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Acoustic directional source based on Penrose quasi-crystal arrangements of metallic rods embedded in a fluid

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ABSTRACT

The directional emission properties of a circular device formed of a small number of metallic rods placed at the nodes of Penrose's tiling are investigated. At the center of the device a non-directional primary source emits monochromatic acoustic waves. In a wide frequency range, the beams come out of the device along preferential directions related to the symmetry of order 5 of the tiling and the non-periodicity in the radial direction. There are some frequencies for which the waves are completely trapped by the device. The shape, the distribution and the intensity of the emitted beams outside are closely dependent on the frequency. A comparison between numerical results and experiments is presented.

1. Introduction

Most of the directional devices designed to control electromagnetic (EM) or acoustic (AC) waves and direct them in specific directions use composite materials that are periodic arrangements (Phononic Crystals) of a medium (the inclusions) embedded in another one (the matrix). Inclusions and matrix have different physical properties. The just mentioned directional devices considered the four-folds symmetry of square lattices of inclusions for EM [1] as well as for AC waves [2–4]. Such a symmetry leads to propagation of AC waves along specific directions (in the case of Ref. [4] four directions of equal intensity making an angle of 90° between them). Moreover, the periodicity introduces strong constraints as to the frequencies to be used and the incidence at which to attack these devices. Depending on whether one is in a frequency band gap or in a frequency pass band, a periodic medium will forbid or allow the travel of AC waves. This work examines an alternative way consisting to use quasi-crystals instead of square symmetry Phononic Crystals, in order to avoid some of the above mentioned constraints.

To include a quasi-periodic translational order they named 'quasi-crystal' (QC) Levine and Steinhardt [5] generalized the notion of crystal. The two fundamental characteristics of QC are (i) long-range quasi-periodic translational order and (ii) long-range crystallographically forbidden orientation symmetry [6]. The icosahedral QC (with 5-fold rotation axes) observed in an Al–Mn alloy obtained by rapid quenching from the melt in 1984 opened a door for new crystallography [7].

In 2004 and 2007, Sutter and Steurer [8] and Sutter-Widmeck et al. [9] used rectangular devices formed of metal rods arranged in Penrose quasi-crystals and immersed in water to study their acoustic properties. They focused on transmittance through these devices to investigate the possibilities to form band gaps.

This work is devoted to the realization of directional AC sources from a primary non-directional source (pinducer) inserted in a circular device made up of identical, distant, parallel circular cylindrical rods (not touching). The rods (either 86 or 186) are arranged so that, in a plane perpendicular to their axes, they occupy the nodes of the Penrose tiling [10]. The device is no longer a Phononic Crystal (obeying one of the orientation symmetry classes of conventional crystals) but a QC obeying the definitions (i) and (ii) with 5-fold rotation axes recalled above.

Penrose tiling is a non-periodic surface covering using two elemental rhombuses described in more detail below. The sides of the rhombuses have the same length, while the internal angles are different. It can also be noted that this kind of tiling has a self-similarity, so that a same geometrical figure can reproduce to infinity, on a larger scale. However, it should be noted that the geometry of our devices (see Fig. 1) is only a detail of the true Penrose tiling. Indeed, the number of rods is reduced. If more rods were to be taken into account, the symmetry of the QC would be modified, becoming then decagonal, according to the studies presented in ref. [8].

Ideally, angular directionality should apply over a wide range of

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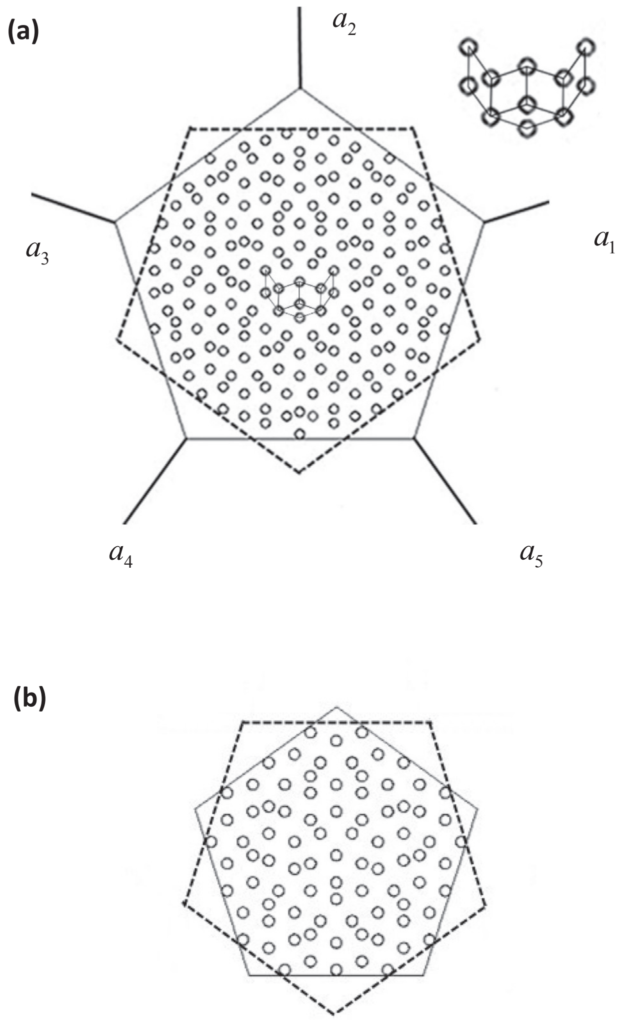

Fig. 1. Geometry of a Penrose quasi-crystal realized with (a) 186 rods and (b) 86 rods. The two basis rhombuses constitutive of the tiling are shown at the right top of figure (a). The axes a_j (j = 1–5) attached to the pentagon (in solid line) are axes of rotation of order 2. A second pentagon (dotted line) can also be considered.

frequencies. The structures that can generate such angular directivity do not currently exist. But, as will be seen, the QCs (similar to the Phononic Crystals) concentrate the AC waves in particular directions in relation to the axes of symmetry. In the case of QC however two very different frequencies will direct the beams along the same axes, while two closer frequencies will direct the beams in different axis systems. This directivity will always depend strongly on the frequency of the AC waves.

The arrangement of this paper is as follows. In Section 2, a description of the QC is briefly summarized. In Section 3, numerical results are presented and discussed based on the case of water immersed steel rods. The choice of the materials results from the relative simplicity of implementing such a device for experimental studies. Section 3.1 presents numerical results on the QC with the central rod replaced by a non-directional monochromatic source. The process of the spatial filtering is analysed. In Section 3.2 one studies the influence on the angular directivity of defects due to the removal of some rods from the QC. Section 4 discusses the effects of adding rods to the periphery of the QC while keeping the character of Penrose's tiling. The absence of periodicity along the radial directions, which can be seen as a form of randomization, plays a role in the QC's ability to direct and focus the AC waves. An averaging process is progressively initiated as the number of rods increases, preventing the formation of large beams outside. Finally, in Section 5 experimental measurements of the AC pressure around the QC are presented. A good agreement between predictions and measurements is found.

2. Geometry of the QC

The QC consists of parallel identical rods of radius a arranged to form a Penrose tiling constrained inside a circle. One can distinguish two basic cells in Fig. 1(a). The first cell is a rhombus with interior angles 36° and 144° and sides equal to $r = 5a$. The second one is a rhombus with interior angles 72° and 108° and same sides r. The spacing between adjacent rods is therefore not regular. These angles are multiples of $2\pi/10$ (i.e. 36°). Given the arrangement of these patterns to form the QC, an axis of rotation of order 5 can be observed. The QC is transformed into itself by the rotations of the dihedral group D_5. This group is that of symmetry rotations of a regular pentagon as shown with solid lines in Fig. 1(a). It is generated by a 5–fold rotation about an axis through the center and perpendicular to the plane of the figure and any one of the five 2–fold rotations about the axes a_j (j = 1, ...,5) in the plane. The group D_5 is of order 10 and the 10 elements can be written in canonical form as $\{e, c, c^2, ..., c^4, b, bc, bc^2, ..., bc^4\}$ with $c^5 = b^2 = (bc)^2 = e$ (e, c and b are the identity, the anticlockwise rotation through $2\pi/5$ and the 5–fold rotation through π, respectively). A second pentagon (in dotted lines) with the same group properties can be exhibited. Note that all the above remarks are valid regardless of the size of the QC (the number of rods involved can change moderately as for example from Fig. 1(a) to (b)). Failure to observe a periodicity does not affect the symmetry properties. Thus, in Fig. 1(b), around the same center and with a smaller number of rods placed at the Penrose nodes, the structure is again inscribed in two pentagons.

3. Numerical simulations

Slip boundary conditions (i.e. continuity of normal displacements and stresses, vanishing of tangential stresses) are applied at the liquid/rod interfaces. Two dimensional finite elements (three node triangles) are used for the computations. The finite element mesh is such that for a given rod, there are 6 internal nodes and 8 nodes at the boundary. The size of the triangles is the same for the inner as well as for the outer domain of the rods. All along the paper, steel rods of diameter 2 mm embedded in water are considered, see Table 1 for the parameters. The QC is surrounded by a large circle of radius 30 cm whose edge is absorbent so as not to activate the reflections (Perfect Matching Layer) [11] (the centers of the large circle and of the QC are confounded). Focusing on the case presented in Fig. 1(b), the filling fraction (or concentration) $\phi = Na^2/R^2 \simeq 0.16$ where $R = 2.3$ cm is the radius of the smallest disc containing the QC and $N = 86$ the number of rods. The removal of the central rod of the QC creates a hole in which one introduces the primary non-directional source (pinducer) emitting monochromatic waves. The study takes place in the range 167.5–619 kHz, which also includes the central frequency (450 kHz) of the pinducer.

3.1. QC with no defects

Consider at first the problem of Fig. 1(b). One examines the

Table 1
Parameters used for the computations.

	Mass density (kg/m^3)	Longitudinal wave speed (m/s)	Transverse wave speed (m/s)
Steel	7850	5834	3225
Water	1000	1480	–

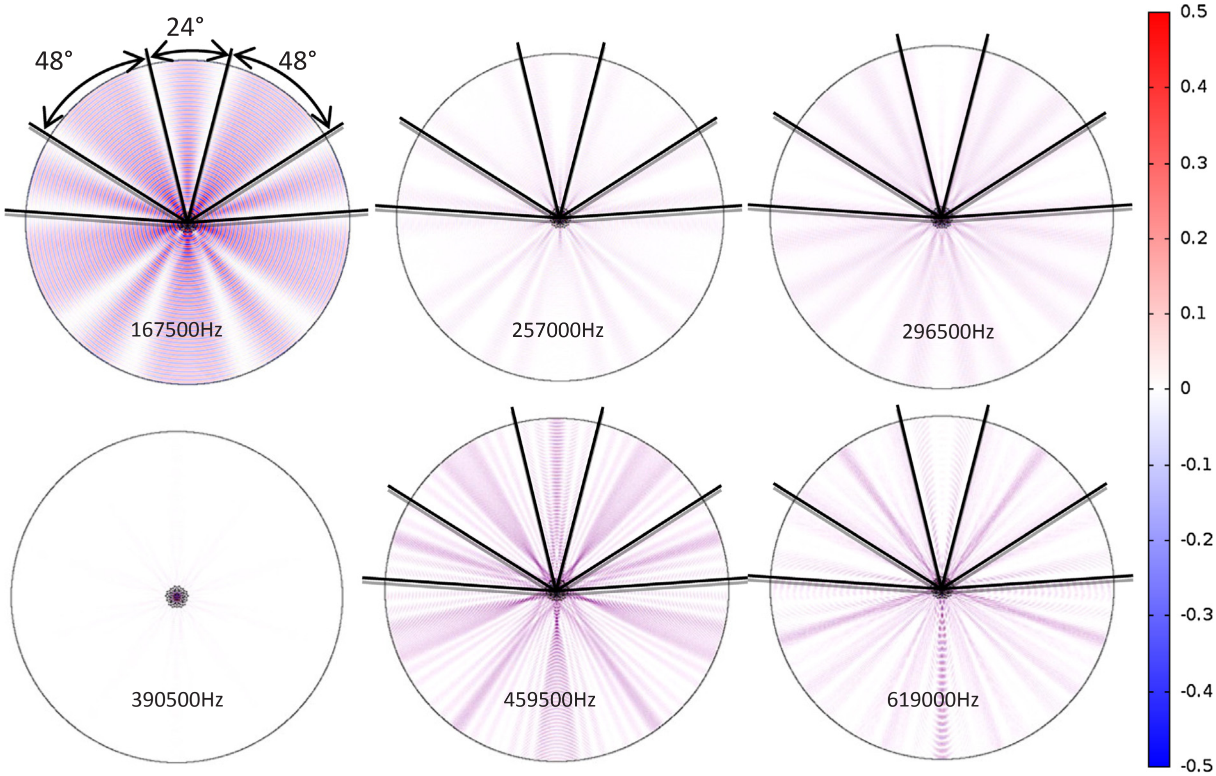

Fig. 2. Distribution of the acoustic pressure at various frequencies around a circular Penrose quasi-crystal of 86 rods (only the central rod was replaced by the pinducer). The superimposed black segments delineate two kinds of angular sectors in relation with the symmetry observed for the quasi-crystal.

distribution of the AC pressure outside the QC in the absence of defects: all the rods except the one that has been replaced by the source are therefore at the nodes of the Penrose tiling. The usual terminology 'pass band' and 'stop band' of phononic crystals can be used here since as shown by some authors, spectral gaps were observed in 2D EM [12] and AC [9,13,14] Penrose quasi-crystals. However the calculation of the band diagrams for the devices used here is not our first concern and is therefore not performed in the paper.

The numerical simulations were carried out by steps of 500 Hz. Fig. 2 shows several distributions of the AC pressure field in pink color (online) outside the QC as the frequency increases. At 167.5 kHz (wavelength in water $\lambda \simeq 0.884$ cm), the pressure field is divided into ten sectors (five 24°-sectors and five 48°-sectors). The boundaries of these sectors are indicated by the black lines in the figure. Those lines form a basic frame reported later on all the figures that follow. A non collimated beam is located in each sector (with the maximum intensity of pressure at the center). It is around this frequency that the simplest distribution for the pressure is met. As the frequency increases to 257 kHz ($\lambda \simeq 0.576$ cm), the beams change shape and previous maxima give way to minima. This is particularly clear using the frame: one goes from ten beams to twenty beams located in the neighborhood of the black lines. At the frequency 296.5 kHz ($\lambda \simeq 0.449$ cm), the number of beams has not increased (there are still twenty). They have only moved away from each other. Using again the frame, it is seen that some beams are centered on the black lines. The 24°-sectors do not contain any beams while the 48°-sectors contain two beams.

A continuous transformation of the pressure distribution is therefore observed as the frequency increases. There are frequencies for which the AC pressure outside the device is nearly nil in all directions. Thus at 390.5 kHz ($\lambda \simeq 0.379$ cm) the QC almost completely traps the waves emitted by the pinducer (no transmission in the surrounding space). This frequency is located in a 'stop band'. The AC wave emitted by the source can be confined and distributed in the device via a set of specular reflections between the rods and scattering by the rods to the fluid matrix. When one moves away from this frequency, the QC re-emits again in privileged directions in relation to its axes of symmetry. A particular case is observed at 459.5 kHz ($\lambda \simeq 0.322$ cm) where there are no less than 3 types of emitted beams (thin, intermediate and broad). Using the frame, some analogies can be seen with the situation prevailing at 167.5 kHz: the broad beams are located in the center of the 48°-sectors, the intermediate beams in the center of the 24°-sectors. Between broad and intermediate beams, there are exactly 4 thin beams. The total number of beams is fifty with 5 fold-pattern of equal intensity for the broad and the intermediate ones. The discussion on this particular case is continued in Section 4.

When passing from 459.5 kHz to 619 kHz ($\lambda \simeq 0.239$ cm), the shape of the beams changes and their number decreases. The beams located in the center of the 48°-sectors become narrower and are almost collimated without significantly altering the intensity of the pressure: a phenomenon of ultra-directivity in 5 very specific directions is observed. For their part, the beams that were in the center of 24°-sectors disappear.

3.2. QC with Schottky-like defects

So far, the only change in the QC has been to replace the central rod with a linear source. This does not change the symmetry of the QC, Fig. 1. But QCs can also find usefulness if one knows how to modify their multi-directional emission properly to transform it into a more selective emission limited to only a few angular sectors. The simplest way to achieve this is to introduce defects in the QC. The point defects are associated with a local rupture of the quasi-periodicity (or periodicity for the PhCs) by withdrawal of one or more rods (vacancy defects or Schottky-like defects). The result can be the presence of trapped AC modes influencing the pressure field outside the QC at discrete frequencies. One will leave aside point defects created by insertion of a rod into an interstice (Frenkel-like defects) or extended defects such as the dislocations.

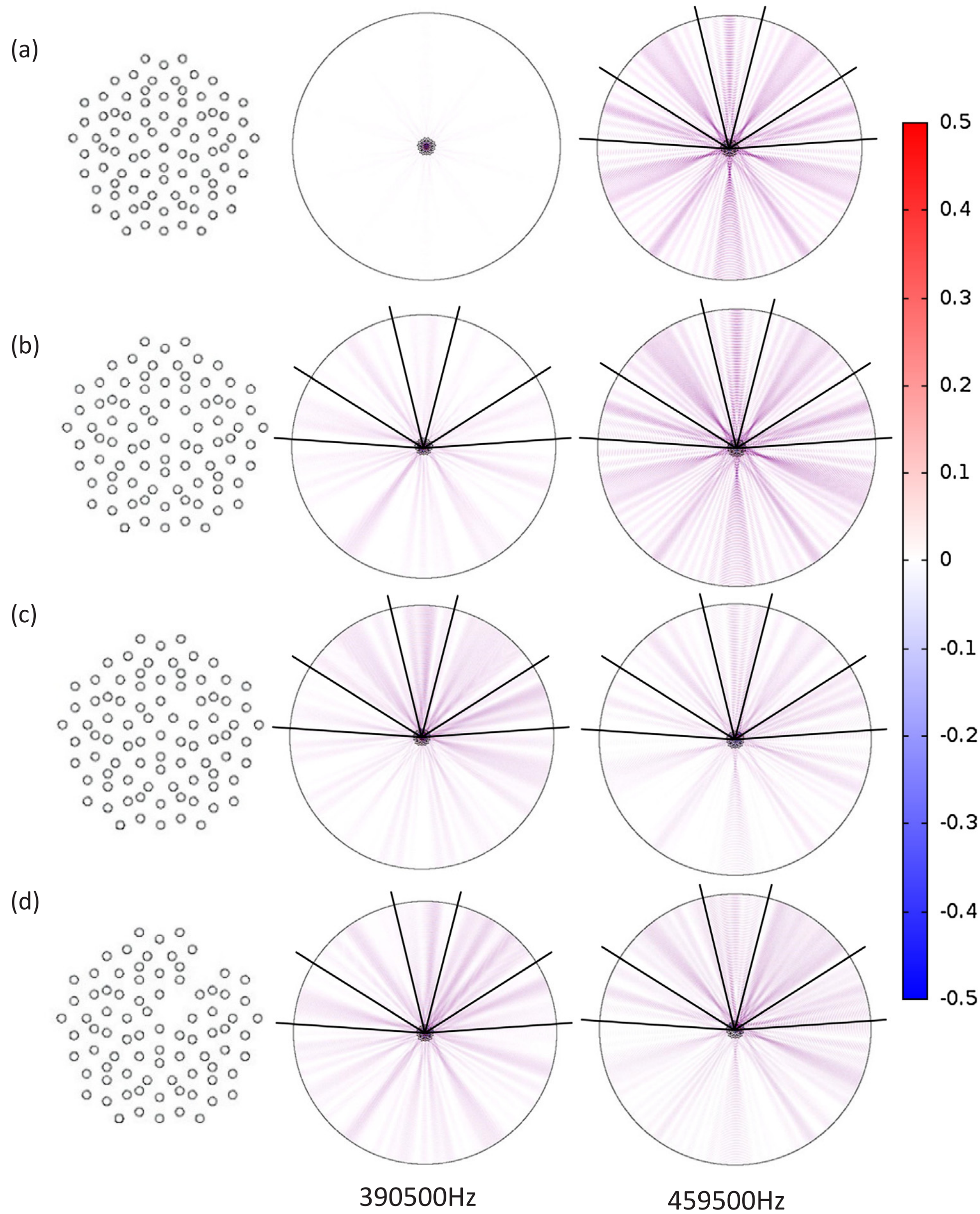

Fig. 3. Distribution of the acoustic pressure at 390 kHz and 459.5 kHz around a circular Penrose quasi-crystal with 86 rods. Case (a) was reported from Fig. 2 whereas cases (b) (c) and (d) were obtained by creating Schottky type defects (withdrawal of rods along an axis of rotation of order 2).

The defects are practiced along one of the 2-axis symmetry (a_j) of the dotted pentagon represented in Fig. 1(b). The effects of these withdrawals on the angular distribution of the pressure field outside the QC are illustrated in Fig. 3, for two of the frequencies for which calculations were already made (Fig. 2): 390.5 kHz and 459.5 kHz. The row (a) of Fig. 3 corresponds to the QC without defect, the rows (b) and (c) to the withdrawal of one of the rods positioned along the (a_j) axis. The rows (d) presents the case where all the rods are withdrawn from this axis**.**

At 390.5 kHz, which corresponds to a frequency in a stop band where the QC does not emit waves to the surrounding fluid, the removal of a single rod allows the QC to transmit the AC waves. It can be observed, however, that the directivity pattern no longer has any relation to the 5–fold rotation symmetry through the center of the QC without defects. This is particularly clear in examples (b) (c) and (d) shown in Fig. 3. In cases (b) and (c), moving the defect from a site to a neighbor while remaining on the axis of symmetry radically changes the distribution of the pressure field. At 459.5 kHz (third column of Fig. 3), the effect produced by the defects is remarkable only when they are not too close to the QC center (see cases (c) and (d)).

The study that has just been presented in Section 3 shows the possibility of several applications such as the preferential transmission of AC waves in specific directions from a primary non-directional source.

4. Effects induced by the number of rods

Let us briefly examine the effect of the QC's size on the angular distribution of the AC pressure outside. To do this we add rods at the periphery of the QC of Fig. 1(b) to obtain the device with 186 rods depicted in Fig. 1(a). Since the number of rods added is small, we do not observe any influence on the axes of symmetry as described in Section 1. The angular distribution produced by each device is shown in Fig. 4. It is no surprising that the two angular distributions differ strongly. This results from the fact that at the considered frequency (here 459.5 kHz) the mapping of the pressure field inside the QC changes as rods are added to the periphery. Indeed, it must be remembered that if the

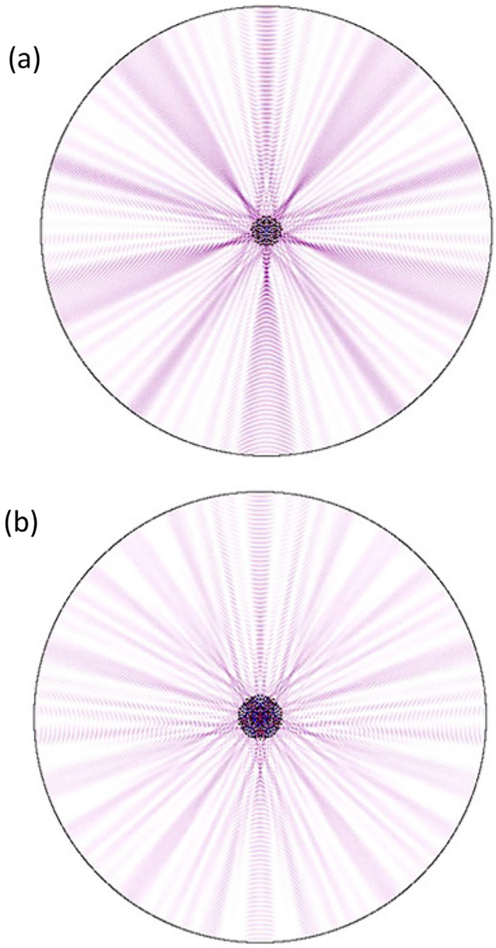

Fig. 4. Distribution of the acoustic pressure outside two circular Penrose quasi-crystals with (a) 86 rods and (b) 186 rods. The operating frequency of the pinducer is 459.5 kHz. In (a) two types of symmetric caustics emerging from the quasi-crystal can be seen. Higher order diffraction beams with respect to the largest beams are also observed. In (b) the adding of rods at the periphery according to Penrose's tiling modifies completely the distribution of the beams.

symmetry remains unchanged (5–fold rotation about an axis through the center), the arrangement of the rods along the radial directions of the devices, although obeying the rules of the Penrose tiling, is for its part irregular. This absence of periodicity plays a role in the QC's ability to direct and concentrate the AC waves. The averaging process of the multiple scattering is progressively initiated as the number of rods increases in the radial directions, preventing the formation of intense beams outside. In Fig. 4, the computer simulations show that it is the QC with the fewest rods (Fig. 4(a)) that produces the simplest network of intense beams outside. One can note that the passing from 86 to 186 rods tends to include long range effects of Penrose's QC (the symmetry of the QC rather than decagonal tends to become pentagonal [8]). This has an effect on the number of beams emitted: they are more numerous and less intense.

Adopting the point of view of geometrical acoustics, it is possible to interpret with simple words certain aspects of Fig. 4(a). Several caustics (envelopes of AC rays that emerge from the "curved discontinuity" delimiting the domain of the QC) can be observed. These caustics are curves to which a family of AC rays is tangent, i.e., envelopes on which AC rays concentrate. The formation of the caustics and of the beams depend on the axes of symmetry of the QC. One can distinguish in Fig. 4(a) two kinds of symmetric caustics (localized near the QC) each delimiting a family of rays giving rise to the more intense beams observed in the 24°- and 48°-sectors. The existence of these caustics means that collimated sound beam will not be realized accurately.

5. Experiments

5.1. Experimental setup

The experimental setup is sketched in Fig. 5. The device supporting the QC is made up of two plastic plates (10 mm thick) drilled at the nodes of the studied Penrose tiling (see Fig. 1b). Steel rods (diameter = 2 mm and length = 200 mm) are inserted in these two perforated plastic plates placed at each end of the rods. They are held in this position, making sure that the cylinders remain parallel. The entire structure can be rotated around an axis coinciding with that passing through the center of the QC. Then we replace the rod at the center of the QC by a Valpey Fisher pinducer (diameter 2.4 mm with a passband of 50 kHz centered at 450 kHz) which will allow to excite acoustic wave inside. Contrary to the structure constituted by the plastic plates and the rods, the pinducer don't rotate.

In reception, we use a classic immersion transducer with of central frequency equal to 500 kHz and 1.5 in. diameter) positioned at 35 cm of the pinducer. This distance is assumed to be in the far-field area. The pinducer, the QC and the receiver are all immersed in water.

A generator HP 33120A sends to the pinducer a signal in the form of a burst of 5 periods at 459.5 kHz frequency and amplified by a factor of 50 dB.

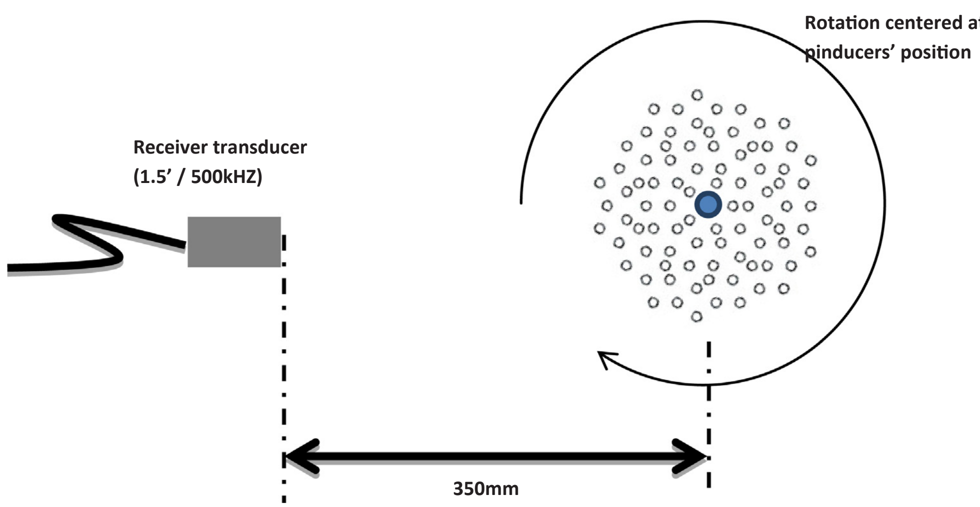

Fig. 5. Experimental setup of the quasi-crystal with 86 rods. The emitting pinducer is placed at the center of the structure (dark disk).

Fig. 6. Angular plot of the maximum-amplitude signal received from the pinducer immersed in water. The signal is emitted at the frequency 459.5 kHz.

5.2. Results and discussion

At first, we characterize the directionality of the pinducer source without the installation of the QC. The detection transducer is fixed and we raised the diagram of emission directivity by performing a rotation of the pinducer at a frequency of 459.5 kHz (Fig. 6). We observe two maximum of emission, the first one at 90° and the second one at 270°. This experiment was made with the same frequency as for the study through the QC. Then we put the pinducer in the QC and direct it such a way as the maximum of the diagram of emission is oriented towards the receiving transducer and maintained in this position.

Fig. 7-a represents the experimental statements of the maximal intensity of the AC waves in water after the generation by the pinducer and transmission through the QC. The direction of the pinducer is maintained at the value of 90° according to the diagram of Fig. 6.

For every angular position (1 degree step) of the structure, we record the average of 150 spreaded signals obtained by the immersion transducer witch is connected to a digital oscilloscope @Lecroy. We thus obtain a diagram of spreading in a plane perpendicular to the axis of rotation of the structure and observe clearly ten privileged directions of reemission of the AC waves such as they were predicted by computation (Fig. 2 at 459.5 kHz).

In Fig. 7-b, the curve is obtained in the same conditions as for Fig. 6-a, except in this case, we created a Schottky-type defect in the QC by removing a row of cylinder situated at the angle equal to 60°. We observe in this case, a general decrease of the intensity of the transmitted waves with a peculiarity in the direction of the defect (60°) which consists of a wide beam which covers an angle about 10° (without defect the reemissions are more directive). We can conclude that at 459.5 kHz, the results of the experiments agree with the numerical simulations.

6. Conclusion

Two dimensional Penrose quasi-crystals formed of metallic rods (the first one with 86 rods, the second with 186 rods) split up the acoustic waves emitted by a primary source placed at center. As the frequency of the waves increases, the number of beams coming out of the device increases while respecting the symmetry of the quasi-crystals.

There is no known EFC (Equi Frequency Contour) [4,15] for quasi-

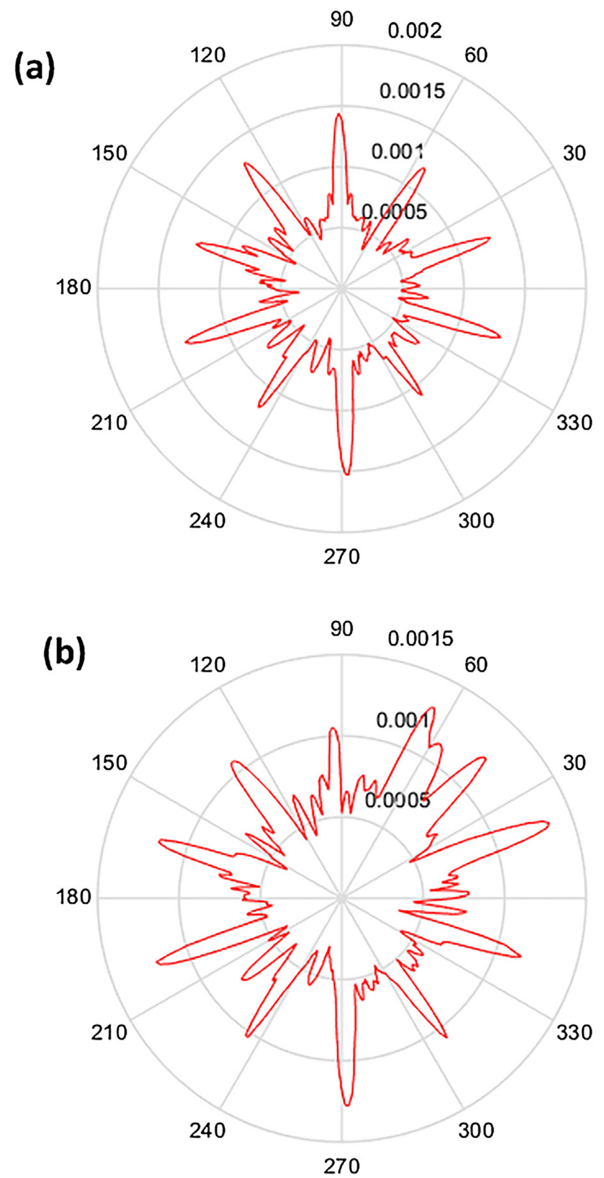

Fig. 7. Angular plot of the maximum-amplitude of the averaged signal received from the water-immersed quasi-crystal (86 rods) with the pinducer placed at the center. The signal is emitted at 459.5 kHz. In (a) no defect was considered, in (b) rods were withdrawn along an axis of symmetry.

crystal contrary to what has been found for a phononic crystal with square pattern. Hence, many of the discussions in Ref. [4] must be handled with care. What can be emphasized is that the collimation of acoustic waves does not occur inside the QC, a fact that is at the opposite of what has been observed for a phononic crystal with square pattern. The collimation can be observed outside the device at certain frequencies (619 kHz for example) and in certain directions only. In such cases, the beams are rather norrow. The presence of defects created by the withdrawal of one or two rods from the quasi-crystal prevents the emission of beams in certain directions. It also allows to control and to create preferential directions of emission from the quasi-crystal to the outer fluid. The device can serve as a directional source or for spatial filtering. Since they exhibit a 360° band gap at certain frequencies, quasicrystals can be used to coat noisy equipments to make them quieter.

Experiments at the frequency 459.5 kHz show that despite the divergence observed numerically for some beams, ten of them are detectable at a great distance (transducer placed at 35 cm of the center of the device). In all cases, a very good agreement is observed between numerical results and experiments.

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ultras.2018.12.014.

References

[1] S. Enoch, B. Gralak, G. Tayeb, Enhanced emission with angular confinement from photonic crystals, Appl. Phys. Lett. 81 (2002) 1588.
[2] J. Mei, C. Qiu, J. Shi, Z. Liu, Enhanced and directional water wave emission by embedded sources, Wave Motion 47 (2010) 131.
[3] Tsung-Tsong Wu, Chung-Hao Hsu, Jia-Hong Sun, Design of a highly magnified directional acoustic source based on the resonant cavity of twodimensional phononic crystals, Appl. Phys. Lett. 89 (2006) 171912.
[4] B. Morvan, A. Tinel, J.O. Vasseur, R. Sainidou, P. Rembert, A.-C. Hladky-Hennion, N. Swinteck, P.A. Deymier, Ultra-directional source of longitudinal acoustic waves based on a two-dimensional solid/solid phononic crystal, J. Appl. Phys. 116 (2014) 214901.
[5] D. Levine, P.J. Steinhardt, Quasicrystals: a new class of ordered structures, Phys. Rev. Lett. 53 (1984) 2477.
[6] A. Katz, M. Duneau, Quasiperiodic patterns, Phys. Rev. Lett. 54 (1985) 2688.
[7] D. Schechtman, I. Blech, D. Gratias, J.W. Cahn, Metallic phase with long range orientational order and no translational symmetry, Phys. Rev. Lett. 53 (1984) 1951.
[8] D. Sutter, W. Steurer, Ultrasonic investigation of phononic Penrose crystals, phys. Stat. Sol. (c) 1 (11) (2004) 2716–2719.
[9] D. Sutter-Widmer, S. Deloudi, W. Steurer, Prediction of Bragg-scattering-induced band gaps in phononic quasicrystals, Phys; Rev. B 75 (2007) 094304.
[10] R. Penrose, The role of aesthetics in pure and applied mathematical research, Bull. Inst. Math. Appl. 10 (1974) 266.
[11] J.-P. Berenger, A *perfectly matched layer* for the absorption of electromagnetic waves, J. Comp. Phys. 114 (1994) 185–200.
[12] M. Bayindir, E. Cubukcu, I. Bulu, E. Ozbay, Photonic band gaps and localization in two-dimensional metallic quasicrystals, Europhys. Lett. 56 (1) (2001) 41–46.
[13] S. He, J.D. Maynard, Eigenvalue spectrum, density of states, and eigenfunctions in a two-dimensional quasicrystal, Phys. Rev. Lett. 62 (1989) 1888.
[14] F.M. de Espinosa, M. Torres, Extended and localized vibrational modes in (1–3) Penrose-like piezocomposites, Appl. Phys. Lett. 65 (1994) 1352.
[15] N. Swinteck, J.O. Vasseur, A.C. Hladky-Hennion, C. Croënne, S. Bringuier, P.A. Deymier, Multifonctional solid/solid phononic crystals, J. Appl. Phys. 112 (2012) 024514.