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Analytical estimations of temperature in a living tissue generated by laser irradiation using experimental data

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ABSTRACT:

This paper presents an analytical approach associated with Laplace transformation, experimental temperature data, and a sequential concept over time to obtain the thermal damage and the temperature in a living tissue due to laser irradiation. The analytical solutions in the Laplace domain are appreciably obtainable. The thermal damage to the tissue is completely assessed by the denatured protein range using the formulation of Arrhenius. Numerical outcomes for temperatures and the thermal damages are graphically introduced. Besides, the comparison between the numerical computations and the existing experimental study shows that a current mathematical model is an effective tool for evaluating the biological heat transfer in biological tissues.

1. Introduction

Recently, the estimation of temperature in biological tissues has been carefully scrutinized by researchers. 'Thermal therapy' has been lately considered one of the best existing alternatives for modern clinical treatments. Among these clinical procedures, the application of moveable thermal sources on the biological tissues considering perfusion rate is conducted in some plastic surgery operations such as removing moles, spots or tattoos using laser radiation or in the thermal actions of the cornea with laser for correcting hyperopia. Since the thermal behavior of the biological tissues depends on some complex phenomena such as blood circulation and metabolic heat generation, some governing equations are respectively developed by researchers. In 1948 (Pennes, 1948), Pennes studied the distribution of temperature in the forearm skin temperature, which meant that the equation could be analyzed by different methods usually used to solve the model of thermal transfer for infinite heat propagation based on heat conduction of classical Fourier. However, in the living tissue, the heat transfer is a more complex procedure. It encompasses several phenomenological mechanizations such as radiation, heat conduction, blood perfusion, metabolic heat generation and phase change. In the biological tissues, the phase changes occur in wide ranges. Modified Penne's bio-heat equation is solved using a different type of numerical methods which are available in the literature (Dillenseger and Esneault, 2010). have used the finite difference approach to examine the improvement of temperature over time in the condition of having an abnormally low body temperature. The finite-decomposition method (Gupta et al., 2013), homotopy perturbation method (Gupta et al., 2010) and Galerkin approach with variation iteration method and finite element Legendre wavelet Galerkin method (Kumar et al., 2015; Yadav et al., 2014). (Zhu et al., 2002) have presented the sedimentation of lighting energy in tissue and the rate process models for the heat injuries resulting in using the theory of diffusion. For laser irradiated cartilages (Díaz et al., 2002). have estimated the solutions of thermos-diffusion models in the tissue by applying the finite element method to evolve the thermal injuries models (Dombrovsky and Timchenko, 2015). have studied the laser-induced hyperthermia of superficial tumors as computational models for radiative transfer, combined heat transfer, and degradation of biological tissues. In these transfers and biological tissues, a novel kinetic model of thermal degradation of biological tissues has been suggested for the first time. The proposed model has been taken into account In the partial regeneration of the tissues due to oxygen supplied by arterial blood perfusion (Ezzat et al., 2014). have presented the fractional modeling of Pennes' bioheat transfer equation (Ezzat et al., 2016). have also studied the tissue responses to fractional transient heating with sinusoidal heat flux condition on the skin surface (Ghanmi and Abbas, 2019). have further presented the analytical study on the fractional transient heating within the skin tissue during the thermal therapy (Phadnis et al., 2016). have studied the numerical thermal response of laser-irradiated biological tissue phantoms embedded with gold nanoshells (Kumar et al., 2016). have used the finite differences method to study the numerical simulation during the

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thermal therapy under dual-phase-lag bioheat transfer model. They have discretized in space variable x using the symmetric difference scheme in the second derivative and central difference scheme in the first derivative (Hobiny and Abbas, 2018). have investigated the theoretical analysis of thermal injuries in skin tissue due to a movable thermal source (Zhao et al., 2005). have studied one-dimensional Pennes' bioheat equation by two-level finite difference scheme. When a real phenomenon regarding thermal transfer in finite medium is investigated, the linear and nonlinear models of heat transfer are expanded and their analytical or numerical solutions are solved by numerous researchers (Abbas, 2007; Abbas and Youssef, 2013; Abbas et al., 2011; Hassan et al., 2018; Marin, 1997; Marin and Craciun, 2017; Zenkour and Abbas, 2014).

The objective of this paper is to introduce the analytical solutions for nonfurrier bio-heat model in a living tissue. The numerical outcomes can be used as a substantiation division for biological tissue interactions such as continuous scanning laser interactions. The comparisons are made with the outcomes obtained in the cases of the absence of thermal relaxation time. Based on the bioheat transfer model, the treatment of hyperthermia is very important in the clinical application which the doctors will take into account concerning the options while the patients will get the interest. Furthermore, the comparison between the numerical computations and the existing experimental study indicates that a current mathematical model is an effective tool for evaluating the biological heat transfer in biological tissues.

2. Mathematical model

In this model, a semi-infinite living biological tissue under thermally insulated, is examined. To look for the various temperatures of the tissue, the transient problem is presented. Based on Cattaneo (1958) for heat flux including the characteristic time τ_o, the basic equation of thermal waves fashion of bioheat equation in skin tissues can be given as (Ahmadikia et al., 2012; Xu et al., 2008):

$$\rho c\left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2}+\left(1+\tau_o\frac{\partial}{\partial t}\right)(Q_b+Q_m+Q_{ext}),\tag{1}$$

where$T$ is the temperature of tissue, τ_o is the thermal delay time, T_b is the temperature of blood, c is the specific heat of tissues, t is the time, ρ is the tissue mass density, k is the thermal conductivity of tissues, Q_b point to the blood perfusion heat source which is given by (Deng and Liu, 2000)

$$Q_b=\omega_b(T_b)\rho_b c_b(T_b-T)\tag{2}$$

where c_b is the blood specific heat, ρ_b is the blood mass density and ω_b is the rate of blood perfusion which was assumed to be a linear function of blood temperature T_b.

$$\omega_b(T_b)=\omega_{bo}(1+\alpha T_b)\tag{3}$$

where α is the constant associated blood perfusion, ω_{bo} is the reference blood perfusion rate and Q_m is the heat generated by metabolic process due to various physiological processes occurring in rest of the body while, Q_{ext} represents the thermal generated per unit volume of tissues. (Gardner et al., 1996) have proposed the following model for a laser heat source:

$$Q_{ext}(x,t)=I_o\mu_a[U(t)-U(t-\tau_p)]\left[C_1e^{-\frac{k_1}{\delta}x}-C_2e^{-\frac{k_2}{\delta}x}\right],\tag{4}$$

where I_o is the laser intensity, μ_a is the absorption coefficient, τ_p is the laser exposure time, and $U(t)$ is the step function, δ is penetration depth, C_1, C_2, k_1 and k_2 are the functions of diffuse reflectance R_d and they are mentioned in (Gardner et al., 1996). The penetration depth is as follows (Gardner et al., 1996):

$$\delta=\frac{1}{\sqrt{3\mu_a(\mu_a+\mu_s(1-g))}},\tag{5}$$

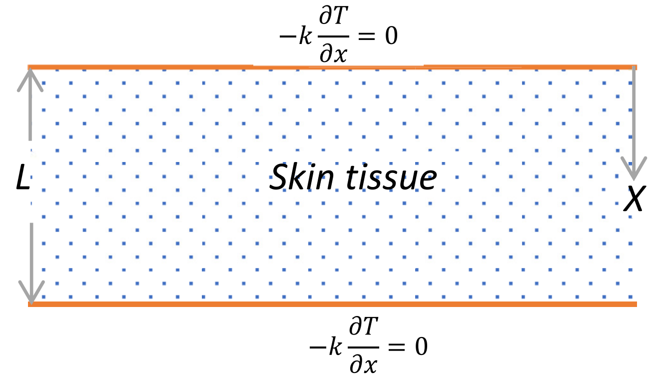

Fig. 1. Schematic of the problem.

where μ_s is the scattering coefficient, and g is the anisotropy factor. The finite domain of living tissue is examined with a thickness L and both sides are assumed to be thermally insulated as in Fig. 1 (Hobiny and Abbas, 2018):

Now, the boundary conditions and initial condition are expressed as

$$-k\frac{\partial T(L,t)}{\partial x}=0,\ -k\frac{\partial T(0,t)}{\partial x}=0,\tag{6}$$

$$T(x,0)=T_b,\ \frac{\partial T(x,0)}{\partial t}=0.0\tag{7}$$

For appropriateness, the dimensionless variables can be expressed through

$$\mathrm{T}'=\frac{\mathrm{T}-\mathrm{T_o}}{T_o},\ T_b'=\frac{\mathrm{T_b}-\mathrm{T_o}}{T_o},\ (t',\tau_o',\tau_p')=\frac{k}{\rho cL^2}(t,\tau_o,\tau_p),\ x'=\frac{x}{L},$$
$$(k_1',k_2')=\frac{L}{\delta}(k_1,k_2),\ f_b=\frac{\rho_b\omega_{bo}c_bL^2}{k},\ f_m=\frac{L^2Q_m}{kT_o},\ f_r=\frac{L^2I_o\mu_a}{kT_o}.\tag{8}$$

In terms of this dimensionless form of variables in (8), the basic equation (1) with boundary conditions (6) and initial conditions (7) can be introduced by the forms (the dash is neglected for appropriateness)

$$\frac{\partial^2 T}{\partial x^2}=\left(1+\tau_o\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t}-f_b(1+\alpha T_b)(T_b-T)-f_m-f_r\phi(x,t)\right),\tag{9}$$

$$\frac{\partial T(0,t)}{\partial x}=0,\ \ \frac{\partial T(L,t)}{\partial x}=0,\tag{10}$$

$$T(x,0)=0,\ \frac{\partial T(x,0)}{\partial t}=0,\tag{11}$$

where $\phi(x,t)=[U(t)-U(t-\tau_p)][C_1e^{-k_1x}-C_2e^{-k_2x}]$.

2.1. Laplace's transform

The Laplace transform for a function $M(x,t)$ can be defined by

$$\bar{M}(x,s)=L[M(x,t)]=\int_0^\infty M(x,t)e^{-st}dt\tag{12}$$

where s is the Laplace transform parameter. Therefore, we can use the initial conditions while the basic equation can be replaced by

$$\frac{d^2\bar{T}}{dx^2}-s_1\bar{T}=-s_2-s_3e^{-k_1x}-s_4e^{-k_2x}\tag{13}$$

with the boundary conditions

$$\frac{\partial\bar{T}(0,t)}{\partial x}=0,\ \frac{\partial\bar{T}(L,t)}{\partial x}=0,\tag{14}$$

where $s_1=(1+s\tau_o)(s+f_b(1+\alpha T_b))$, $s_2=\frac{1}{s}(f_bT_b(1+\alpha T_b)+f_m)$, $s_3=\frac{f_rC_1}{s}(1-e^{-s\tau_p})$ and $s_4=-\frac{f_rC_2}{s}(1-e^{-s\tau_p})$. The general solution $\bar{T}$ of the nonhomogeneous equation (13) is the sum of two solutions. The first one is the complementary solution $\bar{T}_c$ of the associated

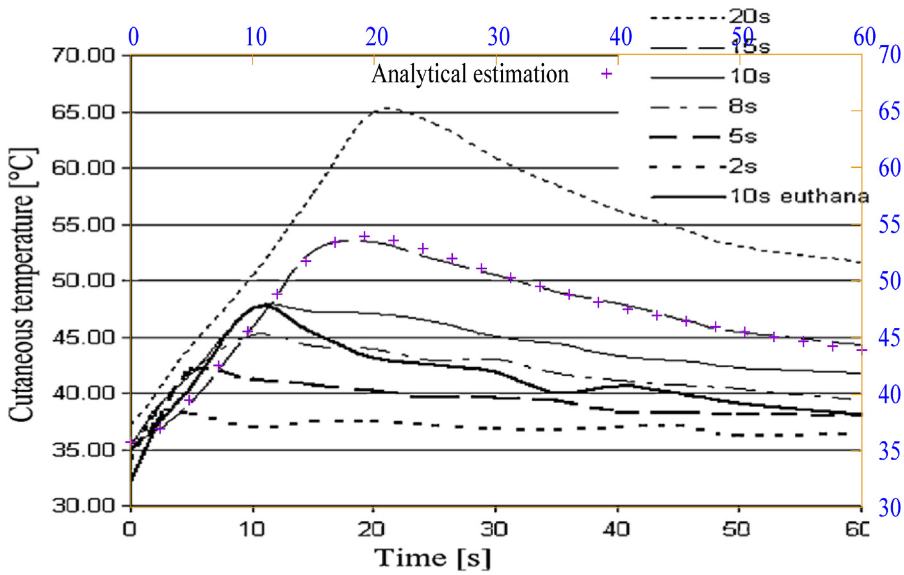

Fig. 2. The comparison of present study with the experimental study (Museux et al., 2012) when.($\tau_o = 3(s)$, $I_o = 122 \times 10^3 (W)(m^{-2})$, $\tau_p = 15(s)$, $\alpha = 0.02$,)

homogeneous equation while the second is the particular solution $\bar{T}_p$ of the nonhomogeneous equation. The general solution of equation (13) can be given by

$$\bar{T}(x, s) = \frac{s_2}{s_1} + B_1 e^{\sqrt{s_1}x} + B_2 e^{-\sqrt{s_1}x} + \frac{s_3}{s_1 - k_1^2} e^{-k_1 x} + \frac{s_4}{s_1 - k_1^2} e^{-k_2 x} \tag{15}$$

To complete the solutions, by using the boundary conditions (14) to determine the constants $B_1$and B_2 that can be given by

$$B_1 = \frac{\mathrm{e}^{-L(k_1+k_2)}(\mathrm{e}^{Lk_2}(\mathrm{e}^{Lk_1} - \mathrm{e}^{L\sqrt{s_1}})k_1(k_2^2 - s_1)s_3 + \mathrm{e}^{Lk_1}(\mathrm{e}^{Lk_2} - \mathrm{e}^{L\sqrt{s_1}})k_1^2 k_2 s_4 - \mathrm{e}^{Lk_1}(\mathrm{e}^{Lk_2} - \mathrm{e}^{L\sqrt{s_1}})k_2 s_1 s_4)}{(\mathrm{e}^{2L\sqrt{s_1}} - 1)\sqrt{s_1}(s_1 - k_1^2)(s_1 - k_2^2)},$$

$$B_2 = \frac{\mathrm{e}^{L\sqrt{s_1}}}{(\mathrm{e}^{2L\sqrt{s_1}} - 1)\sqrt{s_1}} \left(\frac{\mathrm{e}^{-Lk_1}(\mathrm{e}^{L(k_1+\sqrt{s_1})} - 1)k_1 s_3}{k_1^2 - s_1} + \frac{\mathrm{e}^{-Lk_2}(\mathrm{e}^{L(k_2+\sqrt{s_1})} - 1)k_2 s_4}{k_2^2 - s_1} \right).$$

For final solution of temperature distribution, a numerically reversal method is adopted depending on the approximation method of Riemann-sum used to investigate the numerical outcomes. According to this method, for any function in the domain of Laplace may be transformed into the domain of time such as (Tzou, 1996):

$$M(x, t) = \left(Re \sum_{n=0}^{N} (-1)^n \bar{M}\left(x, m + \frac{in\pi}{t}\right) + \frac{1}{2} Re[\bar{M}(x, m)] \right) \frac{e^{mt}}{t}, \tag{16}$$

whereas i is the imaginative number unit and Re is the actual part.

3. Results and discussion

In this section, the variation of temperature in skin tissues under hyperbolic bio-heat model is accurately investigated. For numerical calculations, exemplary values of heat properties for skin tissues are chosen (Askarizadeh and Ahmadikia, 2014)

$$\rho_b = 1060(kg)(m^{-3}),\ c_b = 3860(J)(kg^{-1})(k^{-1}),\ \omega_b = 1.87 \times 10^{-3}(s^{-1}),\ T_b = 36.5\ °\mathrm{C},$$

$$\rho = 1000(kg)(m^{-3}),\ c = 4187(J)(kg^{-1})(k^{-1}),\ k = 0.628\ (W)(m^{-1})(k^{-1}),\ \tau_o = 3.2(s),$$

$$\tau_p = 15(s),\ Q_m = 1.19 \times 10^3 (W)(m^{-3}),\ I_o = 3 \times 10^5 (W)(m^{-2}),\ L = 0.03(m),\ g = 0.9,$$

$$C_1 = 3.09 + 5.44R_d - 2.12e^{-21.5R_d},\ K_2 = 1.63e^{3.4R_d},\ C_2 = 2.09 - 1.47R_d - 2.12e^{-21.5R_d},$$

$$K_1 = 1 - \left(1 - \frac{1}{\sqrt{3}}\right)e^{-20.1R_d},\ R_d = 0.05,\ \mu_a = 40(m^{-1}),\ \mu_s = 12000\ (m^{-1}),\ T_o = 36.5\ °\mathrm{C}.$$

The accurate prognosis of thermal injury to skin tissues is useful for thermal therapy. The evaluation of burn is one of the utmost important attributes in the bio-engineering sciences in skin tissue. To quantify thermal damages, the method improved by Moritz and Henriques (Henriques and Moritz, 1947; Moritz and Henriques, 1947) can be used. The non-dimensional measure of thermal damages index Ω can be given by

$$\Omega = \int_0^t B e^{-\frac{E_a}{RT}} dt \tag{17}$$

where B= $3.1 \times 10^{98} s^{-1}$ is the frequency factor, R= 8.313 J/mol·K is the constant of universal gas and $E_a = 6.28 \times 10^5$ J/mol is the activation energy. The results obtained here are based on the skin tissue with the properties listed above and also the laser parameters and blood properties. Some experimental studies have proved that there are great similarities between human and pig skins, especially in the vascular organization. (Museux et al., 2012) have conducted an experimental study on laser heating of pig skin. Several exposure durations and laser powers have been tested in their work. By using the Xfig program under Linux to merge our results with the experimental results (Museux et al., 2012) at the exposure time of 15 s. As shown in Fig. 2, there is good conformity between the results of the analytical solution and those obtained from the experimental study.

Fig. 3 shows the increment of temperature with respect to the distance x for different values of α. It is clear from the graph that the temperature starts from a maximum value on the surface $x = 0$, then it decreases continuously to the normal temperature $T_b = 36.5°C$. The time histories of surface temperature through four values of $\alpha$are exhibited in Fig. 4. It has been revealed that the temperature starts from the normal temperature T_b and increases with time until the maximum value then decreases to the normal temperature again. Fig. 5 displays the variation of thermal damages with respect to the time t. Predictably,

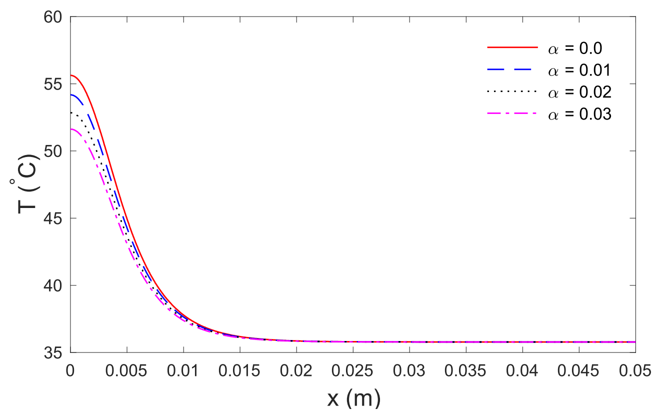

Fig. 3. The distribution of temperature through the distance for different values of α.

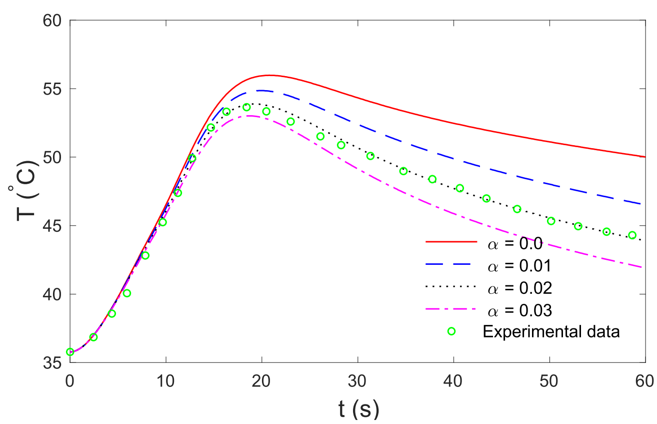

Fig. 4. Temperature history at skin surface for different values of α.

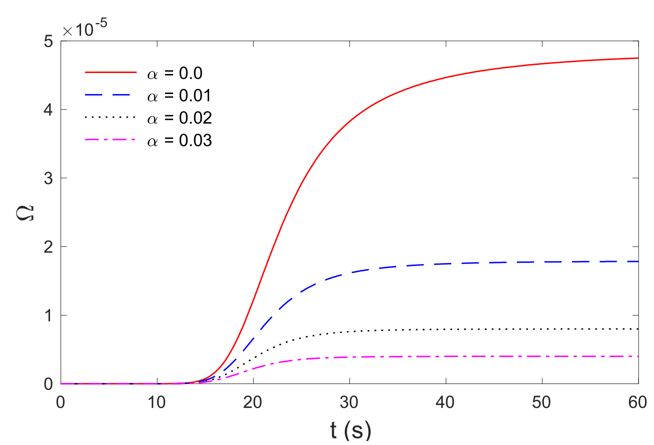

Fig. 5. The variation of thermal damage for different values of α at skin surface $x = 0$.

the parameter α has a great effect on the distribution of field quantities. Figs. 6 and 7 show the effects of thermal relaxation time in the temperature distributions and the results of thermal damages. As expected, the thermal relaxation time effect plays a significant role on the temperature and the thermal damages.

4. Conclusion

Based on hyperbolic bioheat model in biological skin tissues, the behavior of temperature increment and the thermal damages are accurately examined during thermal therapy handling. Analytical solutions are derived for hyperbolic bio-heat transfer subject to laser irradiation. The numerical results obtained by the analytical method are compared with existing experimental data used to verify the accuracy of the numerical solutions. This comparison shows that a current mathematical model is an effective tool for evaluating the biological heat transfer in biological tissues.

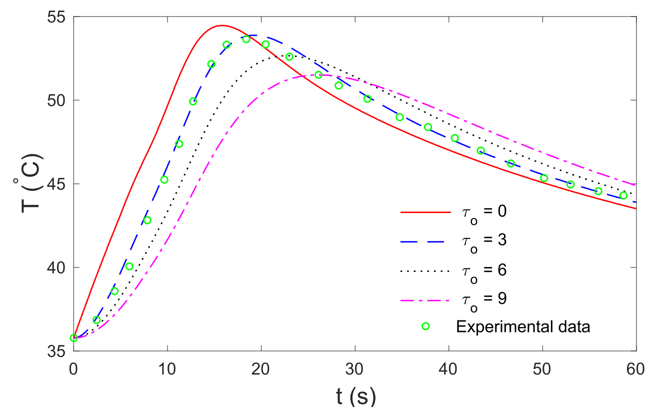

Fig. 6. Temperature history at skin surface for different values of τ_o.

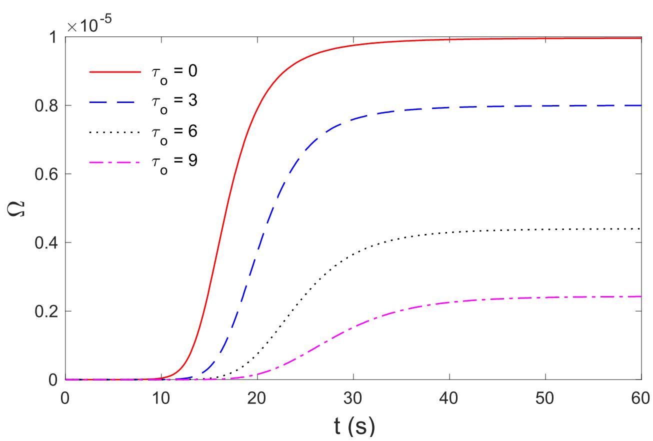

Fig. 7. The variation of thermal damage for different values of τ_o at skin surface $x = 0$.

Acknowledgments

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