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# Characteristics of chemical reaction and convective boundary conditions in Powell-Eyring nanofluid flow along a radiative Riga plate

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## Abstract

A typical base fluid such as water, oil or glycol is poor conductor of heat due to deficient thermo-physical properties. This deficiency is normally addressed by saturation of thermally strong conductive metallic nanoparticles such as Fe, Ti, Hg, Cu, Au into the base fluid resulting a stronger thermal conductivity, electric conductivity, heat and mass flux of the so formulated nanofluid. Nanoparticles having a diameter size less than 100 nano-meter are preferred in this formulation because these nano-sized particles stay suspended into the base fluid for a longer time-period. This communication aims to investigate the salient features of a nanofluid flow along a radiative Riga plate using Powell-Eyring model and convective boundary conditions. The flow model involves the effect of first order chemical reaction as well as the Brownian motion diffusion and Thermophoresis effects. Governing PDEs are transformed into ODEs using suitable transformations. HAM is applied for convergent series solutions to the boundary value problem. Impact of various parameters including Brownian motion, Thermophoresis, modified Hartman number, Lewis and Prandtl number on flow profiles is analyzed graphically. Parameters of physical interest like Skin-friction, Nusselt and Sherwood numbers are illustrated through numerical data. Effect of modified Hartman number is significant

on flow profiles due to involvement of Riga plate. An efficiency is achieved in fluid flow and heat-mass flux through suspension of nanoparticles in base fluid.

Keywords: Applied mathematics, Condensed matter physics, Electromagnetism, Thermodynamics

## 1. Introduction

Recent studies have revealed that high profile cooling is mandatory for most of the technological as well as industrial processes. This high efficiency in heat-mass flux is not easy to achieve with typical fluids having poor thermo-physical properties. The issue was addressed to some extent by the introduction of the concept of nanofluids. A nanofluid is a fluid saturated with highly conductive nano-size metallic particles. These nanoparticles easily stay suspended in the base fluid for sufficiently longer time-period as compared to larger particles. Named after the great work presented by Choi [1] and the subsequent literature, nanofluids are remarkably promising in heat and mass transfer applications especially in micro processing, heat exchangers, aerospace technology, automotives and refrigeration etc. where high level energy devices are involved with a very small size and squeezed shape. Nanofluids are equally important for medical point of view. For example, the treatment of cancer by therapy, hyperthermia, treatment of wounds, surgeries and opening/unblocking the arteries etc. are direly dependent on the applications of nanofluids. Buongiorno [2] applied the idea of nanofluid and developed a mathematical model to explore and analyze the thermal characteristics of base fluids. The model is comprised of two factors, the Brownian motion factor (Nb) and the thermophoresis factor (Nt). Later on, numerous developments have been made in the field of nanofluids that can be seen in reference cited in this text (see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]) and cross references cited therein.

In fluid mechanics the importance of magnetic field holds a key place due to its various applications in enhancement of thermo-physical characteristics of a fluid. Fields such as astrology and earth sciences encounter many fluids that are poor conductors of electricity. Thus, an external agent is always needed to enhance the heat transfer phenomena through better conductivity and other related thermo-physical properties. This external agent could be a magnetic bar or permanently fixed array of such magnets with alternating electrodes. This kind of formulation was first done in Riga by Gailitis and Lielausis [22] and was given the name of Riga plate. The so formulated Riga plate is so convenient that it took no time to get famous in industrial processes involving fluid flow phenomena. Ahmed et al. [23] studied the impact of heating with zero mass flux at the surface. The conduction was eased by using a Riga plate in the model to involve the effect of Lorentz force that decays exponentially with increasing displacement from the surface of

plate. Sheikholeslami et al. [24] studied the impact of Lorentz forces on Marangoni convection of nanofluid. Shafiq et al. [25] studied fluid point flow based on Walters-B model through Riga plate. Adeel et al. [26] analyzed mixed convection of nanofluids with vertically placed Riga plate in the fluid flow. Recently, Rasool et al. [27] reported the impact of thermal radiation on Marangoni convective nanofluid flow towards a Riga plate.

Complicated systems in fluid mechanics undergo various chemical reactions inside the fluid. These reactions are of various types such as homogeneous reactions, heterogeneous reactions etc. having significant impact on the flow profiles. Chaudhary and Merkin [28] presented the impact of chemical reactions on a stagnation point fluid flow utilizing (i) equal and (ii) different diffusivities for auto-catalyst and reactants. Merkin [29] analyzed viscous flow alongside a flat plate considering the effect of reactions. Merkin et al. [30] is an extension of Merkin [29] to interpret the loss of catalyst. Khan and Pop [31, 32] investigated the effect of chemical reactions on viscoelastic fluids. The viscoelastic parameter reduced significantly for the presented model.

Complex rheological fluids are always difficult to be predicted through Navier Stokes equations. Grease, human blood, various paints, typical oil and polymer solutions etc. are typical examples of such fluids. There is a highly non-linear relation between stress and strain in such type of fluids as compared to the Newtonian fluids. To analyze their behavior, various models have been proposed including the Powell-Eyring model that is under consideration in this study. At an extreme shear stress rate, the fluid behaves like a Newtonian fluid however, it behaves like a non-Newtonian fluid at an intermediate shear stress rate. Instead of empirical relation, the extrapolation of constitutive expressions from kinetic theory of gases is a significant characteristic of this model. The model is important for its various industrial applications especially in polymer and chemical engineering processes. Numerous researches have been carried out in the context of Powell-Eyring nanofluid and its engineering and industrial application (see [33, 34, 35, 36, 37, 38] and reference cited therein).

The inspiration for this work is covered by three main aspects. Firstly, to formulate an incompressible nanofluid flow along a radiative Riga-plate in the presence of first order chemical reaction and convective boundary conditions at the surface. There is no literature available on such formulation in the past. Secondly, to interpret the effect of Thermophoresis, Brownian motion and Lorentz force generated by Riga plate on the velocity, temperature and concentration profiles. Finally, to note the variation in skin friction (wall drag force), Nusselt and Sherwood number that are significantly important factors in industrial applications of nanofluids.

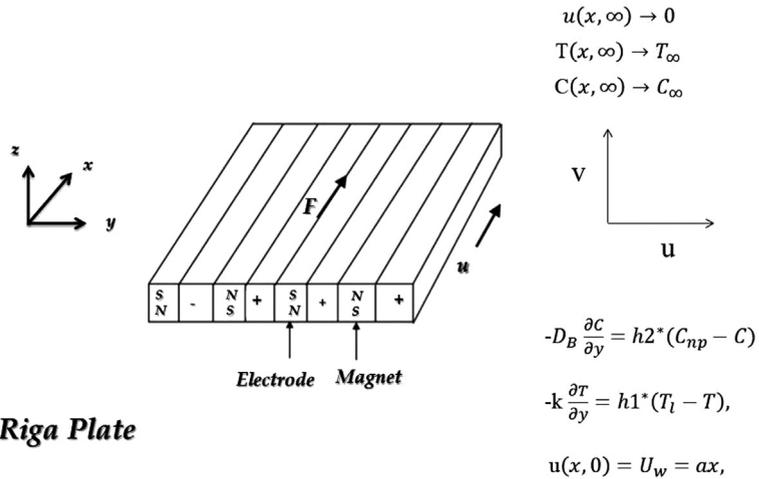


Figure 1. Schematic diagram.

## 2. Model

We assume an incompressible steady Powell-Eyring nanofluid flow along a radiative Riga surface in two-dimensions. Convective boundary conditions for heat and mass are utilized. The surface of Riga plate is heated because of hot fluid having initial temperature  $T_l$  with concentration of nanoparticles  $C_{np}$ , respectively that gives rise to  $h_1^*$  and  $h_2^*$ , the heat and mass transport coefficients. The fluid behaves electrically conducting along  $y$ -axis due to the Lorentz force generated by Riga plate as well as the nanoparticles saturation into the base fluid. The Lorentz force incorporated in the model using Grinberg term decays exponentially with an increasing displacement from the surface of plate. The  $x$ -axis is taken along the surface of Riga plate. The  $y$ -axis is taken normal to it. Schematic diagram can be seen in Figure 1. Following Hayat et al. [39] together with the Grinberg term and the first order chemical reaction term, the boundary value problem has following governing PDEs with subsequent BCs,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( v + \frac{1}{\rho_l \gamma C^*} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho_l \gamma C^{*3}} \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \left( \frac{\pi \cdot j_0 \cdot M_0 \exp\left(\frac{-\pi y}{b}\right)}{8\rho_l} \right), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left( D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right) - \frac{1}{(\rho c)_l} \frac{\partial q_r}{\partial y}, \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial y^2} \right) - K(C - C_\infty), \tag{4}$$

such that,

$$\begin{aligned}
 u = xa, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_1^*(T_l - T), \quad -D_B \frac{\partial C}{\partial y} = h_2^*(C_{np} - C) \quad \text{at } y = 0, \\
 u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty,
 \end{aligned}
 \tag{5}$$

where  $\mu$  represents dynamic viscosity,  $\rho_l$  is used for density of base fluid,  $\nu$  represents kinematic viscosity,  $\gamma$  and  $C^*$  denote the material constants,  $\sigma$  represents the electric-conductivity,  $\alpha$  is the thermal diffusivity,  $k$  is called thermal conductivity of nanofluid,  $(\rho c)_l$  is called the heat capacity of nanofluid,  $D_B$  is used for Brownian diffusion,  $D_T$  is used for thermophoretic diffusion,  $T_\infty$  and  $C_\infty$  are the temperature and concentration distributions away from Riga surface.  $j_0$  is the density of applied current through electrodes,  $M_0$  is the magnetization in magnets,  $b$  is width of magnets and electrodes and  $\tau$  is the ratio between productive heat capacity of fluid and nanoparticles.  $q_r$  represents the radiative heat-flux. Using Rosseland’s approximation and Taylor series expansion with omission of higher order terms beyond the 1st degree, we have,

$$T^4 \cong T_\infty^3(4T - 3T_\infty).
 \tag{6}$$

Eq. (6) in Eq. (7) yields,

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}.
 \tag{7}$$

Using (7) in (3), we have,

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_l} \left( D_B \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right) + \frac{16\sigma^*T_\infty^3}{3k^*(\rho c)_l} \frac{\partial^2 T}{\partial y^2}.
 \tag{8}$$

Define,

$$\begin{aligned}
 u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \\
 \sqrt{\frac{\nu}{a}} \eta = y, \quad \frac{\psi}{x\sqrt{\nu a}} = f(\eta), \\
 \theta(\eta) = \frac{T - T_\infty}{T_l - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_{np} - C_\infty}.
 \end{aligned}
 \tag{9}$$

System (9) in equations (1), (2), (3) and (4), results in following nonlinear ODEs with subsequent boundary conditions,

$$f''' + Kf''' - K\Lambda f''^2 f''' + f f'' - f'^2 + Q \exp(-\beta \eta) = 0,
 \tag{10}$$

$$\left( 1 + \frac{4}{3} Rd \right) \theta'' + Pr \left( Nb \theta' \phi' + f \theta' + Nt \theta'^2 \right) = 0,
 \tag{11}$$

$$\phi'' + Le Pr \left( f \phi' - K_1 \phi \right) + \frac{Nt}{Nb} \theta'' = 0,
 \tag{12}$$

$$f = 0, \quad \frac{\partial f}{\partial \eta} = 1, \quad \frac{\partial \theta}{\partial \eta} = -\alpha_1(1 - \theta(0)), \quad \frac{\partial \phi}{\partial \eta} = -\alpha_2(1 - \phi(0)) \text{ at } \eta = 0, \quad (13)$$

$$\frac{\partial f}{\partial \eta} \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (14)$$

Here  $Pr$  represent the Prandtl factor,  $Q$  is used for modified Hartman number,  $\beta$  is known as dimensionless parameter,  $Rd$  is the symbol of radiation parameter,  $Nb$  is the Brownian motion,  $Nt$  is the Thermophoretic parameter,  $\Lambda$  and  $K$  are fluid parameters,  $Le$  is Lewis number,  $\alpha_1$  and  $\alpha_2$  are heat-mass transfer Biot factors and  $K_1$  represents the chemical reaction, respectively. Mathematically,

$$Pr = \frac{\nu}{\alpha}, \quad Q = \frac{\pi j_0 M_0}{8 \rho_l x a^2}, \quad \beta = \frac{\pi}{b} \sqrt{\frac{\nu}{a}}, \quad Rd = \frac{4\sigma^* T_\infty^3}{kk^*},$$

$$Nb = \frac{(\rho c)_p D_B (C_{np} - C_\infty)}{(\rho c)_l \nu}, \quad Nt = \frac{(\rho c)_p D_T (T_l - T_\infty)}{(\rho c)_l \nu T_\infty}, \quad \Lambda = \frac{u^3}{2\nu x C^{*2}}, \quad (15)$$

$$K = \frac{1}{\mu\gamma C^*}, \quad Le = \frac{\alpha}{D_B}, \quad \alpha_1 = \frac{h_1^*}{k} \sqrt{\frac{\nu}{a}}, \quad \alpha_2 = \frac{h_2^*}{D_B} \sqrt{\frac{\nu}{a}}.$$

Skin-friction (drag force), Nusselt number (heat flux) and Sherwood number (mass flux) are written below,

$$Re_x^{1/2} C_{fx} = \left( (1 + K) f''(0) - \frac{1}{3} K \Lambda f'''(0) \right),$$

$$Re_x^{-1/2} Nu_x = - \left( 1 + \frac{4}{3} Rd \right) \theta'(0), \quad (16)$$

$$Re_x^{-1/2} Sh_x = -\phi'(0),$$

where  $Re_x$  represents the local Reynolds.

### 3. Methodology

Homotopy analysis method (HAM) is independent of small as well as large physical parameters. Thus, it is more efficient as compared to perturbation technique and other conventional methods for solving nonlinear systems. It is applicable in most of nonlinear system of equations developed in science problems, engineering and finance without small or large parameters [40, 41, 42, 43, 44, 45]. For convergent series solutions through HAM, we assume,

$$f_0 = 1 - e^{-\eta}, \quad \theta_0 = \frac{\alpha_1}{1 + \alpha_1} e^{-\eta}, \quad \phi_0 = \frac{\alpha_2}{1 + \alpha_2} e^{-\eta}, \quad (17)$$

$$\mathcal{L}_f = f''' - f', \quad \mathcal{L}_\theta = \theta'' - \theta, \quad \mathcal{L}_\phi = \phi'' - \phi. \quad (18)$$

Such that,

$$\mathcal{L}_f^* [D_1 + D_2 e^\eta + D_3 e^{-\eta}] = 0, \quad \mathcal{L}_\theta^* [D_4 e^\eta + D_5 e^{-\eta}] = 0, \quad \mathcal{L}_\phi^* [D_6 e^\eta + D_7 e^{-\eta}] = 0, \quad (19)$$

where  $D_i$  ( $i = 1 - 7$ ) are calculated through BCs. 0th order problems, are defined as,

$$\mathcal{N}_f = (1 + K) \frac{\partial^3 \tilde{f}}{\partial \eta^3} + \tilde{f} \frac{\partial^2 \tilde{f}}{\partial \eta^2} - \left( \frac{\partial \tilde{f}}{\partial \eta} \right)^2 - K \Lambda \left( \frac{\partial^2 \tilde{f}}{\partial \eta^2} \right)^2 \frac{\partial^3 \tilde{f}}{\partial \eta^3} + Q \exp(-\beta \eta), \quad (20)$$

$$\mathcal{N}_\theta = \left( 1 + \frac{4}{3} Rd \right) \frac{\partial^2 \tilde{\theta}}{\partial \eta^2} + Pr \left( Nb \frac{\partial \tilde{\phi}}{\partial \eta} \frac{\partial \tilde{\theta}}{\partial \eta} + \tilde{f} \frac{\partial \tilde{\theta}}{\partial \eta} + Nt \left( \frac{\partial \tilde{\theta}}{\partial \eta} \right)^2 \right), \quad (21)$$

$$\mathcal{N}_\phi = \frac{\partial^2 \tilde{\phi}}{\partial \eta^2} + Pr Le \left( \tilde{f} \frac{\partial \tilde{\phi}}{\partial \eta} - K_1 \phi \right) + \frac{Nt}{Nb} \frac{\partial^2 \tilde{\theta}}{\partial \eta^2}. \quad (22)$$

Setting  $p = 0, 1$ ,

$$\begin{aligned} \tilde{f}(\eta, p) &= f_0, & \tilde{\theta}(\eta, p) &= \theta_0, & \tilde{\phi}(\eta, p) &= \phi_0, & \text{at } p &= 0, \\ \tilde{f}(\eta; p) &= f, & \tilde{\theta}(\eta, p) &= \theta, & \tilde{\phi}(\eta, p) &= \phi, & \text{at } p &= 1. \end{aligned} \quad (23)$$

From initial guesses to final solutions, the value of  $p$  is varied from 0 to 1. Using Taylor's expansion and assuming that the expanded series converge for  $p = 1$ , the  $m$ th-order problems of deformation are defined below with subsequent boundary conditions,

$$\begin{aligned} \check{\mathcal{R}}_f^m(\eta) &= (1 + K) f_{m-1}''' + \sum_{k=0}^{m-1} f_{m-1-k} f_k'' - \sum_{k=0}^{m-1} f_{m-1-k}' f_k' \\ &\quad - K \Lambda \sum_{k=0}^{m-1} f_{m-1-k}'' \sum_{l=0}^k f_{k-l}'' f_l''' + Q \exp(-\beta \eta), \end{aligned} \quad (24)$$

$$\begin{aligned} \check{\mathcal{R}}_\theta^m(\eta) &= \left( 1 + \frac{4}{3} Rd \right) \theta_{m-1}'' + Pr Nb \sum_{k=0}^{m-1} \theta'_{m-1-k} \phi'_k + Pr \sum_{k=0}^{m-1} f_{m-1-k} \theta'_k \\ &\quad + Pr Nt \sum_{k=0}^{m-1} \theta'_{m-1-k} \theta'_k, \end{aligned} \quad (25)$$

$$\check{\mathcal{R}}_\phi^m(\eta) = \phi_{m-1}''(\eta) + Pr Le \sum_{k=0}^{m-1} f_{m-1-k} \phi'_k - Pr Le K_1 \phi_{m-1}(\eta) + \frac{Nt}{Nb} \theta_{m-1}'', \quad (26)$$

$$\left. \begin{aligned} f_m(0) &= 0, & f'_m(0) &= 0, & \theta'_m(0) - \alpha_1 \theta_m(0) &= 0, & \phi'_m(0) - \alpha_2 \phi_m(0) &= 0, \\ f'_m(\infty) &= 0, & \theta_m(\infty) &= 0, & \phi_m(\infty) &= 0, \end{aligned} \right\} \quad (27)$$

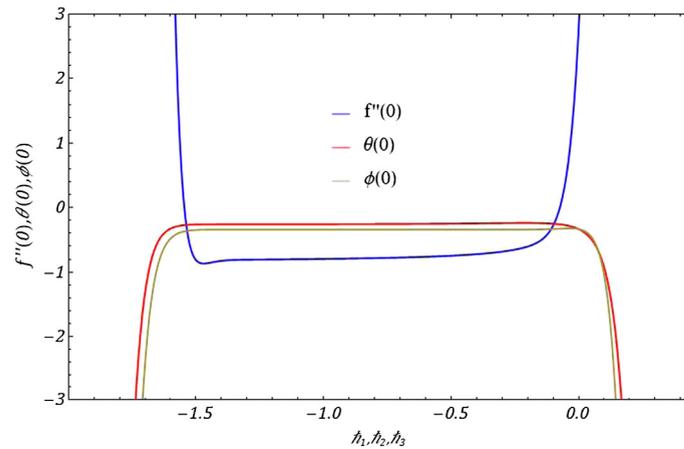
where,  $\chi_m=1$ , for  $m > 1$  and 0 otherwise. The general solutions are expressed below,

$$f_m(\eta) = f_m^*(\eta) C D_1 + D_2 e^\eta + D_3 e^{-\eta}, \quad (28)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + D_4 e^\eta + D_5 e^{-\eta}, \quad (29)$$

$$\phi_m(\eta) = \phi_m^*(\eta) + D_6 e^\eta + D_7 e^{-\eta}, \quad (30)$$

where  $D_i$  ( $i = 1 - 7$ ) are found through BCs and asterisk is used for special solutions.



**Figure 2.** Convergence interval of  $f$ ,  $\theta$  and  $\phi$ .

#### 4. Analysis

The series solutions obtained through HAM involve auxiliary parameters. The parameters are called convergence controlling parameters and a speedy convergence is dependent on best choice of these parameters. The HAM-curves on the bases of suitable values for auxiliary parameters are plotted in Figure 2 for the three flow profiles velocity, temperature and concentration, respectively. The meaningful intervals of convergence for the three flow profiles are  $[-1.5, -0.5]$ ,  $[-1.5, 0.00]$  and  $[-1.50, 0.00]$ , respectively.

#### 5. Results & discussion

In this section we explore the influence of various fluid parameters on the flow profiles. Fluid parameters like modified Hartman factor  $Q$ , dimensionless parameter  $\beta$ , fluid parameters  $K$  and  $\Lambda$ , the heat and mass Biot numbers  $\alpha_1, \alpha_2$ , Brownian motion factor  $Nb$  and Thermophoretic parameter  $Nt$ , radiation parameter  $Rd$ , Prandtl factor  $Pr$ , Lewis number  $Le$  and chemical reaction  $K_1$  are of significant importance. Figures 3, 4 and 5 present the influence of  $\beta$ ,  $K$  and  $Q$  on the velocity profile. The dimensionless parameter  $\beta$  significantly reduces the flow profile. The enhanced values of  $\beta$  continuously enhance the fluid viscosity that results in slow motion of the fluid and the profile declines. Figure 4 display the influence of  $K$  on velocity field and corresponding boundary layer. An enhancement in velocity is noticed towards augmented values of the respective fluid parameter.  $K$  is inverse of fluid viscosity  $\mu$ . An enhancement in  $K$  results in reduction of viscosity of the fluid that ultimately, enhances the velocity profile. The modified Hartman factor  $Q$  results in enhancement in velocity field as displayed in Figure 5. The Lorentz forces generated by Riga plate parallel to the surface result in more surface tension. This tension, agitates the fluid to flow with enhanced velocity that results in augmenting

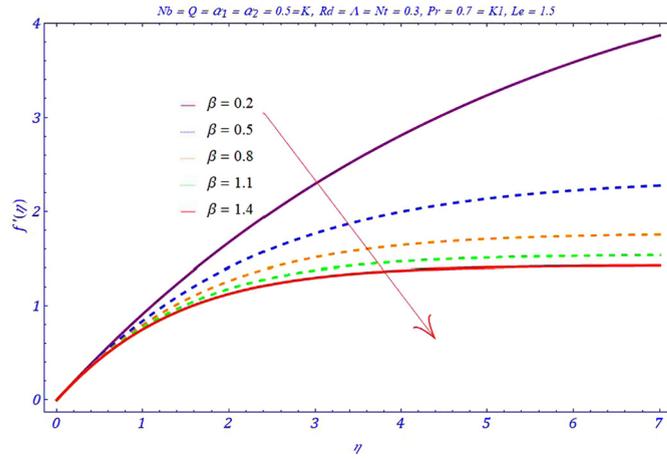


Figure 3. Impact of  $\beta$  on  $f'(\eta)$ .

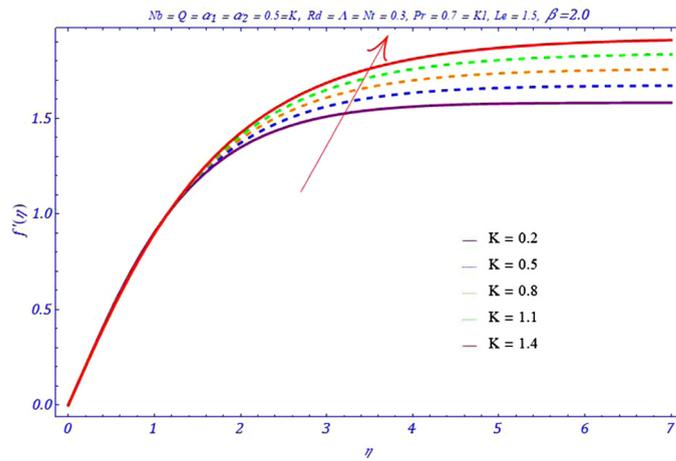


Figure 4. Impact of  $K$  on  $f'(\eta)$ .

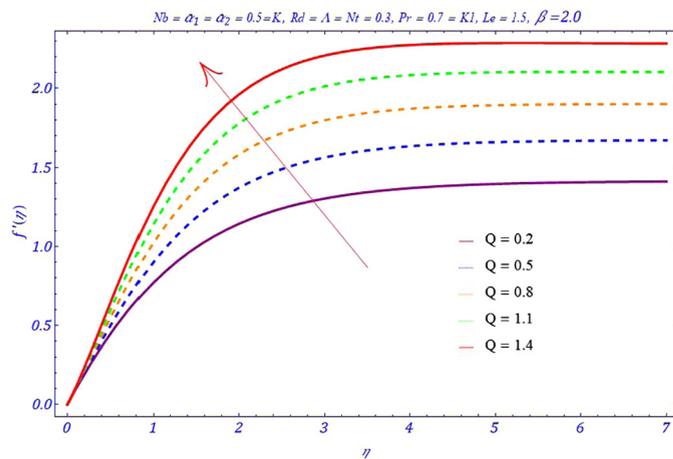
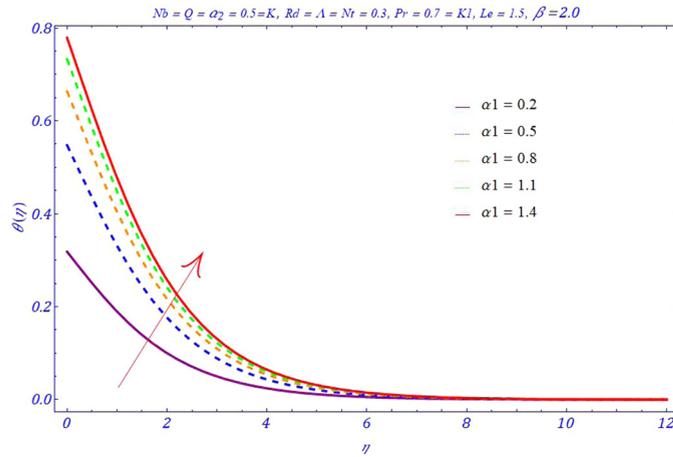
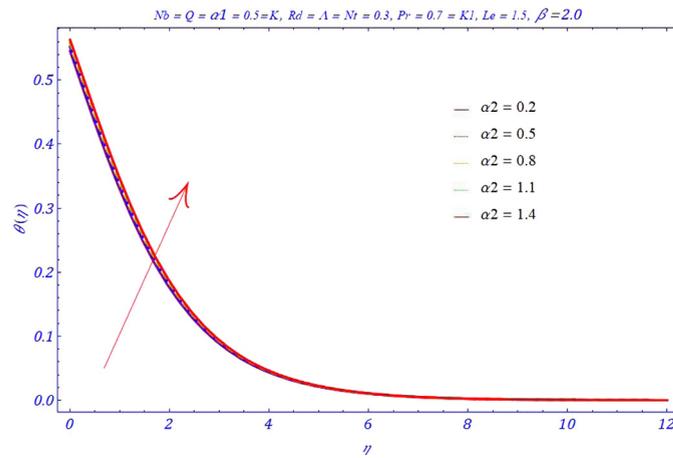


Figure 5. Impact of  $Q$  on  $f'(\eta)$ .



**Figure 6.** Impact of  $\alpha_1$  on  $\theta(\eta)$ .



**Figure 7.** Impact of  $\alpha_2$  on  $\theta(\eta)$ .

the corresponding boundary layer. Figures 6, 7, 8, 9, 10, 11, 12 and 13 are the graphs of various fluid parameters on thermal profile. In particular, Figure 6 presents the influence of  $\alpha_1$  on temperature profile. Enhancement in  $\alpha_1$  results in a more stronger convection that rises the temperature profile. Consequently, the associated boundary layer augments for enhanced values of  $\alpha_1$ . Figure 7 presents the influence of  $\alpha_2$  on the temperature profile. The enhanced  $\alpha_2$  results in increased mass convection. This enhancement results in rise of the temperature profile and corresponding boundary layer. An augment in  $K$  results in reduction of temperature and corresponding boundary layer as portrayed in Figure 8.

The dynamic viscosity involved in  $\gamma$  reduces with enhanced values of  $K$  that reduces the temperature profile. Both, the Brownian motion factor and Thermophoresis effect result in rise of the temperature profile as presented in Figure 9 and Figure 10. The un-predictive motion of fluid particles enhances with a stronger Brownian motion effect under the action of a strong Thermophoretic force generated by

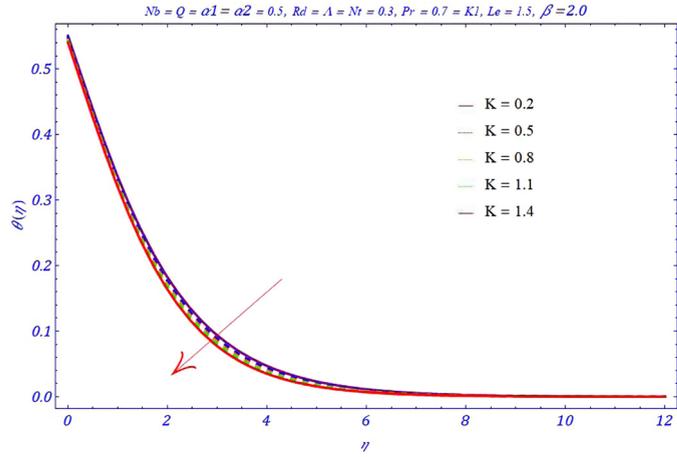


Figure 8. Impact of  $K$  on  $\theta(\eta)$ .

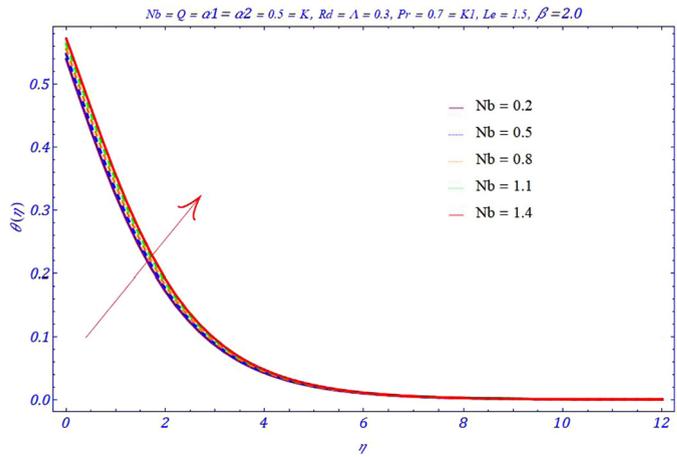


Figure 9. Impact of  $Nb$  on  $\theta(\eta)$ .

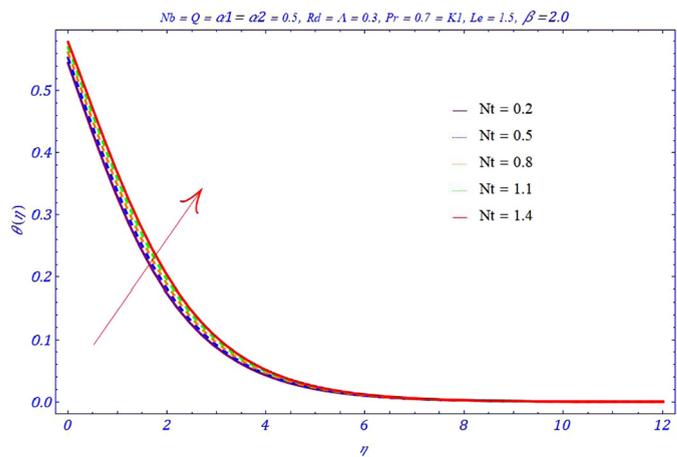


Figure 10. Impact of  $Nt$  on  $\theta(\eta)$ .

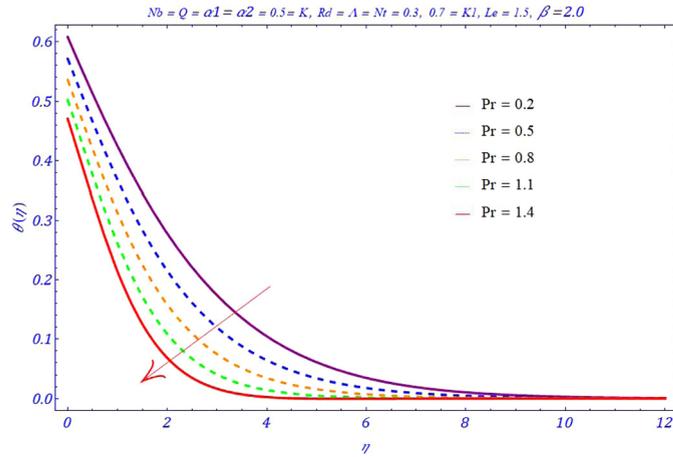


Figure 11. Impact of  $Pr$  on  $\theta(\eta)$ .

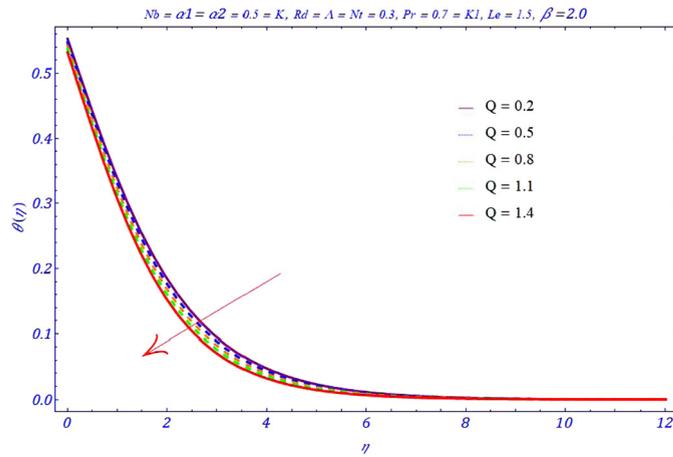


Figure 12. Impact of  $Q$  on  $\theta(\eta)$ .

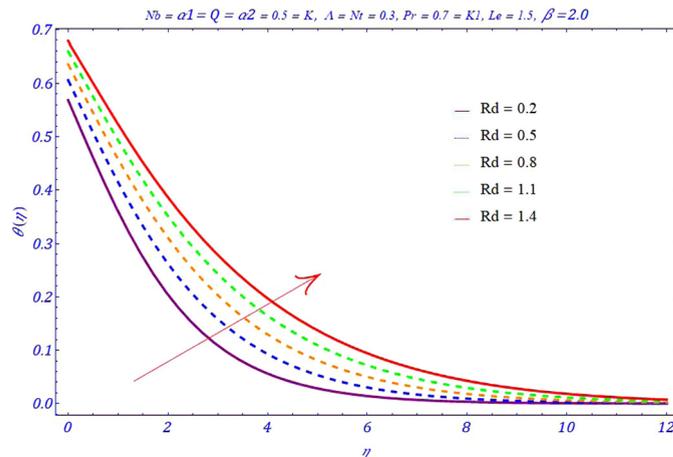


Figure 13. Impact of  $Rd$  on  $\theta(\eta)$ .

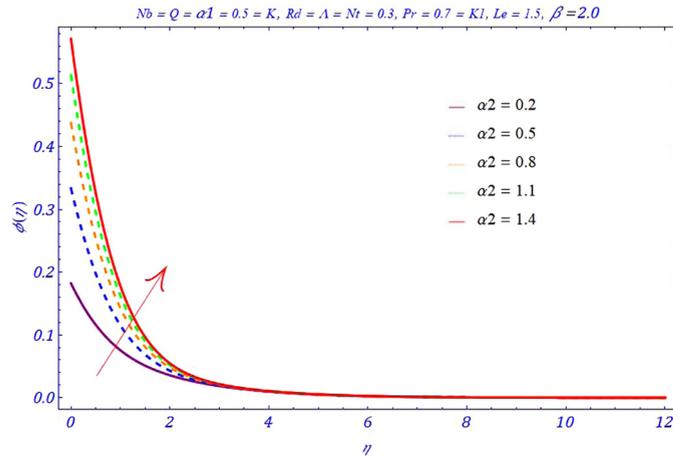


Figure 14. Impact of  $\alpha_2$  on  $\phi(\eta)$ .

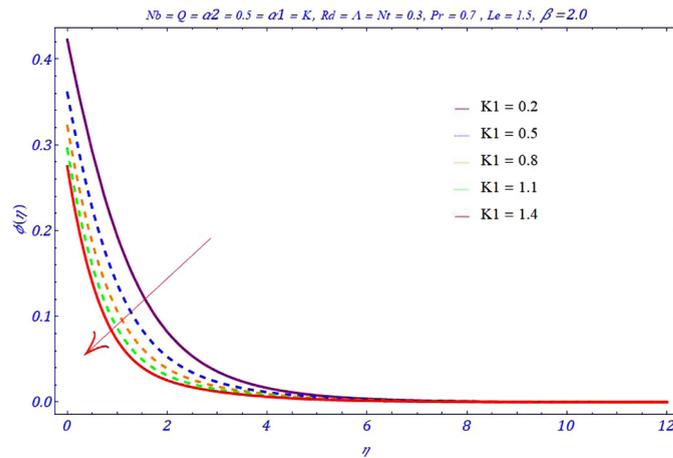


Figure 15. Impact of  $K_1$  on  $\phi(\eta)$ .

the stronger Thermophoresis factor. Consequently, the temperature profile and associated boundary layer increase with augmented values of the  $Nb$  and  $Nt$ , respectively. The momentum to mass diffusivity ratio directly influence the temperature profile portrayed in Figure 11. A decreasing behavior is noticed towards augmented values of the involved parameter i.e.  $Pr$ . The stronger modified Hartman number results in decline of temperature profile and corresponding boundary layer as visualized in Figure 12. Figure 13 presents the effect of Radiation parameter on thermal profile. The enhanced values of radiation parameter result in enhancement of the temperature due to a stronger heat source. Figures 14, 15, 16 and 17 are the graphs of various parameters against the concentration profile. The enhanced  $\alpha_2$  results in enhancement of concentration as displayed in Figure 14. The Biot factor is related to the mass convection. The stronger mass Biot number results in higher concentration profile and enhanced thickness of boundary layer. Figure 15 is the impact of chemical reaction on the concentration profile. The enhanced values

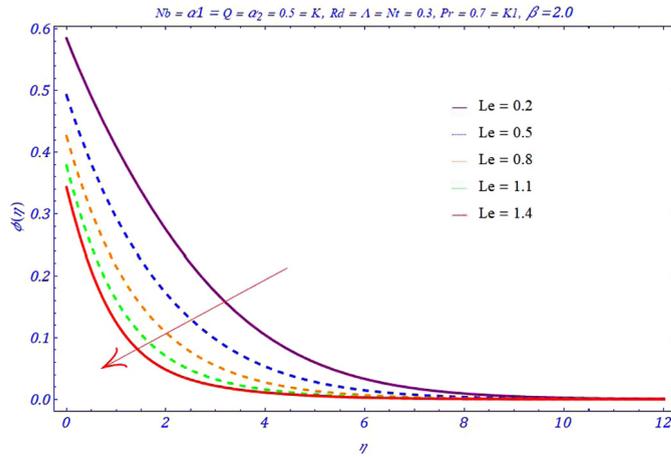


Figure 16. Impact of  $Le$  on  $\phi(\eta)$ .

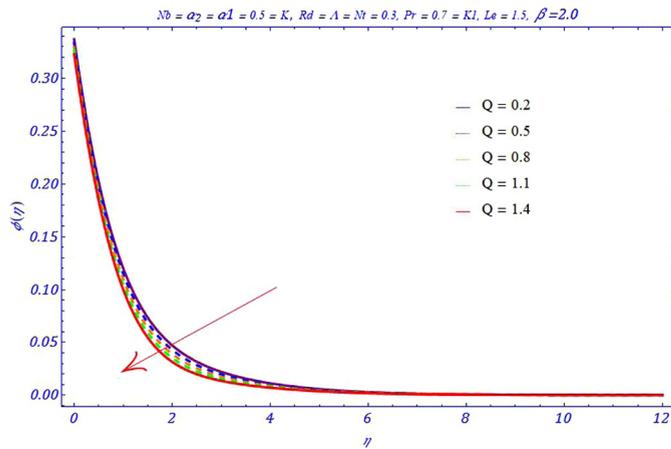


Figure 17. Impact of  $Q$  on  $\phi(\eta)$ .

of  $K_1$  result in an enhanced penetration of fluid particles near the surface that reduces the associated boundary layer. Figures 16 and 17 are the graphs of Lewis number and modified Hartman numbers against concentration profile, respectively. The concentration profile is found as a decreasing function of both the parameters. Table 1 is compiled on the basis of data obtained through numerical simulation in Mathematica 9.0 using HAM based code. The results are presented to analyze the behavior of skin friction against pertinent parameters. The drag force (skin-friction) is found as an increasing function of  $\beta$  and  $K$ . However, the force reduces with augmented values of the modified Hartman number. Data presented in Table 2 is obtained for Nusselt and Sherwood numbers for various pertinent parameters. The values are varied from 0.0 to 1.0. Clearly, the heat flux enhances with augmented values of  $Q$ ,  $Rd$  and  $K_1$ . It reduces with augmented values of  $\beta$ ,  $\Lambda$ ,  $Nb$  and  $Nt$ . The mass flux enhances with augmented values of  $Q$ ,  $Nb$ ,  $Rd$  and  $K_1$ .

**Table 1.** Numerical data of Skin-friction (Drag force) for various fluid parameters.

$\beta$	$K$	$Q$	$-\text{Re}_x^{1/2} C_{fx}$
0.0	0.3	0.3	0.63628
0.5			0.79809
1.0			0.87679
2.0	0.0	0.3	0.39629
	0.5		0.50698
	1.0		0.61517
2.0	0.3	0.0	1.0920
		0.5	0.46026
		1.0	0.11377

**Table 2.** Numerical data of Nusselt number (heat flux) and Sherwood number (mass flux) for different values of fluid parameters.

$\beta$	$Q$	$\Lambda$	$Nb$	$Nt$	$Rd$	$K_1$	$\text{Re}_x^{-1/2} Nu_x$	$\text{Re}_x^{-1/2} Sh_x$
0.0	0.5	0.3	0.5	0.3	0.3	0.7	0.3459	0.3469
0.5							0.3305	0.3363
1.0							0.3252	0.3327
2.0	0.0	0.3	0.5	0.3	0.3	0.7	0.3181	0.3280
	0.5						0.3213	0.3301
	1.0						0.3245	0.3323
2.0	0.5	0.0	0.5	0.3	0.3	0.7	0.3214	0.3302
		0.5					0.3213	0.3301
		1.0					0.3211	0.3301
2.0	0.5	0.3	0.1	0.3	0.3	0.7	0.3286	0.2724
			0.6				0.3195	0.3326
			1.1				0.3104	0.3380
2.0	0.5	0.3	0.5	0.0	0.3	0.7	0.3269	0.3432
				0.5			0.3176	0.3222
				1.0			0.3084	0.3052
2.0	0.5	0.3	0.5	0.3	0.0	0.7	0.2509	0.3269
					0.5		0.3636	0.3317
					1.0		0.4608	0.3350
2.0	0.5	0.3	0.5	0.3	0.3	0.0	0.3210	0.2528
						0.5	0.3212	0.3154
						1.0	0.3216	0.3462

## 6. Conclusions

We considered an in-compressible Powell-Eyring nanofluid flow along a radiative Riga surface in two-dimensions. Convective conditions have been utilized along with the first order chemical reaction effect. Following are salient conclusions of this numerical study:

- The velocity field enhances for augmented values of modified Hartman number whereas a reduction is noticed in thermal and solutal boundary layers.
- The enhanced  $K_1$  effect reduces the nanoparticles concentration near the radiative Riga surface.
- Enhancement in radiation effect enhances the thermal boundary layer.

- Temperature is decreasing function of  $Pr$ ,  $Q$  and  $K$ .
- Both  $\alpha_1$  and  $\alpha_2$  are increasing factors for temperature profile. An enhancement is noticed in concentration profile with enhancement in mass Biot number.
- Brownian motion factor and Thermophoresis result in enhancement of thermal boundary layer.
- Concentration profile declines for stronger values of Lewis number.
- The wall drag force (skin-friction) is found a mounting function of  $\beta$  and  $K$ . However, the force reduces with augmented values of the  $Q$ .
- The heat flux enhances with augmented values of  $Q$ ,  $Rd$  and  $K_1$ . It reduces with augmented values of  $\Lambda$ ,  $Nb$ ,  $\beta$ , and  $Nt$ .
- The mass flux enhances with augmented values of  $Nb$ ,  $Rd$ ,  $Q$ , and  $K_1$ .

## Declarations

## Author contribution statement

Ghulam Rasool: Conceived and designed the analysis; Analyzed and interpreted the data; Wrote the paper.

Ting Zhang: Contributed analysis tools or data.

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## Additional information

No additional information is available for this paper.

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