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An analytical study on the fractional transient heating within the skin tissue during the thermal therapy

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ABSTRACT

In the present paper, the bioheat equation under fractional derivatives is used to study the thermal damage within the skin tissue during the thermal therapy. Basically, the analytical solutions in the Laplace domain are easily obtainable. The influences of the fractional derivative and moving heat source velocity on the temperature of skin tissues and the thermal injuries are precisely investigated. The outcomes show that the fractional bioheat model are reduced to the hyperbolic and parabolic bioheat models when the fractional order parameter is equal to one and the relaxation time is close to zero respectively. The thermal injuries to the tissue are assessed by the denatured protein range using the formulation of Arrhenius. The numerical outcomes of thermal injuries and temperatures are graphically introduced. In conclusion, a parametric analysis is devoted to the identification of an appropriate procedure for selecting important design variables to reach effective heating in hyperthermia treatment.

1. Introduction

Recently, the estimation of transient temperatures in biological tissues has been under the focus of researchers. The so called ‘thermal therapy’ has been lately considered one of the best existing alternatives for modern clinical treatments. Vastly, the methods of thermal treatments have been used for modern clinical treatments such as hyperthermia (Mahjoob and Vafai, 2009), laser tissue soldering (Gabay et al., 2011) and laser surgery (Zhou et al., 2009). Among these clinical procedures, the application of movable thermal sources on biological tissues in the presence of variable perfusion rates affects plastic surgery operations such as removing moles, spots or tattoos using laser radiation or in the thermal action of the cornea with laser for correcting hyperopia. Since the thermal behavior of biological tissues depends on some complicated phenomena such as blood circulation and metabolic heat generation, some governing equations have been developed by researchers. In 1948, Pennes (1948) studied the distribution of temperature in the forearm. The time-dependent heat transfer equation can be analyzed by usually using different methods to solve the model of the thermal transfer for infinite heat propagation based on heat conduction of classical Fourier i.e.

$$q(\boldsymbol{x}, t) = -k\nabla T(\boldsymbol{x}, t)$$

where q and ∇T are heat flux and temperature gradient at the same instant time and space. As a matter of fact, heat still spreads at a finite rate in living biological tissue because it has a highly inhomogeneous internal structure. To disband the paradox that happened in the Penne's bioheat equation, heat waves model of bio-heat transfer is introduced depending on the thermal wave constitutive relation given as follows: (Cattaneo, 1958; Vernotte, 1958)where τ_o is the thermal lagging time. q and ∇T at the same point ($\boldsymbol{x}$) are in various instants of time. As the value of τ_o tends to zero, the thermal wave model is reduced to Penne's model. The thermal wave model gives a suitable required thermal data for describing temperature distribution in living biological tissue. Many current models of physical processes have been successfully modified by using the fractional calculus. It can be safely said that the all the integral theories and fractional derivatives have been created in the second half of the 20th century. In other words, definitions and different methods of fractional derivatives have become the main object of numerous investigations. By utilizing the Taylor-Riemann series expansion of time fractional order, Ezzat et al., 2014, 2016 have presented a new model of fractional bio-heat equation using the equation of fractional heat conduction:

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Modified Penne's bio-heat equation is solved using different types of numerical methods available in literature such as the finite-decomposition method (Gupta et al., 2013), homotopy perturbation method (Gupta et al., 2010) Galerkin approach with variation iteration method and the finite element Legendre wavelet Galerkin method (Kumar et al., 2015; Yadav et al., 2014). Analytical solution is more interesting than experimental and numerical computations because of its lower outlay and exact evaluation. For instance, Dillenseger and Esneault (2010) have used the finite difference approach to study the improvement of temperature over time in hypo-thermia. Using the theory of diffusion, Zhu et al. (2002) have estimated the deposition of light energy in tissue for the thermal injuries possible. For laser irradiated cartilages, Diaz et al. (Díaz et al., 2002) have presented the solution of thermos-diffusions models in the tissue by applying the finite element method to evolve the thermal injuries models. Abbas, 2014a, 2014b, 2015a, 2015b has investigated some problems of thermoelasticity due to the movable thermal source. In other words (Marin and Öchsner, 2017; Abbas and Youssef, 2013; Marin, 1997; Abd-Alla and Abbas, 2002; Abbas, 2014c; Ezzat, 2012; Sherief et al., 2010), use various techniques to solve some thermoelastic problems. To our knowledge, there is no analytical solutions for fractional bioheat models using a movable thermal resource. Hobiny and Abbas (2018) have studied the theoretical analysis of thermal injuries in skin tissues due to a movable thermal source.

The objective of the present paper is to introduce the exact solutions for the fractional bio-heat model in skin tissues. The numerical outcomes can be used as a substantiation division for biological tissues interactions such as continuous scanning laser interactions. The comparisons are made with the outcomes obtained in the case of the absence of fractional order parameter.

1.1. Statement of the problem

To investigate the variance of the temperature of the tissue considering a semi-infinite human skin tissue under thermally insulated, the transient problem is presented. The skin tissue which are initially at a constant temperature $T_b = 37\,°C$ is heated by moving heat source. Based on Cattaneo (1958) and Ezzat et al. (Ezzat et al., 2014), the general form of the thermal waves pattern of fractional bioheat equation in skin tissues can be given as

$$k\nabla^2 T = \left(1 + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c\frac{\partial T}{\partial t} + \omega_b\rho_b c_b(T - T_b) - Q_m - Q_{ext}\right),\quad 0 < \alpha \le 1 \tag{1}$$

Taking into consideration the definition of fractional derivative which can be expressed by:

$$\frac{\partial^\alpha h(\mathbf{r},t)}{\partial t^\alpha} = \begin{cases} h(\mathbf{r},t) - h(\mathbf{r},0), & \alpha \to 0 \\ I^{\alpha-1}\frac{\partial h(\mathbf{r},t)}{\partial t}, & 0 < \alpha < 1 \\ \frac{\partial h(\mathbf{r},t)}{\partial t}, & \alpha = 1 \end{cases} \tag{2}$$

$$I^\nu h(r,t) = \int_0^t \frac{(t-s)^\nu}{\Gamma(\nu)} h(r,s)ds,\ \nu > 0 \tag{3}$$

$$\lim_{\nu\to 1}\frac{\partial^\nu h(\mathbf{r},t)}{\partial t^\nu} = \frac{\partial h(\mathbf{r},t)}{\partial t} \tag{4}$$

The various values of the fractional parameter $0 < \alpha \le 1$ cover two types of conductivity, $0 < \alpha < 1$ for low conductivity and $\alpha = 1$ for normal conductivity. Where Γ is the gamma function, T_b is the blood temperature, T is the tissues temperature, ρ is the tissue mass density, ρ_b is the blood mass density, τ_o is the thermal relaxation time, Q_{ext} is the moving line heat source, c is the specific heat of tissue, ω_b is the blood perfusion rate, k is the tissue thermal conductivity, Q_m is the metabolic thermal generations in skin tissue, t is the time and c_b is the blood specific heat. The variation of temperature is considered in skin tissues with moving heat source for more perceptive to the heat wave propagation behavior of fractional bio-heat transfer equation. The one-dimensional model of the fractional bio-heat transfer equation in an unbounded medium is established. So that the fractional bioheat transfer model using outer thermal resource can be expressed by (Ezzat et al., 2014):

$$k\frac{\partial^2 T}{\partial x^2} = \left(1 + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho c\frac{\partial T}{\partial t} + \omega_b\rho_b c_b(T - T_b) - Q_m - Q_{ext}\right) \tag{5}$$

1.2. Initial and boundary conditions

Two initial conditions are substantial to dub the physical model, so as to get the solution of the bio-heat equation:

$$\frac{\partial T(x,0)}{\partial t} = 0.0\,,\quad T(x,0) = T_b \tag{6}$$

Both lower and upper surfaces are supposed to be thermally isolated as the boundary conditions.

$$-k\frac{\partial T(L,t)}{\partial x} = 0,\quad -k\frac{\partial T(0,t)}{\partial x} = 0 \tag{7}$$

For appropriateness, the dimensionless variables can be expressed by

$$\mathrm{T}' = \frac{\mathrm{T}-\mathrm{T_b}}{T_b},\ (t',\tau_o') = \frac{\omega_b\rho_b c_b}{\rho c}(t,\tau_o),\ x' = \sqrt{\frac{\omega_b\rho_b c_b}{k}}\,x,\ (Q_m', Q_{ext}') = \frac{(Q_m, Q_{ext})}{\omega_b\rho_b c_b T_b} \tag{8}$$

In terms of this non-dimensional form of variables in (8), equations (5)–(12) can be introduced by the forms (the dash has been neglected for appropriateness).

$$\frac{\partial^2 T}{\partial x^2} = \left(1 + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\partial T}{\partial t} + T - Q_m - Q_{ext}\right) \tag{9}$$

$$T(x,0) = 0\ ,\quad \frac{\partial T(x,0)}{\partial t} = 0 \tag{10}$$

$$\frac{\partial T(0,t)}{\partial x} = 0\,,\quad \frac{\partial T(L,t)}{\partial x} = 0 \tag{11}$$

The dimensionless of the external heat source $Q_{ext}(x,\ t)$ is a moveable thermal resource which can be given by (Abbas, 2014a):

$$Q_{ext}(x,\ t) = Q_o\delta(x - vt) \tag{12}$$

where Q_o is constant, v is constant velocity and δ is the delta function.

1.3. Laplace's transform

The Laplace transform for a function $\mathrm{M}(x,\ t)$ can be define by

$$\bar{\mathrm{M}}(x,s) = L[M(x,t)] = \int_0^\infty \mathrm{M}(x,t)e^{-st}dt,\ s > 0 \tag{13}$$

where s is the Laplace transform parameter. Hence, the essential equations can be replaced by

$$\frac{d^2\bar{T}}{dx^2} = (1+s)\left(1 + \frac{s^\alpha\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\bar{T} - \frac{Q_m}{s} - \left(1 + \frac{s^\alpha\tau_o^\alpha}{\Gamma(\alpha+1)}\right)\frac{Q_o}{v}e^{-\frac{sx}{v}} \tag{14}$$

$$\frac{d\bar{T}(0,s)}{dx} = 0.0\,,\quad \frac{d\bar{T}(L,s)}{dx} = 0.0 \tag{15}$$

The general solution $\bar{T}$ of the nonhomogeneous equation (14) combines two solutions. The first one is the complementary solution $\bar{T}_c$ of the associated homogeneous equation while the second one is the particular solution $\bar{T}_p$ of the nonhomogeneous equation. The general solution of equation (14) can be given by

$$\bar{T}(x, s) = B_1 e^{-\beta x} + B_2 e^{\beta x} + \frac{\gamma}{\beta^2} - \frac{\varepsilon}{\zeta^2 - \beta^2} e^{-\zeta x} \tag{16}$$

where $\beta^2 = (1+s)\left(1 + \frac{s^\alpha \tau_o^\alpha}{\Gamma(\alpha+1)}\right)$, $\varepsilon = \left(1 + \frac{s^\alpha \tau_o^\alpha}{\Gamma(\alpha+1)}\right)\frac{Q_o}{v}$, $\zeta = \frac{s}{v}$ and $\gamma = \frac{Q_m}{s}$.

To complete the solution, we have to know the constants B_1 and B_2, by using the boundary conditions (15) which can be given by

1.4. Evaluation of thermal injury

The accurate prognosis of thermal injury to skin tissues is useful for thermal therapy. The evaluation of burn is one of the utmost important attributes in the bio-engineering sciences in skin tissue. To quantify thermal damages, the method improved by Moritz and Henriques (Henriques and Moritz, 1947; Moritz and Henriques, 1947) can be used. The non-dimensional measure of thermal damages index Ω can be given by

$$\Omega = \int_0^t B e^{-\frac{E_a}{RT}} dt \tag{17}$$

where B= $3.1 \times 10^{98} s^{-1}$ is the frequency factor, R= 8.313 J/mol·K is the constant of universal gas and $E_a = 6.28 \times 10^5$ J/mol is the activation energy.

1.5. Numerical results and discussion

In this section, the variation of temperature in skin tissues under fractional bio-heat model is studied under the modified external moving heat source. For numerical calculations, exemplary values of heat properties for skin tissues are chosen (Askarizadeh and Ahmadikia, 2014)

The computations are made using MATLAB (R2017a) software and the outcomes are presented graphically. For the final solution of temperature distribution, a numerically reversal method is adopted depending on the approximation method of Riemann-sum used to investigate the numerical outcomes. According to this method, for any function in the domain of Laplace may be transformed into the domain of time such as:

$$\mathrm{M}(x, t) = \left(Re \sum_{n=0}^{N} (-1)^n \bar{M}\left(x, m + \frac{in\pi}{t}\right) + \frac{1}{2} Re[\bar{M}(x, m)] \right) \frac{e^{mt}}{t} \tag{18}$$

whereas Re is the actual part and i is the imaginative number unit. For quicker assemblage, numerically methods decided that $m = \frac{4.7}{t}$ that satisfying the above equation (Tzou, 1996). This mathematical model based on Pennes bioheat transfer has been founded with the interfaces and suitable boundary conditions. The perfusion, metabolic and conducting thermal resource term are utilized in the formulation. The outer thermal resource impact on the surface of skin is integrated. A slab of tissues 5 cm thick and its normal temperature is $T_b = 37°C$. For the aim of studying the effect of fractional parameter α on the temperature and the thermal injury, results are presented using the graphs as in Figs. 1–6 while, Figs. 7–9 show the effect of the speed of moving heat resource v. By comparing figures of solutions obtained under the fractional Pennes model and Pennes model, important phenomena are noted as in Figs. 1–3. The solid line ($-$) refers to fractional Pennes model ($\alpha = 0.1$) and the dotted line ($\cdots$) refers to Pennes model. Fig. 1 shows the variation of temperature with respect to the distance x at $t = 2.5$ *minute* when the velocity of heat source remain to constant $v = 0.3$. It is clear from the graph that the temperature increases till maximum value after the starting, then it decreases continuously to the normal temperature $T_b = 37°C$. The time histories of surfaces temperature with and without fractional derivative are exhibited in Fig. 2. It has been revealed that the temperature starts from the normal temperature T_b and increases with the time until the maximum value then decreases to the normal temperature again. Fig. 3 displays the variation of thermal injury with respect to the time t. Clearly, the time histories of the thermal injury obtained from the two models are very different. Figs. 4–6 show the effects of fractional order parameter in the temperature and the thermal injury. As expected, the fractional order parameter has a great effect on

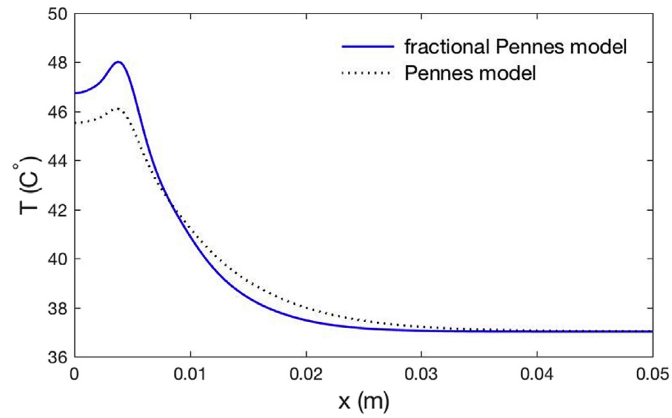

Fig. 1. Temperature profile in skin tissue with and without the fractional derivative.

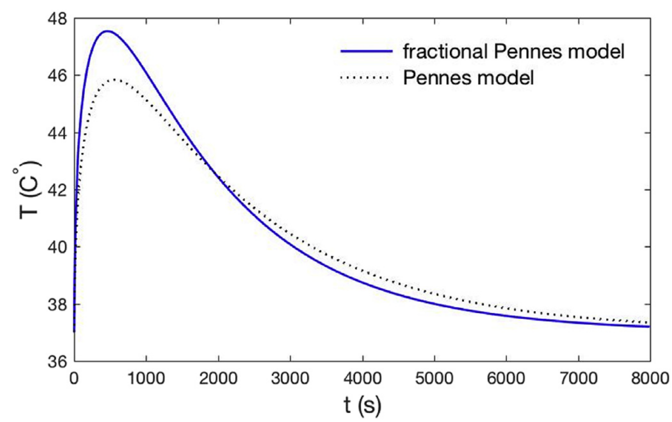

Fig. 2. Temperature history at skin surface with and without the fractional derivative.

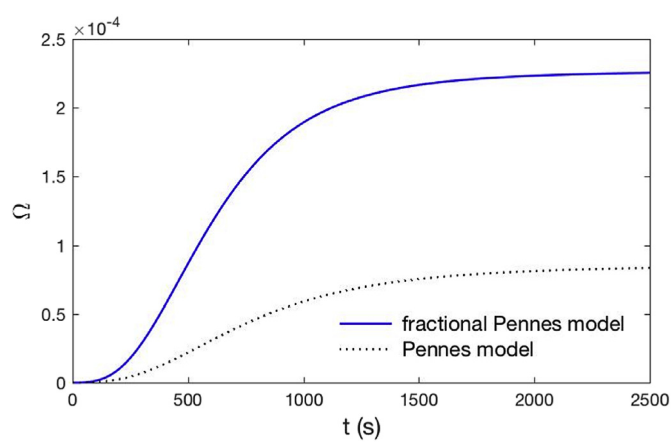

Fig. 3. The variation of the thermal damage at skin surface with and without the fractional derivative.

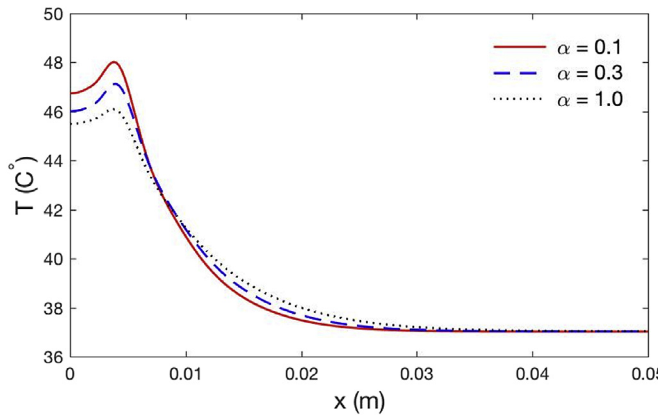

Fig. 4. Temperature profile in skin tissue for different value of α.

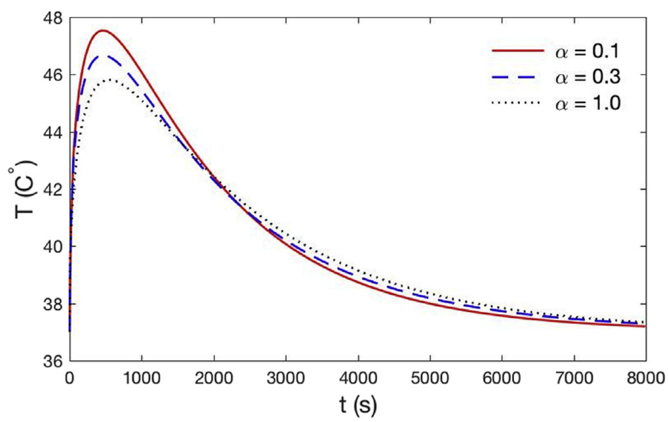

Fig. 5. Temperature history at skin surface for the different value of α.

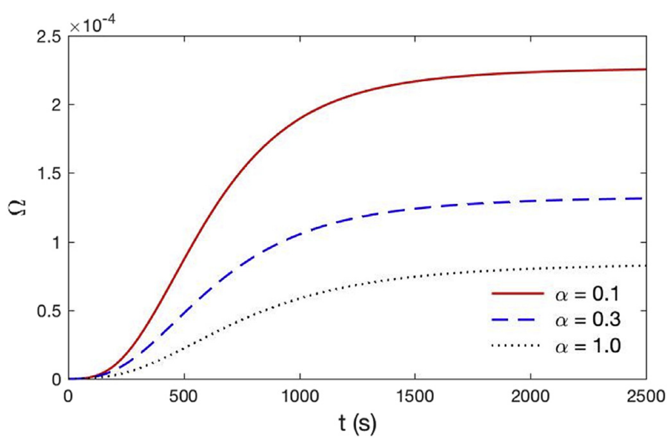

Fig. 6. The variation of the thermal damage with time for the different value of α.

the distribution of field quantities. Based on fractional Pennes model, the comparison of temperature response among three types of heat source speeds with respect to the distance x is conducted as in Fig. 7. In this case, it has been remarked that the mobile heat source increases the temperature of skin surface which relies on the heat source velocity. Gradually, the temperature rises till it reaches a summit value at a certain spot. Then the temperature lessens continuously to the normal level. The time histories of surfaces temperature through various values of the mobile thermal resource speeds are exhibited in Fig. 8. Consequently, the temperature change provokes a reducing valley-to-peak value of temperature in the oriented tissues. Fig. 9 displays the velocity

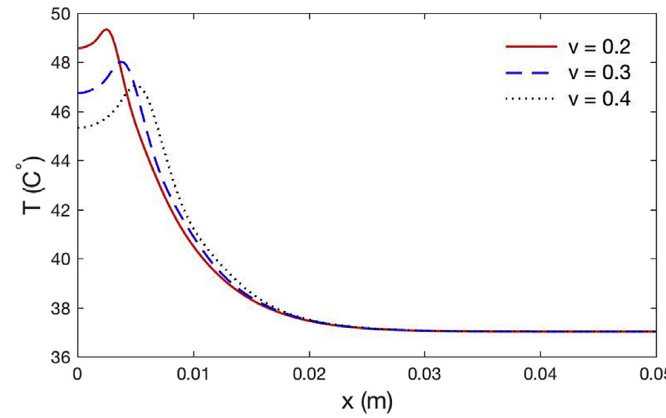

Fig. 7. Temperature profile in skin tissue due to the various moving heat source velocities v.

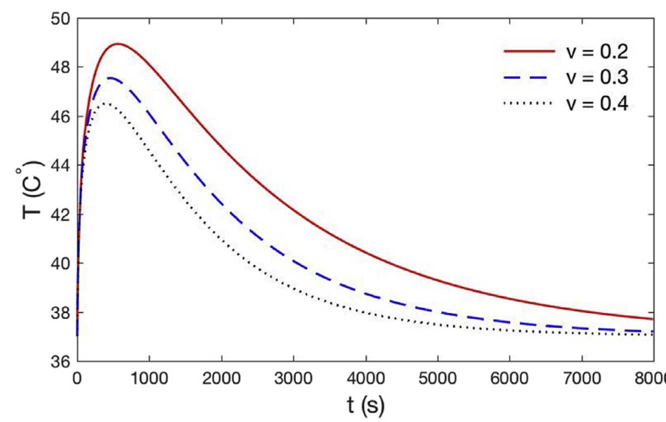

Fig. 8. Temperature history at skin surface due to the various moving heat source velocities v.

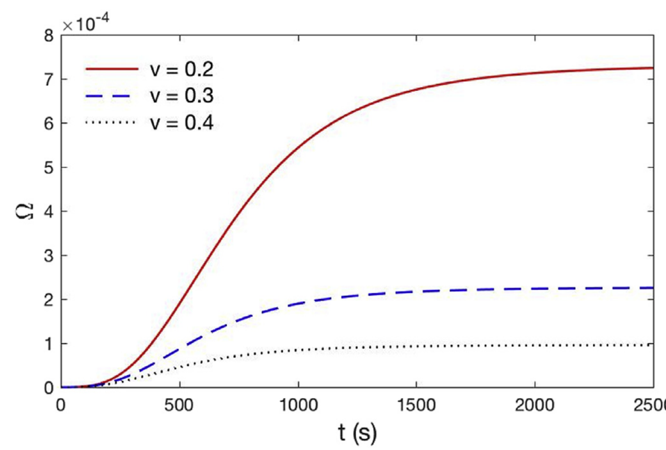

Fig. 9. The variation of thermal damage with time due to various the moving heat source velocities v.

impact of the mobile thermal resource on the thermal injury to skin tissues. It reveals that the thermal injury decreases during the increase of heat source speed.

2. Conclusion

The fractional Pennes bio-heat model is derived and their influences are presented in the applications of the thermal therapy. It can be concluded from the above investigation that the Pennes bio-heat model causes greater temperature rise compared to the fractional Pennes bioheat model.

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