Asymmetric transmission of elastic shear vertical waves in solids

Longhai Zeng[a], Jun Zhang[a,b,*], Yaolu Liu[a], Youxuan Zhao[a], Ning Hu[a,*]

[a] College of Aerospace Engineering, Chongqing University, Chongqing 400044, China
[b] Chongqing Key Laboratory of Heterogeneous Material Mechanics, Chongqing University, Chongqing 400044, China

ABSTRACT

In this work, we propose an asymmetric transmission structure (ATS) for elastic shear vertical (SV) waves in solids, which has been relatively unexplored. The ATS is constituted by a metasurface and a phononic crystal (PC) possessing a directional band gap. While the metasurface aims to redirect the incident wave, the PC acts as a directional filter. The metasurface is composed of a stacked array of composite plates with two connecting parts made of different materials. To examine the performance of the designed ATS, full numerical simulations have been conducted. The numerical results indicate that the proposed ATS offered a relatively broad working frequency band and had a one order of magnitude difference in terms of transmission between the positive and negative incidences. Our study provides an alternative method to control elastic SV waves and could benefit applications in various fields, such as Micro-Electro-Mechanical System (MEMS), in which thin plates are frequently used components.

1. Introduction

Asymmetric energy flow, a hallmark of electric diodes, has revolutionized modern communication and electronic devices. Asymmetric energy transmission is of importance in other physical fields as well and has been demonstrated in optics [1], therm-dynamics [2–4], acoustics [5–11], and elasto-dynamics [12–16].

There are primarily two ways to realize asymmetric transmission of waves: either by introducing nonlinearity into the system to break the time-reversal symmetry or using asymmetric linear structures to break the spatial symmetry. In general, an asymmetric transmission structure mainly contains two parts: a component to change the behavior (i.e., propagation directions, modes, and frequencies) of waves and a filter. For the sake of brevity, the first part is named "rectifier" in this work. The filter is usually selected as a PC [17] with a full or partial band gap. More specifically, to break the time-reversal symmetry, the rectifier is usually made of a nonlinear material [5,12,18,19] which functions to modulate frequencies. Additionally, a PC is well designed to prohibit the fundamental wave instead of the after-induced higher order harmonic waves. Thus the asymmetric transmission is realized as the fundamental wave comes from different sides. In some cases [15,16], the rectifier and the filter are integrated together through a nonlinear periodic structure. In these cases, the periodic structure usually inherits a loading-dependent band structure, and frequencies of the incident waves are preserved.

To break the spatial symmetry, various ingenious designs of asymmetric linear structures [13,20–27] have been proposed. Among them, the ATS constituted by a metasurface [28] in addition to a PC has been confirmed to be an effective tool [24,29], in which the metasurface is designed to redirect incident waves and the PC possesses a directional band gap. The idea is that, the incident waves would be redirected after passing through the designed metasurface and subsequently fall within the pass or the forbidden band of PCs. On the contrary, waves incident from the opposite direction would be directly forbidden or allowed by the PC. For example, Song et al. [24] have designed such an ATS for acoustic waves, in which the metasurface is made of slits filled with two noble gases (argon and xenon) [30], and Shen et al. [29] have proposed a design in which a metamaterial with near-zero index is selected as the filter. In the designs by Yuan et al. [31] and He et al. [32], the role of metasurfaces is replaced by waveguides. It is worth noting that existing reports are mainly focused on acoustic waves, whereas studies on elastic waves have not been extensively reported, possibly due to the complexity caused by the coupling between longitudinal and transverse waves existing in solids.

In this work, a corresponding ATS constituted by a metasurface and a PC for elastic SV waves in solids was proposed. The metasurface was constituted by a stacked array of composite plates, which were composed of two connecting parts made of two different materials. A phononic crystal with a square lattice was selected, which has a directional band gap between 64.5 kHz and 84.5 kHz for the shear modes. Although the size considered in this work is at the macro-scale, such a design can be scaled down to the micro- and nano-scale as well.

* Corresponding authors.
E-mail addresses: mejzhang@cqu.edu.cn (J. Zhang), ninghu@cqu.edu.cn (N. Hu).
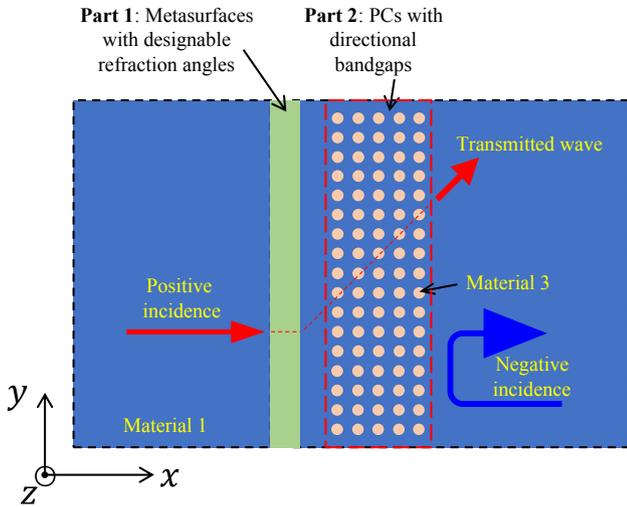
Fig. 1. Two dimensional (2D) schematic of the ATS for elastic SV waves in solids, which is composed of two parts including a metasurface and a PC with directional band gaps.
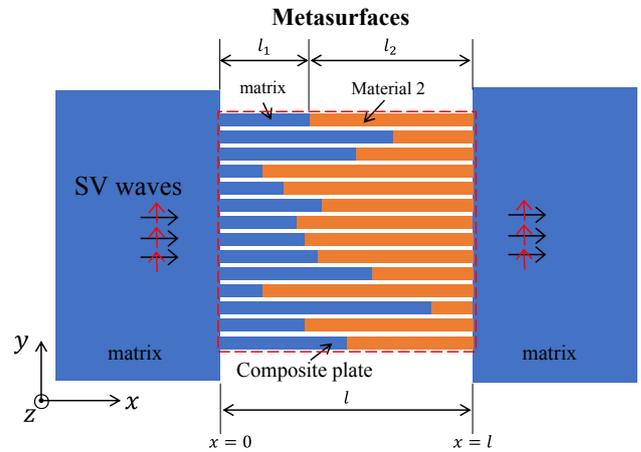

Fig. 2. Two dimensional diagram of the metasurfaces acting as the role to re-direct the incident wave, which are made of parallel thin composite plates connected at both ends to the supporting bases. Different colors represent different materials. Adapted from [33], copyright Zhang et al. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Therefore, our study is beneficial to the Micro-Electro-Mechanical System (MEMS) in which thin plates are the typical components and specific waves, e.g., elastic SV waves, should be controlled at certain frequency ranges.

The remaining sections of the paper are organized as follows. Two components of the ATS including a metasurface and a PC are introduced and analyzed in Section 2. Full numerical simulations validating the asymmetric transmission performance of the ATS are described in Section 3. Conclusions are presented in Section 4.

2. Structure of the ATS

In this work, the designed ATS for elastic SV waves was composed of two parts as shown in Fig. 1. One is a metasurface which works to redirect the incident wave, and the other is a phononic crystal with directional band gaps which plays the role as a filter [5,8].

2.1. Metasurfaces

The metasurfaces used in the current ATS are those proposed by the authors in the recently published paper [33]. Fig. 2 shows a 2D schematic of the metasurface, which is composed of an array of parallel thin composite plates connected at both ends to the half-space solids. All the composite plates are constituted by two connecting parts made of two different materials. As a result, varying the length ratio of the two connecting parts is capable of tuning the travelling time of flexural waves through these composite plates, subsequently of controlling the phase shifts of transmitted waves.

In this work, the length ratio of the two connecting parts of the composite plates, l_1/l_2, is defined as α, which varies from 0 to 1. The materials for the two parts were denoted as material 1 and 2, respectively. Among them, material 1 was selected as aluminum, and material 2 was a platonic material, with detailed material properties were listed in Table 1.

Throughout the paper, the general sizes for the composite plates are set as $h = 5\,\text{mm}$, $l = 50\,\text{mm}$, and $a = 0.5\,\text{mm}$, where h is the thickness of plates (in the y direction), l denotes the total length in the x direction,

Table 1
Materials [33] used for metasurfaces.

No.	Young's modulus/Gpa	Mass density/kg/m³	Poisson's ratio
1	70.00	2700.00	0.33
2	500.00	2000.00	0.31

and a represents the gap between adjacent plates. Only the normal incident waves were considered in this work.

The theoretical transmissivity and phase of transmitted waves changing with α at three different frequencies have been illustrated in Fig. 3. It is confirmed that the composite plates have a good transmission performance and full phase control of transmitted waves within $[0\ 2\pi]$ can be achieved. For comparison, a numerical model illustrated in Fig. 4 has been established and the calculated transmissions and phase shifts using COMSOL® MultiPhysics software have also been plotted in Fig. 3. The numerical results are found to be in good agreement with analytical results. According to the generalized Snell's law [34] such a metasurface is capable of steering elastic SV waves in solids.

As an illustration, a metasurface with a prescribed refraction angle of 45° has been designed based upon the aforementioned composite-plate technique. Fig. 5 shows the curl of simulated displacements for the metasurface under a normal incidence of a plane elastic SV wave. As shown in the inset of Fig. 5, the metasuface was constituted by five identical periodically distributed sub-structures. Each of them was comprised of 11 composite plates numbered 1–11. Values of α for these 11 composite pates were determined by our proposed theoretical model in Ref. [33] and tabulated in Table 2. In the numerical simulations, the exterior zone marked as PML in Fig. 5 was set as PMLs to mimic the non-reflecting boundaries. A uniform displacement in the y direction was applied on the dashed line AB to generate an incident plane SV wave. The numerical results show the incident wave was indeed changed by 45° anti-clockwise, the same as the theoretically prescribed value.
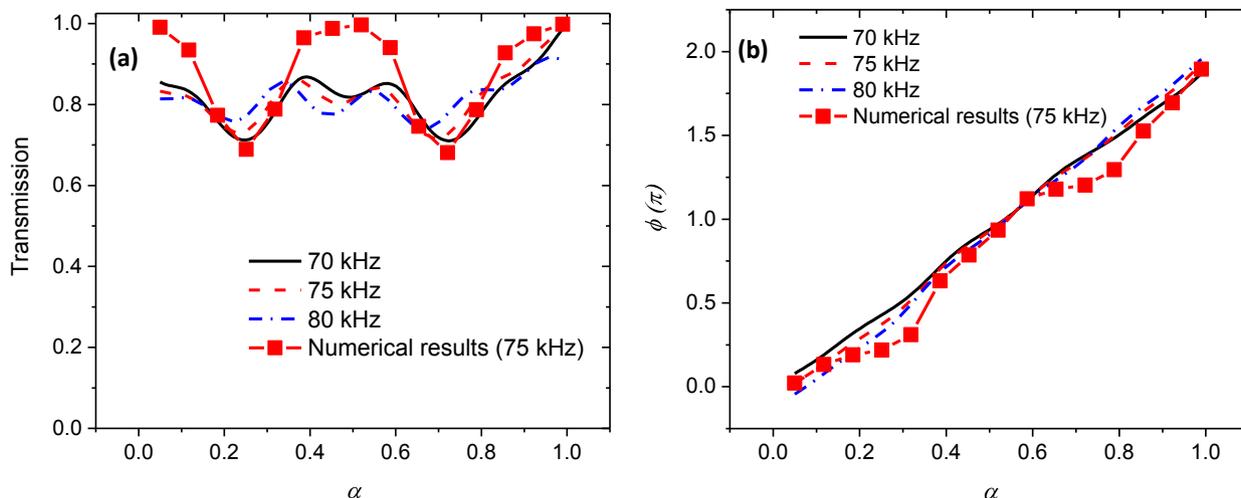
Fig. 3. Theoretically [33] and numerically calculated (a) transmission and (b) phases of the transmitted waves as a function of α.
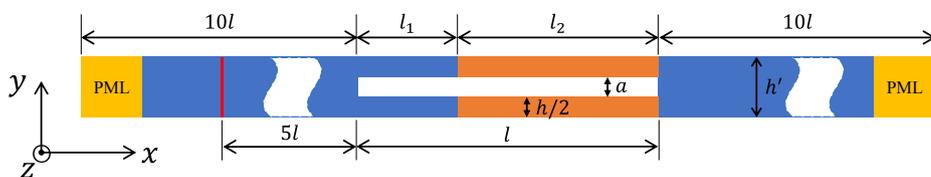

Fig. 4. A numerical model of unit cells used to calculate the transmission spectrum of elastic SV waves through composite plates in the designed metasurfaces. The periodic boundary conditions have been applied on the top and bottom edges and a uniform displacement in the y direction was applied on the red line to generate a plane SV wave. Perfectly matched layers (PMLs) were set at the two extreme ends to yield non-reflecting boundaries.

2.2. Pcs with directional band gaps

The PC used in this work was a common 2D crystal constituted by cylindrical fibers embedded in a matrix with a square lattice. The material for the matrix is the same as material 1 constituting composite plates in the metasurfaces. The cylindrical fibers are made of a material with a Young's modulus $E = 31.5$ GPa, mass density $\rho = 54,000$ kg/m^3,

and Poisson's ratio $\nu = 0.33$, denoted as material 3 in this work. Actually, the PCs with the required directional band gaps can be obtained through topology optimization techniques [32]. Fig. 7 shows the calculated band structure for such a PC with a 20% volume fraction of fibers with a radius of 2 mm. As indicated in Fig. 6, curves of the lowest order denote the shear mode. It is clear that there exists a directional band gap for elastic SV waves in the ΓX direction, as the doubled-arrow
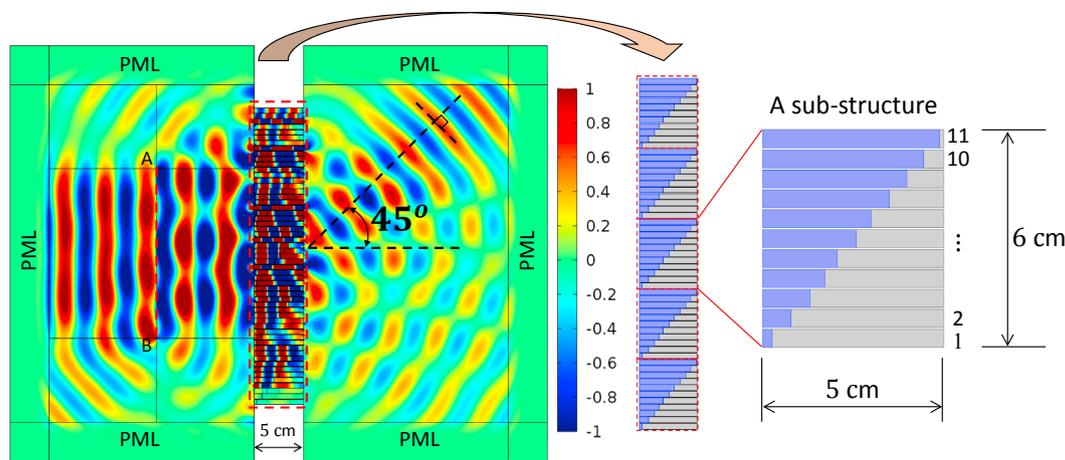

Fig. 5. The performance of a designed metasurface which changes the direction of incident SV waves by 45° anti-clockwise at 75 kHz.

Table 2
Parameters of the 11 composite plates constituting the metasurface with a re-
fraction angle of 45° at 75 kHz.

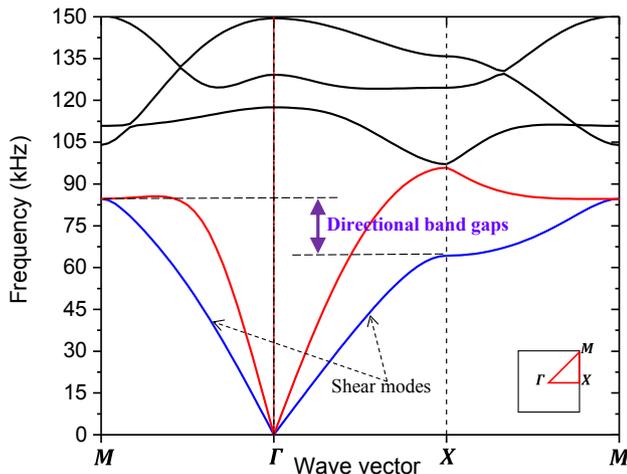| Plate	α	Φ (π)	$|T|$
1	0.0504	0.0220	0.8327
2	0.1562	0.2090	0.7886
3	0.2638	0.3956	0.7442
4	0.3426	0.5825	0.8525
5	0.4132	0.7695	0.8322
6	0.5155	0.9562	0.8231
7	0.5993	1.1433	0.8138
8	0.7006	1.3300	0.7160
9	0.7973	1.5169	0.8251
10	0.8887	1.7036	0.8890
11	0.9757	1.8905	0.9684

Fig. 6. Band structure for the mixed mode of elastic waves in a 2D PC with a
square lattice. The radius of the fibers is 2 mm and the volume fraction of fibers
is 20%.

line shown in Fig. 6. The directional band gap starts from 64.5 kHz and
ends at about 84.5 kHz.

3. Numerical validation of the performance of the ATS

In this work, the metasurface designed at the end of the Section 2.1
and the PC considered in the Section 2.2 were combined to yield an ATS
for elastic SV waves. The structure of such an ATS is plotted in Fig. 1. To
examine its performance, a series of full numerical simulations using
COMSOL® MultiPhysics software were conducted. The numerical model
is plotted in Fig. 7, in which the metasurface was constituted by 9 sub-
structures the same as those used in Fig. 5, and 10 columns of fibers
along the x direction were employed for the PC. The zone marked by
blue color was set as PMLs to yield non-reflecting boundaries. A uni-
form displacement in the y direction was applied on the dashed line AB
to generate the incident SV wave. For the negative incidence, the da-
shed loading-line AB was located on the right hand side of the PC. To
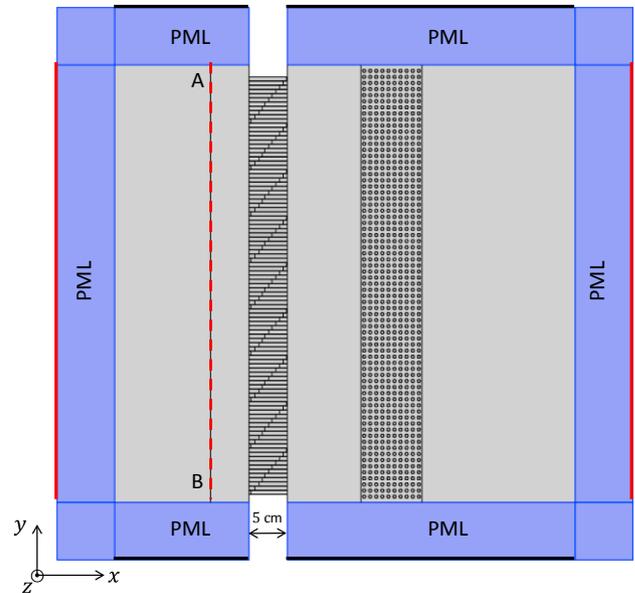avoid rigid motion, the movement in the x direction was fixed for the

Fig. 7. Numerical models used to examine transmission performance of elastic
SV waves through the designed ATS.

two thick red lines and the movement in the y direction for the four
thick black lines. All the domains were discretized using plane-strain
elements.

Fig. 8 shows the simulated curl of displacements for both the posi-
tive and negative incidence at three frequencies. It is obvious that a
portion of the incident wave passed through the designed ATS posi-
tively. Moreover, the transmitted wave had an angle of 45° with respect
to the x direction, since the metasurface had a refraction angle of 45°.
On the contrary, waves incident from the negative direction were en-
tirely prohibited. To quantitatively compare the transmission perfor-
mance, values of the normalized curl of displacements on the black
dash lines illustrated in Fig. 8 are averaged and plotted in Fig. 9. It is
found that, the transmission rate was one order of magnitude higher in
the positive direction than that in the negative direction. It indicates
that the current designed ATS works in agreement with the theoretical
expectation and has a relatively broad range of working frequencies.

4. Conclusions

We have presented an ATS for elastic SV waves in solids, which was
comprised of a metasurface and a PC. The metasurface was constituted
by a series of parallel composite plates with two connecting parts made
of different materials. The PC with a square lattice was selected, which
was required to possess a directional band gap. A metasurface with a
refraction angle of 45° based on our proposed composite-plate tech-
nique has been designed. Then, this metasurface and the PC constituted
an ATS for elastic SV waves. Full numerical simulations indicate that
the transmissivity of the designed ATS was one order of magnitude
higher in the positive direction than that in the negative direction
within the interested frequency range. In conclusion, the current ATS
provides an alternative way to realize asymmetric transmission of
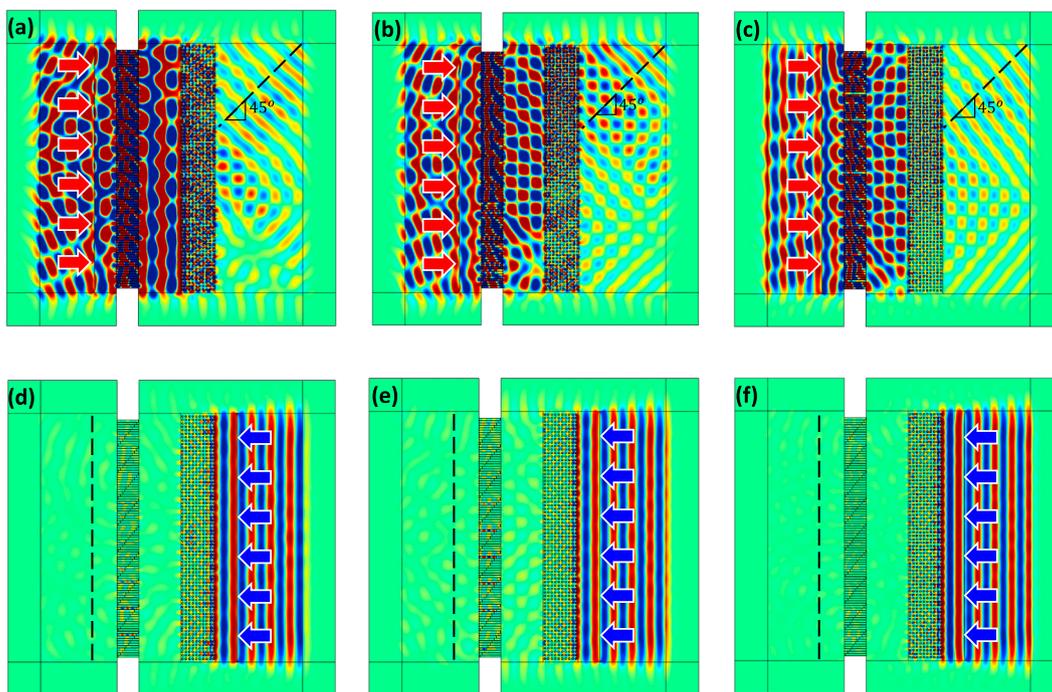elastic SV waves in solids.

Fig. 8. Curls of the simulated displacements at (a and d) 71 kHz, (b and e) 75 kHz, and (c and f) 80 kHz for the positive (→) and negative (←) incidence.
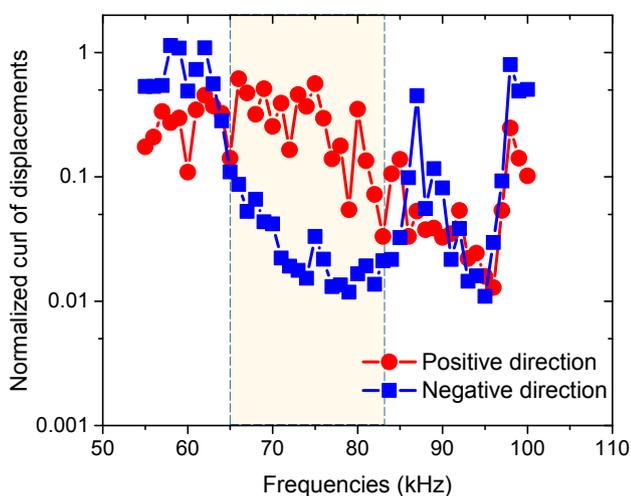

Fig. 9. Transmission spectrum of the designed ATS for elastic SV waves under the positive and negative incidence.

Acknowledgements

References

[1] X. Huang, Y. Lai, Z.H. Hang, H. Zheng, C.T. Chan, Dirac cones induced by accidental degeneracy in photonic crystals and zero-refractive-index materials, Nat. Mater. 10 (2011) 582–586.
[2] B. Li, J. Lan, L. Wang, Interface thermal resistance between dissimilar anharmonic lattices, Phys. Rev. Lett. 95 (2005) 104302.
[3] C.W. Chang, D. Okawa, A. Majumdar, A. Zettl, Solid-state thermal rectifier, Science 314 (2006) 1121–1124.
[4] W. Kobayashi, Y. Teraoka, I. Terasaki, An oxide thermal rectifier, Appl. Phys. Lett. 95 (2009) 3.
[5] B. Liang, X.S. Guo, J. Tu, D. Zhang, J.C. Cheng, An acoustic rectifier, Nat. Mater. 9 (2010) 989–992.
[6] Q. Wang, Y. Yang, X. Ni, Y.L. Xu, X.C. Sun, Z.G. Chen, L. Feng, X.P. Liu, M.H. Lu, Y.F. Chen, Acoustic asymmetric transmission based on time-dependent dynamical scattering, Sci. Rep. 5 (2015) 10880.
[7] F. Li, P. Anzel, J. Yang, P.G. Kevrekidis, C. Daraio, Granular acoustic switches and logic elements, Nat. Commun. 5 (2014) 5311.
[8] X.F. Li, X. Ni, L. Feng, M.H. Lu, C. He, Y.F. Chen, Tunable unidirectional sound propagation through a sonic-crystal-based acoustic diode, Phys. Rev. Lett. 106 (2011) 084301.
[9] Y. Li, C. Shen, Y. Xie, J. Li, W. Wang, S.A. Cummer, Y. Jing, Tunable asymmetric transmission via lossy acoustic metasurfaces, Phys. Rev. Lett. 119 (2017) 035501.
[10] Y.J. Guan, H.X. Sun, J.P. Xia, S.Q. Yuan, Broadband asymmetric acoustic transmission in a single medium by an array of heat sources, J. Phys. D-Appl. Phys. 50 (2017).
[11] B. Liang, W.W. Kan, X.Y. Zou, L.L. Yin, J.C. Cheng, Acoustic transistor: Amplification and switch of sound by sound, Appl. Phys. Lett. 105 (2014) 083510.
[12] C. Fu, B. Wang, T. Zhao, C.Q. Chen, High efficiency and broadband acoustic diodes, Appl. Phys. Lett. 112 (2018) 051902.
[13] L. Cao, Y. Xu, B. Assouar, Z. Yang, Asymmetric flexural wave transmission based on dual-layer elastic gradient metasurfaces, Appl. Phys. Lett. 113 (2018) 183506.
[14] J.J. Chen, X. Han, G.Y. Li, Asymmetric Lamb wave propagation in phononic crystal slabs with graded grating, J. Appl. Phys. 113 (2013) 184506.
[15] J.G. Cui, T.Z. Yang, L.Q. Chen, Frequency-preserved non-reciprocal acoustic propagation in a granular chain, Appl. Phys. Lett. 112 (2018) 181904.
[16] C. Liu, Z. Du, Z. Sun, H. Gao, X. Guo, Frequency-preserved acoustic diode model with high forward-power-transmission rate, Phys. Rev. Appl 3 (2015) 064014.
[17] Y.-Z. Wang, Y.-S. Wang, Active control of elastic wave propagation in nonlinear phononic crystals consisting of diatomic lattice chain, Wave Motion 78 (2018) 1–8.
[18] B. Liang, B. Yuan, J.C. Cheng, Acoustic diode: rectification of acoustic energy flux in one-dimensional systems, Phys. Rev. Lett. 103 (2009) 104301.

[19] Z.-N. Li, Y.-Z. Wang, Y.-S. Wang, Nonreciprocal phenomenon in nonlinear elastic wave metamaterials with continuous properties, Int. J. Solids Struct. 150 (2018) 125–134.

[20] X. Zheng, X. Wei, Y. Xu, Q. Menglu, C. Qian, One-way propagation of acoustic waves through a periodic structure, Appl. Phys Exp. 11 (2018) 027301.

[21] R.Q. Li, B. Liang, Y. Li, W.W. Kan, X.Y. Zou, J.C. Cheng, Broadband asymmetric acoustic transmission in a gradient-index structure, Appl. Phys. Lett. 101 (2012) 263502.

[22] X. Zhu, X. Zou, B. Liang, J. Cheng, One-way mode transmission in one-dimensional phononic crystal plates, J. Appl. Phys. 108 (2010) 124909.

[23] Y. Li, J. Tu, B. Liang, X.S. Guo, D. Zhang, J.C. Cheng, Unidirectional acoustic transmission based on source pattern reconstruction, J. Appl. Phys. 112 (2012) 064504.

[24] A.L. Song, T.N. Chen, X.P. Wang, L.L. Wan, Waveform-preserved unidirectional acoustic transmission based on impedance-matched acoustic metasurface and phononic crystal, J. Appl. Phys. 120 (2016) 085106.

[25] C. Li, M. Ke, Y. Ye, S. Xu, C. Qiu, Z. Liu, Broadband asymmetric acoustic transmission by a plate with quasi-periodic surface ridges, Appl. Phys. Lett. 105 (2014) 023511.

[26] Z. He, S. Peng, Y. Ye, Z. Dai, C. Qiu, M. Ke, Z. Liu, Asymmetric acoustic gratings, Appl. Phys. Lett. 98 (2011) 083505.

[27] Y.F. Zhu, X.Y. Zou, B. Liang, J.C. Cheng, Acoustic one-way open tunnel by using metasurface, Appl. Phys. Lett. 107 (2015) 113501.

[28] B. Assouar, B. Liang, Y. Wu, Y. Li, J.C. Cheng, Y. Jing, Acoustic metasurfaces, Nat. Rev. Mater. 3 (2018) 460–472.

[29] C. Shen, Y. Xie, J. Li, S.A. Cummer, Y. Jing, Asymmetric acoustic transmission through near-zero-index and gradient-index metasurfaces, Appl. Phys. Lett. 108 (2016) 223502.

[30] J. Mei, Y. Wu, Controllable transmission and total reflection through an impedance-matched acoustic metasurface, New J. Phys. 16 (2014) 123007.

[31] B. Yuan, B. Liang, J.C. Tao, X.Y. Zou, J.C. Cheng, Broadband directional acoustic waveguide with high efficiency, Appl. Phys. Lett. 101 (2012) 043503.

[32] J. He, Z. Kang, Achieving directional propagation of elastic waves via topology optimization, Ultrasonics 82 (2017) 1.

[33] J. Zhang, X. Su, Y. Pennec, Y. Jing, X. Liu, N. Hu, Wavefront steering of elastic shear vertical waves in solids via a composite-plate-based metasurface, J. Appl. Phys. 124 (2018) 164505.

[34] N. Yu, P. Genevet, M.A. Kats, F. Aieta, J.-P. Tetienne, F. Capasso, Z. Gaburro, Light propagation with phase discontinuities: generalized laws of reflection and refraction, Science 334 (2011) 333–337.