

The comprehensive cognitive reliability of the experts for alternative a_i can then be written as:

$$\delta_i = \sum_{k=1}^{l}\lambda^k\delta_i^k. \tag{14}$$

To improve the levels of cognitive familiarity and thus mitigate decision-making risks, a minimum level of cognitive reliability (denoted by $\bar{\delta}^G$) is required from experts with respect to the alternatives, whereby $\delta_i = \sum_{k=1}^{l}\lambda^k\delta_i^k \geq \bar{\delta}^G$, where δ_i is obtained from the solutions of Eqs. M1 and M2. In addition, to enhance the achievement of maximum levels of comprehensive group consistency with respect to the alternatives and thus to increase the acceptability of the decision-making, we use $max\theta^G$ as the objective function of the model. Then, together with the aforementioned constraints, a third linguistic optimization model (M3) is established for determining the segmentation points $\alpha_1, \ldots, \alpha_{M-1}$ and π^G.

$$\max\theta^G = \frac{1}{mnl(l-1)}\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]$$
$$s.t.\begin{cases}\delta_i = \sum_{k=1}^{l}\lambda^k\delta_i^k \geq \bar{\delta}^G,\\ \alpha_{(M-1)/2} + (\alpha_{(M+1)/2} - \alpha_{(M-1)/2})\pi^G = M/2,\\ \tau_D^G \leq \alpha_{i_G} - \alpha_{i_G-1} \leq \tau_U^G,\\ 0 \leq \pi^G \leq 1, 0 \leq \alpha_{i_G} \leq M,\\ \alpha_0 = 0, \alpha_M = M, i_G = 1, \ldots, M.\end{cases} \tag{M3}$$

In Eq. M3, the first inequality indicates that the comprehensive cognitive reliability of the experts with respect to alternative a_i is not less than the required level of reliability. The second one denotes the value of $g_{(M+1)/2}$, and the third one indicates the range of the interval length of g_{i_G}. As with the first constraint included in Eq. M2,

$\sum_{k=1}^{l} \lambda^k \delta_i^k \geq \bar{\delta}^G$ should be obtained from the region in which the rest of the constraints in Eq. M3 are situated.

Theorem 3 *An optimal solution exists in model (M3).*

Proof Let the feasible region of model (M3) be denoted as Ω_3. In Eq. 14, supposing that $\sigma^k = \hbar\theta^{Hk}/(2\sum_{k=1}^{l}\theta^{Hk}) + \ell\lambda^{\delta k} + (1-\hbar-\ell)\lambda^{sk}$; then, in Ω_3, we have

$$\bar{\delta}^G \leq \delta_i = \sum_{k=1}^{l}\lambda^k\delta_i^k$$
$$= \sum_{k=1}^{l}\left\{\frac{\hbar \sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]}{2\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]} + \sigma^k\right\}\delta_i^k$$
$$\leq 1.$$

When $\hbar = 0$, all of the variables in the constraint has no meaning so that we have $\hbar > 0$. The constraint is intended to be obtained from the region that contains the rest of the constraints. It can then be transformed into

$$0 \leq \sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\delta_i^k - a\right)\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]$$
$$\leq M\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|a - \delta_i^k\right|,$$
$$0 \leq \sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(b - \delta_i^k\right)\left[M - \left|\sum_{i_G=1}^{M}\left(\varepsilon_{ij,i_G}^{Gk} - \varepsilon_{ij,i_G}^{Go}\right)[\alpha_{i_G-1} + (\alpha_{i_G} - \alpha_{i_G-1})\pi^G]\right|\right]$$
$$\leq M\sum_{k=1}^{l}\sum_{o=1,o\neq k}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|b - \delta_i^k\right|,$$

where $a = 2 \times (\bar{\delta}^G - \sum_{k=1}^{l}\rho^k\delta_i^k)/\hbar$, and $b = 2 \times (1 - \sum_{k=1}^{l}\rho^k\delta_i^k)/\hbar$. In this way, the first constraint is bounded by the other constraints, and proving that Ω_3 is non-empty and bounded is not difficult. □

Like the objective function of Eq. M2, the objective function of Eq. M3 is a continuous function of region Ω_3 and thus, according to the extreme value theorem of multivariate functions [35], must attain a maximum in Ω_3. An optimal solution therefore exists for model (M3).

The next step is to obtain $\alpha_1, \ldots, \alpha_{M-1}$ and π^G using Eq. M3. From Eqs. 13 and 14, we have λ^k and δ_i. Let $\hat{g}_i^G$ be the aggregated assessment of alternative a_i, which is expressed as:

$$\hat{g}_i^G = \sum_{k=1}^{l}\sum_{j=1}^{n}\lambda^k w_j^k \hat{g}_{ij}^k. \tag{15}$$

From the above discussion, it is evident that the solutions from Eqs. M1 and M2 form the premise of Eq. M3, since the optimized variables from Eqs. M1 and M2 are both utilized as known parameters for solving (M3).

Using Eqs. M1, M2, and M3, aggregated results can be obtained for two types of values: aggregated grades $\hat{g}_i^G$ and aggregated cognitive reliability δ_i. Producing an accurate decision based on these two kinds of values requires the development of a selection method.

Selection Method Based on Two Aggregated Values

Suppose that R_i denotes the aggregated values of alternative a_i and contains two aggregated values: aggregated grades $\hat{g}_i^G$ and aggregated cognitive reliability δ_i. From Eqs. 14 and 15, we have

$$R_i = \left(\hat{g}_i^G, \delta_i\right). \tag{16}$$

In general, the alternative with the largest $\hat{g}_i^G$ should be selected as the best candidate. However, to reduce decision-making risk, decision-makers also require the alternative with a higher δ_i. When a selection is made based only on $\hat{g}_i^G$ without consideration of δ_i, decision-makers are likely to face high levels of decision-making risk in the future, which they usually try to avoid. Thus, alternatives should be selected with consideration given to both $\hat{g}_i^G$ and δ_i. Suppose that there are two alternatives denoted by a_p and a_q $(p, q = 1, \ldots, m,\ p \neq q)$; seven situations then exist, which can be represented as follows:

(1) $\hat{g}_p^G < \hat{g}_q^G, \delta_p \leq \delta_q$; (2) $\hat{g}_p^G < \hat{g}_q^G, \delta_p > \delta_q$; (3) $\hat{g}_p^G = \hat{g}_q^G, \delta_p < \delta_q$; (4) $\hat{g}_p^G = \hat{g}_q^G, \delta_p = \delta_q$; (5) $\hat{g}_p^G = \hat{g}_q^G$, $\delta_p > \delta_q$; (6) $\hat{g}_p^G > \hat{g}_q^G, \delta_p \geq \delta_q$; (7) $\hat{g}_p^G > \hat{g}_q^G, \delta_p < \delta_q$.

Clearly, if situation (1) or (3) applies, we have $a_p \prec a_q$; if situation (5) or (6) applies, we have $a_p \succ a_q$; if situation (4) applies, we have $a_p \equiv a_q$. However, if situation (2) or (7) applies, a direct comparison is difficult. To deal with these two situations, we propose the following new ranking method.

In R_i, cognitive reliability δ_i intrinsically indicates expert certainty that the aggregated grade value of alternative a_i is $\hat{g}_i^G$. Experts generally assign the most likely grade information to each alternative according to their knowledge and experience. Thus, δ_i can be regarded as the probability of $\hat{g}_i^G$, which is considered the most probable value of alternative a_i. With $\hat{g}_i^G \in [0, M]$, the probability distribution for $\hat{g}_i^G$ is not clearly known. Since $\hat{g}_i^G$ is the most likely value of alternative a_i, we consider that a grade value closer to $\hat{g}_i^G$ has a probability that is higher but less than that of $\hat{g}_i^G$; this characteristic conforms to features of the triangle ambiguity function. A probability distribution function is thus established based on the triangle ambiguity function, as follows:

1. When $\hat{g}_i^G = 0$, $f_i(x) = 0$.
2. When $\hat{g}_i^G \geq M/2$,

$$f_i(x) = \begin{cases} 0, & x \neq [0, M] \\ \frac{\delta_i x}{\hat{g}_i^G}, & x \in (0, \hat{g}_i^G] \\ -\frac{\delta_i x}{\hat{g}_i^G} + 2\delta_i. & x \in (\hat{g}_i^G, M]. \end{cases}$$

3. When $0 < \hat{g}_i^G < M/2$,

$$f_i(x) = \begin{cases} 0, & x \neq [0, M] \\ \frac{\delta_i (x - \hat{g}_i^G)}{M - \hat{g}_i^G} + \delta_i, & x \in (0, \hat{g}_i^G] \\ \frac{\delta_i (M - x)}{M - \hat{g}_i^G}. & x \in (\hat{g}_i^G, M]. \end{cases}$$

Suppose that the situation involving alternatives a_p and a_q is $\hat{g}_p^G < \hat{g}_q^G$ and $\delta_p > \delta_q$, and that the probability distribution functions of a_p and a_q are valued at $f_p(x)$ and $f_q(x)$. Clearly, $f_p(x)$ and $f_q(x)$ cannot be compared when $\hat{g}_p^G \neq \hat{g}_q^G$, so making a comparison requires that a suitable reference interval can be chosen. Because an alternative with higher grades is a better candidate for the decision-makers, we set the comparable interval as $[\hat{g}_p^G, M]$. The cumulative probabilities of $f_p(x)$ and $f_q(x)$ are then calculated within the interval $[\hat{g}_p^G, M]$; i.e., $F_{pp}(x) = \int_{\hat{g}_p^G}^{M} f_p(x)dx$, $F_{qp}(x) = \int_{\hat{g}_p^G}^{M} f_q(x)dx$. The formulas used for generating $F_{pp}(x)$ and $F_{qp}(x)$ are presented in Appendix A of the Appendices.

After obtaining $F_{pp}(x)$ and $F_{qp}(x)$, we can make a direct comparison: if $F_{pp}(x) < F_{qp}(x)$, then $a_p \prec a_q$; if $F_{pp}(x) = F_{qp}(x)$, then $a_p \equiv a_q$; if $F_{pp}(x) > F_{qp}(x)$, then $a_p \succ a_q$. Every two alternatives are then compared to each other in order to obtain a ranked order of alternatives.

The following steps outline the details of the proposed method:

Step 1 Form an ELMAGDM problem. l experts are selected to give their linguistic cognitions g_{ij}^k, c_{ij}^k, and h_j^k $(g_{ij}^k \in G, c_{ij}^k \in C, h_j^k \in H, i = 1, \ldots, m, j = 1, \ldots, n, k = 1, \ldots, l)$ for m alternatives with respect to n attributes, where G, H, and C, respectively, denote the linguistic sets of alternative grades, attribute weights, and cognitive familiarity of the experts.

Step 2 Determine the cognitive familiarity and cognitive reliability of the experts. In agreement with the entropy method, linguistic variables indicating the cognitive familiarity of the experts are quantified from Eq. M1, enabling values for the cognitive reliability δ_{ij}^k of the experts to be obtained.

Step 3 Generate the attribute weights. Through the execution of Eq. M2, linguistic variables denoting the attribute weights are quantified so that w_j^k and δ_i^k can be obtained using Eqs. 4 and 5 based on the solutions from Eqs. M1 and M2.

Step 4 Generate alternative grades. Using the optimized variables from Eq. M1 and M2, M3 is constructed in order to quantify the linguistic variables for the alternative grades. Next, comprehensive weights for the experts are calculated from Eq. 13. Equations 14 and 15 are then applied in order to determine the aggregated cognitive reliability δ_i and the aggregated grades $\hat{g}_i^G$.

Step 5 Make a selection from the alternatives. According to the selection approach which involves the aggregated assessment $\hat{g}_i^G$ and the aggregated cognitive reliability δ_i, a ranked order of the alternatives is generated, which constitutes a solution to the ELMAGDM problem based on consideration of cognitive reliability.

Case Study

This section describes the use of a numerical example as a demonstration of the feasibility and rationality of the proposed method. To illustrate its advantages, the new method is also compared to the five methods previously proposed in [14, 28, 30, 37, 38].

Background Description

By boosting employment and economic and technological innovation, SMEs play a significant role in the economic and social development of a nation [39]. In many countries, governments have applied a series of policies directed at increasing the financial support given to local SMEs [40]. For the best use of funds and the selection of the most suitable SMEs, investors must employ an appropriate selection method. For example, in China, because of the large number of SMEs that require funding, higher-credit SMEs must be selected to utilize the money efficiently. SME selection is often considered a multi-attribute decision-making problem (MADM), which is frequently characterized by interdependent criteria derived from inaccurate or uncertain information [41]. Such an issue is often referred to as decision-making uncertainty [37]. The successful selection of an SME from several choices requires the use of information systems such as service-oriented architecture (SOA) technologies in order to manage the large volume of data obtained from heterogeneous sources [42]. The use of SOA enables an investor to grasp real-time cognition about SME performance in order to make a decision [43], but SOA cannot be employed for solving decision-making problems associated with a system. Boumahdi et al. [30] therefore proposed their SOA+d approach, which combines SOA with the decision support systems of services. Selections are made using MADM methods, such as the analytic hierarchy process (AHP) [30, 44], the fuzzy VIKOR method [38], and fuzzy multicriteria analysis combined with a similarity method [45] used to make a selection. However, the approaches presented in these studies omit consideration of the reliability of the various information sources employed during the decision-making process, and thus enhance investment risk. For investors, uncertain information can affect decision-making reliability and ultimately lead to high degrees of investment risk and fund loss. To overcome these limitations, we therefore applied the proposed method to SME selection involving the use of uncertain information.

As explained by Sohna et al. [46] as well as Ju and Sohn [47], the SME selection process involves an examination of higher-credit alternatives with respect to 16 individual attributes (f_j, $j = 1, \ldots, 16$) that can be sorted into four categories: management, technology, marketability, and profitability. The attributes pertaining to management reflect the abilities of leaders in various areas (e.g., knowledge management, technological experience, management ability, funding supply, and human resources). The attributes pertaining to management reflect the abilities of leaders with respect to a variety of areas, such as knowledge management, technological experience, management ability, funding supply, and human resources. Attributes that pertain to technology can be gauged according to the technological development environment, the technological development output, new technologies, technological superiority, and technological commercialization. Marketability is measured based on market potential, market characteristics, and product competitiveness. Profitability is often assessed with reference to sales schedules, business progress, and investment returns.

In China, along with an investment company from Beijing, the local government of Anshan (a city in Liaoning province) has established a project for funding SMEs in Anshan, with the intention of selecting an SME from a number of industries. The high-technology industry is the primary target for this project. However, many start-up SMEs also require funds. If all SMEs were to be funded, the total amount required would far exceed the planned amount. For this reason, only one SME was selected from all of the companies. Preliminary screening eliminated most of the SMEs. The complex evaluation system that was employed left only three potential candidates to choose from: a_1, a_2, and a_3. Two of these are software management businesses. Although various types of selection methods exist, the government cannot collect complete information about all of the candidates because the SME selection process is complex and professional and because obtaining accurate values for some attributes is difficult. On the other hand, experts find it convenient to use linguistic variables to express their cognitions. For these reasons, the application of the proposed method is appropriate in this case.

The government invited three experts (e^1, e^2, and e^3): one each from a governmental agency, a supervision organization, and a relevant company. Their priori weights are provided as $\lambda^{s1} = \lambda^{s2} = \lambda^{s3} = 1/3$. A linguistic set representing the alternative grades is established as $G = \{g_1 = \text{very poor}, g_2 = \text{poor}, g_3 = \text{relatively poor}, g_4 = \text{average}, g_5 = \text{relatively good}, g_6 = \text{good}, g_7 = \text{very good}\}$. A linguistic set denoting the attribute weights is recorded as $H = \{h_1 = \text{very unimportant}, h_2 = \text{unimportant}, h_3 = \text{relatively unimportant}, h_4 = \text{average}, h_5 = \text{relatively important}, h_6 = \text{important}, h_7 = \text{very important}\}$. Another linguistic set describing the cognitive familiarity of the experts is expressed as $C = \{c_1 = \text{very low}, c_2 = \text{low}, c_3 = \text{average}, c_4 = \text{high}, c_5 = \text{very high}\}$. The initial information given by experts is shown in Appendix B of the Appendices.

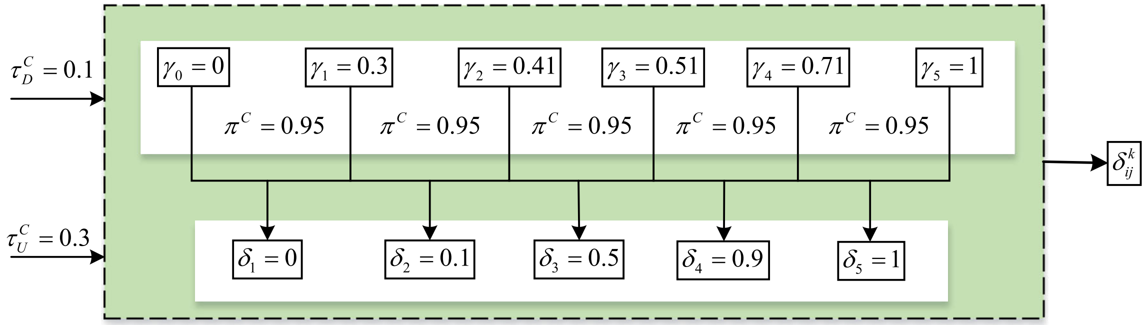


Fig. 4 Equation M1 solution

The goal of stipulating the linguistic cognitions of the experts was to make full use of the information available so that the proposed method can be applied to produce a final decision characterized by a high degree of reliability and group consistency.

Computation Process and Analysis of Results

Based on the ELMAGDM method, we solved this case by applying the following five steps. Some solutions were obtained using the MATLAB optimization toolbox.

Step 1 Form an ELMAGDM problem. As described for this case, three experts are selected for assessing the three alternatives with respect to 16 attributes. Three linguistic sets are provided: alternative grades, attribute weights, and the levels of cognitive familiarity of the experts, where $M = 7$, $N = 7$, and $L = 5$. This case thus falls within the definition of an ELMAGDM problem. Through the application of Definition 2, the three kinds of linguistic variables are transformed into binary connection numbers.

Step 2 Generate values for the cognitive familiarity and the corresponding reliability of the experts. To prevent the development of an uneven interval distribution, the interval length threshold is set to vary by 50% relative to that of an even distribution; i.e., $\tau_U^C = 0.3$, and $\tau_D^C = 0.1$. Figure 4 presents the solution produced by Eq. M1. The distribution is intensive over natural intervals but is scattered on both sides, which is consistent with the characteristics of a normal distribution without loss of the principle of universality. Applying (M1) to solve the problem is therefore reasonable. Definition 3 can be applied in order to obtain a value for the cognitive reliability δ_{ij}^k of expert e^k for alternative a_i with respect to attribute f_j ($i = 1, 2, 3$; $j = 1, \ldots, 16$; $k = 1, 2, 3$).

Step 3 Determine the attribute weights. As was the case for determining τ_U^C and τ_D^C, $\tau_U^H = 0.215$ and $\tau_D^H = 0.072$. The minimum requirement for the degree of cognitive reliability of the experts with respect to every attribute is set as $\bar{\delta}^H = 70\%$, which is dependent on the experience of experts in the SME industry. It should be noted that if the value of each δ_{ij}^k is generally low or if $\bar{\delta}^H$ is set at a high level, Eq. M2 might not find a feasible solution. In that case, initial cognitions should be adjusted by specific experts. Based on Eq. M2, quantified linguistic variables representing the attribute weights are obtained (Fig. 5). The normalized weight w_j^k of attribute f_j given by expert e^k is generated ($j = 1, \ldots, 16$, $k = 1, 2, 3$), and the cognitive reliability of expert e^k with respect to alternative a_i is determined as $\delta_1^1 = 0.794$, $\delta_2^1 = 0.771$, $\delta_3^1 = 0.787$, $\delta_1^2 = 0.911$, $\delta_2^2 = 0.897$, $\delta_3^2 = 0.904$, $\delta_1^3 = 0.976$, $\delta_2^3 = 0.895$, and $\delta_3^3 = 0.953$.

Step 4 Generate alternative grades. As with the determination of τ_U^C and τ_D^C, τ_U^G and τ_D^G are set as 1.5 and 0.5, respectively. Since no clear preference exists for this

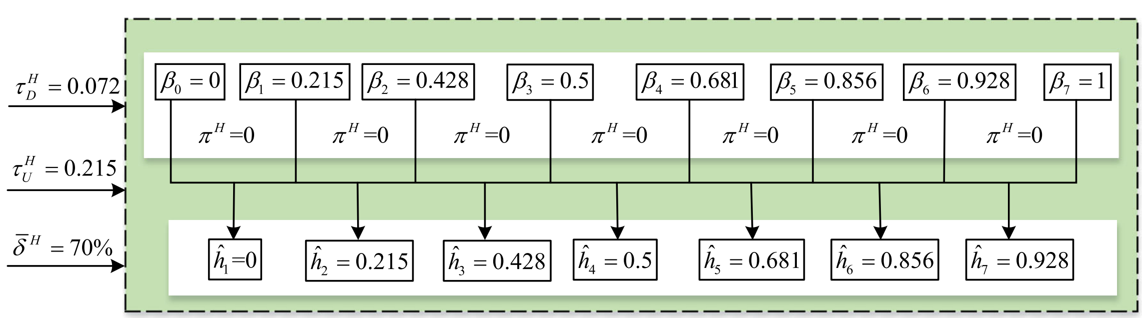


Fig. 5 Equation M2 solution

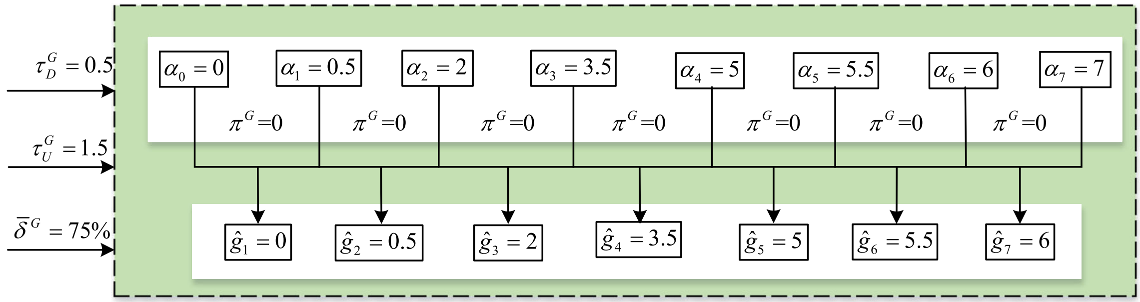


Fig. 6 Equation M3 solution

case, we have $\hbar = \ell = 1/3$. The minimum requirement for the level of reliability of each alternative is set as $\bar{\delta}^G = 75\%$. The Eq. M3 solution is then obtained, as indicated in Fig. 6. When Eq. 13 is applied, the comprehensive weights for the experts are computed as $\lambda^1 = 0.32$, $\lambda^2 = 0.336$ and $\lambda^3 = 0.344$. Using Eqs. 14 and 15, the aggregated cognitive reliability values are established as $\delta_1 = 0.861$, $\delta_2 = 0.858$, and $\delta_3 = 0.905$, and those for the aggregated grades as $\hat{g}_1^G = 4.689$, $\hat{g}_2^G = 4.713$, and $\hat{g}_3^G = 5.536$.

Step 5 Make a selection from the alternatives. Using the selection method, the ranked order of the alternatives is obtained as follows. (1) For SMEs a_1 and a_2, $F_{11} = 1.5$ and $F_{21} = 1.5$, implying that $a_1 \equiv a_2$. (2) For a_2 and a_3, $F_{22} = 1.48$ and $F_{32} = 1.84$, and thus $a_2 \prec a_3$. (3) For a_1 and a_3, $F_{11} = 1.15$ and $F_{31} = 1.85$, and we have $a_1 \prec a_3$. The ranked order of the SMEs is consequently $a_1 \equiv a_2 \prec a_3$.

Based on the ranked order, SME a_3 exhibits the best credit performance, while SMEs a_1 and a_2 belong to the "average" level and SME a_3 is categorized as being at the "good" level. In particular, SME a_3 presents advantages with respect to some attributes (e.g., funding supply, human resources, technological development environment, new technologies, technological superiority, and technological commercialization). This assessment is produced because the weights of these attributes are higher than the average value of all of the attribute weights, implying that these attributes have a stronger impact on the overall performance of the alternatives. If SMEs a_1 and a_2 wish to obtain the investment, they should improve the performance of these attributes in order to be assessed on par with SME a_3. In general, in terms of SME selection, the proposed method can be deemed both rational and feasible.

With the proposed method, when the values of the three types of linguistic variables are known, all of the algorithms used during the aggregation process can be easily executed using constants. The computational complexity of the proposed method thus relies heavily on the solution methods employed in the three optimization models ((M1), (M2) and (M3)). For the case study, we used the sequential linear programming (SLP) method to obtain solutions from the three models, in accordance with each type. Of the different optimization techniques available, the SLP method is very popular owing to its conceptual simplicity and simple algorithm [48, 49]. The computational complexity of this method is strongly affected by the number of variables and move limits under consideration [50]. In the three models presented here, the maximum number of variables is seven, and the number of optimized variables is unaffected by the number of experts, alternatives, or attributes. Hence, if the number of optimized variables is fixed, the computational complexity of the three models changes only insignificantly with an increasing number of other parameters. An additional feature is that because the distance between the upper and lower limits of the variables is quite short (maximum value = 7), move limits are largely undefined during the solving process. For these reasons, the SLP method is applied for the solution process in the three models. With the SLP method, the three models produce solutions within a short time frame (maximum time period = 10 s), with the maximum amount of generator memory used being measured at 26. The result is that the proposed method is easy to implement using real-world applications and is less computationally complex.

Sensitivity Analysis

In this work, cognitive familiarity is deemed to be a measure of cognitive reliability. To provide a clear illustration of the need for cognitive reliability and to demonstrate the rationality of the proposed selection method, we further analyzed the influence of cognitive reliability and of alternative grades on a cumulative probability F. For simplification purposes, we directly discuss the effects of aggregated cognitive reliability δ_i and of aggregated grades

$\hat{g}_i^G$ on F. δ_i is a type of variable with respect to $\hat{c}_{ij}^k$ ($\hat{g}_{ij}^k$ and $\hat{h}_j^k$ are fixed), and $\hat{g}_i^G$ is varied in relation to $\hat{g}_{ij}^k$ (other factors are left unchanged). Accordingly, δ_i and $\hat{g}_i^G$, rather than $\hat{c}_{ij}^k$ and $\hat{g}_{ij}^k$, are the salient features of the analysis. We take SMEs a_1 and a_2 as examples to guide our discussion. Figure 7 shows the impact of both δ_i and $\hat{g}_i^G$ on F.

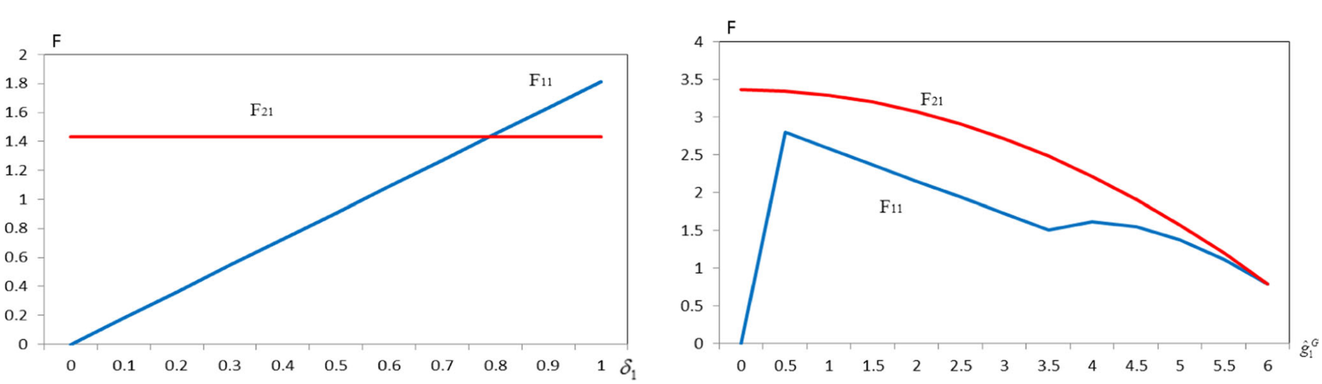
In Fig. 7a, for F_{11} and F_{21}, the aggregated grade $\hat{g}_1^G$ of SME a_1, the $\hat{g}_2^G$ of SME a_2, and the cognitive reliability δ_2 of SME a_2 are all fixed. Different values of δ_1 (describing SME a_1) hence lead to different F_{11} values, and a larger δ_1 results in a larger F_{11}. In the figure, the intersection of F_{11} and F_{21} denotes that $a_1 \equiv a_2$. As indicated in Fig. 7a, we can clearly conclude that cognitive reliability has a substantial impact on the decision-making results and thus serves as a positive reference based on which administrators can make reliable decisions.

Figure 7b shows the effect of $\hat{g}_1^G$ on F when δ_1, δ_2 ($\delta_1 = \delta_2$), and $\hat{g}_2^G$ are fixed. To ensure that $\hat{g}_1^G \leq \hat{g}_2^G$, $\hat{g}_2^G$ is assigned the maximum value: $\hat{g}_2^G = 6$. As $\hat{g}_1^G$ increases, the gap between F_{11} and F_{21} tends to decrease, and the difference between them gradually declines to 0. It should be noted that inflection points can be observed in the figure because different equations are used for computing F_{11} and F_{21} when $\hat{g}_1^G$ belongs to different intervals; i.e., $0 = \hat{g}_1^G < M/2 < \hat{g}_2^G$, $0 < \hat{g}_1^G < M/2 < \hat{g}_2^G$, and $M/2 \leq \hat{g}_1^G < \hat{g}_2^G$. As shown in Fig. 7b, the variation in F that results from different $\hat{g}_1^G$ values conforms to conventional wisdom (a higher grade value for one alternative denotes better performance from that alternative), thus illustrating the rationality of the proposed selection method.

Comparative Analysis and Discussion

For further exploration of the advantages of the proposed method, we compared our method (designated Method 1) with two existing approaches that have been the focus of theoretical [14, 28] and practical [37, 38] research. The two theoretical methods are referred to as Method 2 [14] and Method 3 [28]; the two existing practical application approaches are identified as Method 4 [37] and Method 5 [38]. Method 2, which omits consideration of reliability and group consistency, is used mainly for solving group decision-making problems that involve incomplete weights, and Method 5 is employed for studying Web service selection in SOA based on consideration of confidence levels and the expertise of decision-makers during the selection process. The two methods are suited for group decision-making when no special requirements are placed on group consistency. Method 4 is applied for the examination of system service selection for Chinese SMEs under uncertainty. The method is suited for use with multi-attribute decisions to be made by one decision-maker, and its relatively low computational complexity makes it easy to execute. Method 3, which lacks consideration of group consistency, takes into account expert information and is thus comparable to the proposed method. These four methods are therefore useful for making comparisons with our method. Table 2 lists the detailed comparative results obtained using these methods. A further comparison involved contrasting the computations of group consistency for the five methods using Eq. 5 (Table 2). It should be noted that, because Methods 4 and 5 use different scoring rules, we first translated these two methods so that they follow the same scoring rule used in our method and so that all five methods would be based on seven grades.

As indicated in Table 2, a comparison of the two theoretical approaches (Methods 2 and 3) revealed that the primary difference with respect to the three ranked orders produced is the positioning of alternatives a_1 and a_2. The ranked order of the two alternatives obtained from Method 2 is $a_1 \prec a_2$ and that obtained from Method 3 is $a_1 \prec a_2$ while that obtained using our method is $a_1 \equiv a_2$. The



(a) Impact of δ_i on F

(b) Impact of $\hat{g}_i^G$ on F

Fig. 7 Impact of δ_i and $\hat{g}_i^G$ on cumulative probability F

Table 2 Results of the comparison

		Method 1	Method 2 [14]	Method 3 [28]	Method 4 [37]	Method 5 [38]
Aggregated values	a_1	$R_1 = (4.689, 0.861)$	–	$R_1 = (4.73, 0.75)$	$\hat{g}_1^G = 5.05$	–
	a_2	$R_2 = (4.713, 0.858)$	–	$R_2 = (5.09, 0.68)$	$\hat{g}_2^G = 4.97$	–
	a_3	$R_3 = (5.536, 0.905)$	–	$R_3 = (6.29, 0.7)$	$\hat{g}_3^G = 5.26$	–
Group consistency	a_1	6.52	6.04	6.04	5.8	5.91
	a_2	6.44	5.96	5.96	5.78	5.86
	a_3	6.75	6.5	6.5	6.36	6.42
Ranked order		$a_1 \equiv a_2 \prec a_3$	$a_2 \prec a_1 \prec a_3$	$a_1 \prec a_2 \prec a_3$	$a_2 \prec a_1 \prec a_3$	$a_2 \prec a_1 \prec a_3$

cognitive reliability with Method 1 is also shown to be greater than that with Method 3, thereby reducing decision-making risks. Method 2, on the other hand, does not include consideration of cognitive reliability. In addition, greater group consistency, which improves the acceptability of the decision-making, is obtained with Method 1 than with either Method 2 or Method 3. The ranked order obtained using the proposed method is therefore more credible.

When the results using the proposed method are compared to those produced by the two practical methods (Methods 4 and 5), the ranked orders are found to be different. Based on the aggregated values of Methods 1 and 4, the assessments of alternatives a_1 and a_2 are very similar, so making a selection using only the aggregated grade values of the alternatives is thus insufficient. However, this issue can be addressed if the cognitive reliability of the experts is taken into account. As can be seen in Table 2, the performance of alternatives a_1 and a_2 is identical, which indicates greater validity. The table further reveals that the group consistency of Method 1 is superior to that of either Method 4 or Method 5 (Table 2), which means that the acceptability of the decision-making is also enhanced by the use of the new method.

Compared to the other four methods, our method can facilitate decision-making characterized by a higher level of cognitive reliability and group consistency. The advantages evident in the results are derived mainly from the computation process. To illustrate the enhancements provided by the proposed method in greater detail, we conducted a further comparison from the perspective of basing a selection on the initial information, linguistic quantification, weighting of the experts, and aggregated values. (1) With respect to the initial information, in our method, in contrast with other methods, the cognitive familiarity of the experts is included in consideration as an initial linguistic variable to be employed for making a reliable decision. (2) Regarding linguistic quantification, linguistic terms are quantified objectively using three granulation optimization models that take into account group consistency and cognitive reliability, thus limiting the amount of information loss during the calculation process. (3) The proposed method applies cognitive reliability, group consistency, and subjective information in determining the expert weighting, making the decision-making more objective and comprehensive. (4) With the proposed selection approach, two types of aggregated values are obtained and combined, thus facilitating comprehensive decision-making.

In addition to the above quantitative comparisons, we also compared the proposed method to the AHP technique described in [30] from a qualitative perspective. The method described in [30] adds decision-making functions to SOA and then applies the AHP approach in order to make a selection for suppliers. The AHP is suitable for both group and individual decision-making. It produces problem solutions based on a hierarchy and thus follows clear hierarchical logic. Table 3 provides the results of the comparison of our proposed method with the method proposed in [30]. It is apparent that our method offers a number of advantages: it measures initial information from multiple dimensions, introduces a more objective calculation process, produces more comprehensive results, and supports a higher level of decision-making accuracy.

The comparisons with other approaches have highlighted the following advantages of the proposed method: (1) The cognitive familiarity of the experts is considered as a measure of cognitive reliability, which offers multiple references with respect to evaluating results and which is helpful for facilitating reliable decision-making. (2) The three proposed linguistic granulation optimization models take into account both group consistency and cognitive reliability. These models enable higher levels of group consistency and cognitive reliability to be obtained, rendering the results produced more credible; in cases requiring a higher level of cognitive reliability, when no feasible solution is reached, the level of cognitive familiarity

Table 3 Differences between the proposed method and the AHP approach

Items compared	The proposed approach	AHP approach [30]
Initial forms of information provided by experts	Linguistic terms for cognitive familiarity, alternative grades, and attribute weights	Linguistic terms for attribute and alternative judgments
Methods of transforming the linguistic variables	Linguistic granulation optimization method based on group consistency and cognitive reliability	Predefined assignment values
Information regarding the weights assigned to experts	Cognitive reliability, group consistency, and prior subjective information	Individual decision-making without consideration of expert weights
Decision results	Ranked order of alternatives based on aggregated grade and cognitive reliability values	Ranked order of alternatives
Qualitative and quantitative analyses	Analysis more quantitative in nature	Analysis more qualitative in nature

is too low to satisfy the required degree of reliability, and the initial information should be adjusted in order to ensure a sufficiently high level of decision-making reliability. (3) The weights assigned to experts are measured based on consideration of group consistency, cognitive reliability, and subjective information, thus making them more objective. It should be noted that the quantity of expert information considered in Method 3 can involve the use of ineffective information, and that cognitive familiarity is an indicator of the effectiveness of the information held by the experts. The use of cognitive familiarity is thus more appropriate for describing experts.

Conclusion

We have studied decision-making problems characterized by uncertainty (e.g., SME selection based on the subjective attitudes of decision-makers) by exploring the possibility of incorporating consideration of cognitive reliability and group consistency. Three initial linguistic terms are expressed in terms of alternative grades, attribute weights, and cognitive familiarity. Cognitive reliability is determined from cognitive familiarity calculated using the first optimization model (M1). Then, two additional optimization models ((M2) and (M3)) are proposed as a means of generating attribute weights and alternative grades based on group consistency and cognitive reliability. Since the aggregated results include values for alternative grades and cognitive reliability, the proposed selection method combines these two values to enable a clear comparison among alternatives. Comparisons involving SME selection and previously published methods demonstrate that the proposed method can satisfy decision-making requirements with higher levels of group consistency and reliability. Based on our sensitivity analysis, cognitive reliability has a substantial effect on selection results and can serve as a positive reference for administrators who wish to enhance the reliability of their decisions; it thus should be considered during the fusion process.

In addition to its application for SME selection, our proposed method can be applied to other decision-making problems involving uncertain information and subjective expressions. Further studies should be conducted with the goal of developing approaches that can be applied to dynamic group decision-making involving non-cooperative behavior. Methods of information inference should also be studied in an effort to enhance the accuracy of the cognitive reliability values. These issues could be investigated in future work.

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Compliance with Ethical Standards

Conflict of interests The authors declare that they have no conflict of interest.

Research Involving Human Participants and/or Animals This article does not describe any studies involving human participants or animals performed by any of the authors.

Appendix A: Calculation of $F_{pp}(x)$ and $F_{qp}(x)$

Five situations are considered for the computation of $F_{pp}(x)$ and $F_{qp}(x)$:

1. If $0=\hat{g}_p^G<\hat{g}_q^G<M/2$, then $F_{pp}(x)=0$ and

$$F_{qp}(x) = \int_0^{\hat{g}_q^G}\left[\frac{\delta_q\left(x-\hat{g}_q^G\right)}{M-\hat{g}_q^G}+\delta_q\right]dx + \int_{\hat{g}_q^G}^{M}\frac{\delta_q(M-x)}{M-\hat{g}_q^G}dx = \frac{\delta_q\left[M^2-2\left(\hat{g}_q^G\right)^2\right]}{2\left(M-\hat{g}_q^G\right)}.$$

2. If $0=\hat{g}_p^G<M/2\le\hat{g}_q^G$, then $F_{pp}(x)=0$ and

$$F_{qp}(x) = \int_0^{\hat{g}_q^G}\frac{\delta_q x}{\hat{g}_q^G}dx+\int_{\hat{g}_q^G}^{M}\left(\frac{-\delta_q x}{\hat{g}_q^G}+2\delta_q\right)dx = \frac{\delta_q\left[4M\hat{g}_q^G-M^2-2\left(\hat{g}_q^G\right)^2\right]}{2\hat{g}_q^G}.$$

3. If $0<\hat{g}_p^G<\hat{g}_q^G<M/2$, we have

$$F_{pp}(x) = \int_{\hat{g}_p^G}^{M}\frac{\delta_p(M-x)}{M-\hat{g}_p^G}dx=\delta_p\left[\frac{Mx}{M-\hat{g}_p^G}\Bigg|_{\hat{g}_p^G}^{M}-\frac{x^2}{2\left(M-\hat{g}_p^G\right)}\Bigg|_{\hat{g}_p^G}^{M}\right]=\frac{\delta_p\left(M-\hat{g}_p^G\right)}{2},$$

$$\begin{aligned}F_{qp}(x) &= \int_{\hat{g}_p^G}^{\hat{g}_q^G}\left[\frac{\delta_q\left(x-\hat{g}_q^G\right)}{M-\hat{g}_q^G}+\delta_q\right]dx+\int_{\hat{g}_q^G}^{M}\frac{\delta_q(M-x)}{M-\hat{g}_q^G}dx\\ &= \delta_q\left[\frac{x^2}{2\left(M-\hat{g}_q^G\right)}+\frac{\left(M-2\hat{g}_q^G\right)x}{M-\hat{g}_q^G}\right]\Bigg|_{\hat{g}_p^G}^{\hat{g}_q^G}+\delta_q\left[\frac{Mx}{M-\hat{g}_q^G}-\frac{x^2}{2\left(M-\hat{g}_q^G\right)}\right]\Bigg|_{\hat{g}_q^G}^{M}\\ &= \frac{\delta_q\left[M^2-2M\hat{g}_p^G-\left(\hat{g}_p^G\right)^2+4\hat{g}_p^G\hat{g}_q^G-2\left(\hat{g}_q^G\right)^2\right]}{2\left(M-\hat{g}_q^G\right)}.\end{aligned}$$

4. If $0<\hat{g}_p^G<M/2\le\hat{g}_q^G$, we have

$$F_{pp}(x) = \int_{\hat{g}_p^G}^{M}\frac{\delta_p(M-x)}{M-\hat{g}_p^G}dx=\frac{\delta_p\left(M-\hat{g}_p^G\right)}{2},$$

$$F_{qp}(x) = \int_{\hat{g}_p^G}^{\hat{g}_q^G}\frac{\delta_q x}{\hat{g}_q^G}dx+\int_{\hat{g}_q^G}^{M}\left(\frac{-\delta_q x}{\hat{g}_q^G}+2\delta_q\right)dx = \frac{\delta_q\left[4M\hat{g}_q^G-M^2-2\left(\hat{g}_q^G\right)^2-\left(\hat{g}_p^G\right)^2\right]}{2\hat{g}_q^G}.$$

5. If $M/2<\hat{g}_p^G<\hat{g}_q^G$, we have

$$F_{pp}(x) = \int_{\hat{g}_p^G}^{M}\left(\frac{-\delta_p x}{\hat{g}_p^G}+2\delta_p\right)dx = \frac{\delta_p\left[4M\hat{g}_p^G-M^2-3\left(\hat{g}_p^G\right)^2\right]}{2\hat{g}_p^G},$$

$$F_{qp}(x) = \int_{\hat{g}_p^G}^{\hat{g}_q^G}\frac{\delta_q x}{\hat{g}_q^G}dx+\int_{\hat{g}_q^G}^{M}\left(\frac{-\delta_q x}{\hat{g}_q^G}+2\delta_q\right)dx = \frac{\delta_q\left[4M\hat{g}_q^G-M^2-2\left(\hat{g}_q^G\right)^2-\left(\hat{g}_p^G\right)^2\right]}{2\hat{g}_q^G}.$$

Appendix B: Initial Linguistic Terms Given by the Experts

$$e^1=\begin{bmatrix}(g_6,c_5,h_6),(g_7,c_5,h_6),(g_5,c_4,h_6),(g_6,c_4,h_6),(g_4,c_5,h_7),(g_6,c_4,h_6),(g_6,c_5,h_5),(g_7,c_5,h_7),\\(g_5,c_2,h_6),(g_6,c_5,h_6),(g_6,c_3,h_5),(g_6,c_2,h_7),(g_5,c_4,h_6),(g_4,c_5,h_6),(g_7,c_4,h_7),(g_6,c_3,h_6);\\(g_5,c_5,h_6),(g_6,c_4,h_6),(g_6,c_4,h_6),(g_6,c_5,h_6),(g_5,c_5,h_7),(g_6,c_3,h_6),(g_6,c_4,h_5),(g_7,c_5,h_7),\\(g_6,c_3,h_6),(g_7,c_5,h_6),(g_5,c_3,h_5),(g_6,c_2,h_7),(g_4,c_3,h_6),(g_5,c_5,h_6),(g_6,c_5,h_7),(g_6,c_3,h_6);\\(g_6,c_5,h_6),(g_6,c_4,h_6),(g_5,c_4,h_6),(g_6,c_4,h_6),(g_6,c_5,h_7),(g_6,c_4,h_6),(g_6,c_5,h_5),(g_7,c_4,h_7),\\(g_6,c_3,h_6),(g_6,c_4,h_6),(g_5,c_2,h_5),(g_6,c_2,h_7),(g_5,c_3,h_6),(g_6,c_5,h_6),(g_7,c_5,h_7),(g_6,c_4,h_6).\end{bmatrix};$$

$$e^2 = \begin{bmatrix} (g_5,c_5,h_6), (g_6,c_5,h_5), (g_5,c_5,h_7), (g_4,c_4,h_6), (g_6,c_5,h_6), (g_4,c_4,h_6), (g_6,c_5,h_6), (g_4,c_5,h_6), \\ (g_4,c_4,h_7), (g_5,c_5,h_6), (g_4,c_3,h_6), (g_5,c_3,h_5), (g_5,c_4,h_6), (g_6,c_5,h_7), (g_5,c_5,h_6), (g_5,c_4,h_7); \\ (g_4,c_5,h_6), (g_6,c_4,h_5), (g_4,c_5,h_7), (g_4,c_4,h_6), (g_6,c_5,h_6), (g_5,c_4,h_6), (g_5,c_5,h_6), (g_4,c_5,h_6), \\ (g_5,c_4,h_7), (g_5,c_4,h_6), (g_5,c_4,h_6), (g_5,c_3,h_5), (g_4,c_5,h_6), (g_5,c_4,h_7), (g_6,c_5,h_6), (g_4,c_3,h_7); \\ (g_6,c_5,h_6), (g_7,c_5,h_5), (g_5,c_5,h_7), (g_6,c_5,h_6), (g_6,c_5,h_6), (g_7,c_4,h_6), (g_6,c_5,h_6), (g_5,c_5,h_6), \\ (g_6,c_4,h_7), (g_5,c_5,h_6), (g_6,c_3,h_6), (g_5,c_2,h_5), (g_5,c_5,h_6), (g_6,c_5,h_7), (g_6,c_5,h_6), (g_5,c_4,h_7). \end{bmatrix};$$

$$e^3 = \begin{bmatrix} (g_5,c_5,h_6), (g_6,c_5,h_5), (g_6,c_5,h_6), (g_4,c_5,h_6), (g_5,c_5,h_6), (g_4,c_5,h_6), (g_6,c_5,h_7), (g_4,c_5,h_6), \\ (g_4,c_5,h_7), (g_5,c_5,h_6), (g_5,c_4,h_6), (g_6,c_4,h_6), (g_6,c_4,h_7), (g_5,c_5,h_7), (g_5,c_4,h_5), (g_5,c_5,h_7); \\ (g_6,c_5,h_6), (g_6,c_4,h_5), (g_5,c_4,h_6), (g_5,c_4,h_6), (g_4,c_5,h_6), (g_3,c_4,h_6), (g_5,c_5,h_7), (g_3,c_5,h_6), \\ (g_4,c_4,h_7), (g_4,c_5,h_6), (g_5,c_4,h_6), (g_5,c_2,h_6), (g_4,c_4,h_7), (g_4,c_5,h_7), (g_6,c_4,h_5), (g_5,c_5,h_7); \\ (g_6,c_5,h_6), (g_6,c_5,h_5), (g_5,c_5,h_6), (g_6,c_5,h_6), (g_6,c_5,h_6), (g_5,c_5,h_6), (g_6,c_5,h_7), (g_5,c_5,h_6), \\ (g_6,c_4,h_7), (g_6,c_5,h_6), (g_6,c_5,h_6), (g_5,c_4,h_6), (g_6,c_5,h_7), (g_5,c_3,h_7), (g_6,c_5,h_5), (g_5,c_5,h_7). \end{bmatrix}.$$

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