



A new way to calculate productivity of five point well pattern at high water cut stage



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ABSTRACT

This paper presents a new way to obtain the instantaneous productivity of five point well pattern in low permeability reservoirs at high water cut stage. Different from the traditional analysis methods or semi-analytical methods, this paper takes the distribution of remaining oil into consideration, and uses the successive steady state method and 'six step' to obtain the numerical solution, which is accurate and simple.

The effects of parameters including fracture length (the injection well and the production well) and pressure difference between the injection well and the production well were investigated in detail, which have a significant impact on instantaneous productivity. The advantage of the numerical solution is able to understand productivity at different times, and easy to incorporate well spacing and the optimal pressure difference. The amount of computation that this new method needs is very small, and it can also compute quickly and efficiently.

1. Introduction

Water flooding is an effective way to maintain reservoir pressure and has been widely used to improve oil recovery [1]. At present, most of the water flooding reservoirs in China are still at high water cut stage. Due to the heterogeneity of the reservoir, the water flooding degree of different regions is different, resulting in uneven distribution of residual oil at high water cut stage [2]. Therefore, the heterogeneity of the remaining oil must be considered when calculating the instantaneous productivity of the five point well pattern at high water cut stage.

Due to the low porosity and permeability, water flooding is generally used for EOR in low permeability reservoirs, and it is developed in the form of area well pattern. The predecessors have done a lot of research on the productivity of anisotropic area well patterns, and have made much progress. For example, Raghavan [3], Dietz [4], Ramey [5], Larsen [6] and others studied the problem of single well productivity in closed reservoirs, and they also proposed the conversion method from single well to multiple well productivity. Blasingame [7] et al. used the curve fitting method to conduct a preliminary analysis of the area well pattern productivity. Watson [8] et al. used the pull inversion method to obtain the inverse nine-point well pattern productivity calculation model. Luo Wanjing et al introduced the shape factor to transform the uncertain current problem into a constant flow problem, and used the superposition principle and mirror inversion to build the relationship between the

productivity and the pressure difference [9]. Du Dianfa et al. used the equivalent seepage resistance method to derive the productivity model of the vertical well [10]. Xu Qingyan et al. combined with the superposition principle, and improved the equivalent caliper model, and established a productivity model for the joint development of vertical and horizontal wells [11]. He Ying et al. used the triangular flow tube method to give an analytical solution for the productivity of fractured wells in low permeability rectangular reservoirs [12]. Liu Hailong et al. combined with the pressure drop superposition principle and flow tube integral method, and established a low permeability reservoir steady-state five point well pattern productivity model [13]. Literature research found that the method of solving area well pattern is mainly focused on pull inversion method [3, 4, 5], conformal transformation method [7, 8], equivalent seepage resistance method [9, 11], flow tube integral method [10, 13], numerical simulation method [12] and split flow field method [6].

Although there are many methods to solve area well pattern productivity and the established models are well deduced, there are still some shortcomings, which is showed as below.

- ① The seepage model is based on Darcy flow, however, the fluid flow in the low permeability reservoirs does not obey the Darcy flow, which means the seepage model can not use Darcy flow any more.

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- ② Most of the low permeability reservoirs is now at high water cut stage, and the remaining oil is uneven distributed, and the heterogeneity of the remaining oil must be considered.
- ③ The derivation and calculation are cumbersome.
- ④ When solving the area well pattern productivity, the approximate processing method is usually used to obtain the analytical solution, which limits the accuracy of the productivity of the area well pattern.
- ⑤ The numerical simulation method needs to establish the corresponding geological model, and also needs history fitting. Although the precision is high, it is very time consuming.

Therefore, it is urgent to establish a new and rapid calculation method for area well pattern. This paper presents a new method to accurately and quickly obtain the instantaneous productivity of the five point well pattern in low permeability reservoirs at high water cut stage.

2. Model

Due to the large seepage resistance of low permeability reservoirs, hydraulic fracturing is required for reservoir reconstruction. Both production and injection wells need to be sprayed to a certain extent. Therefore, the physical model of a five point well pattern is shown in

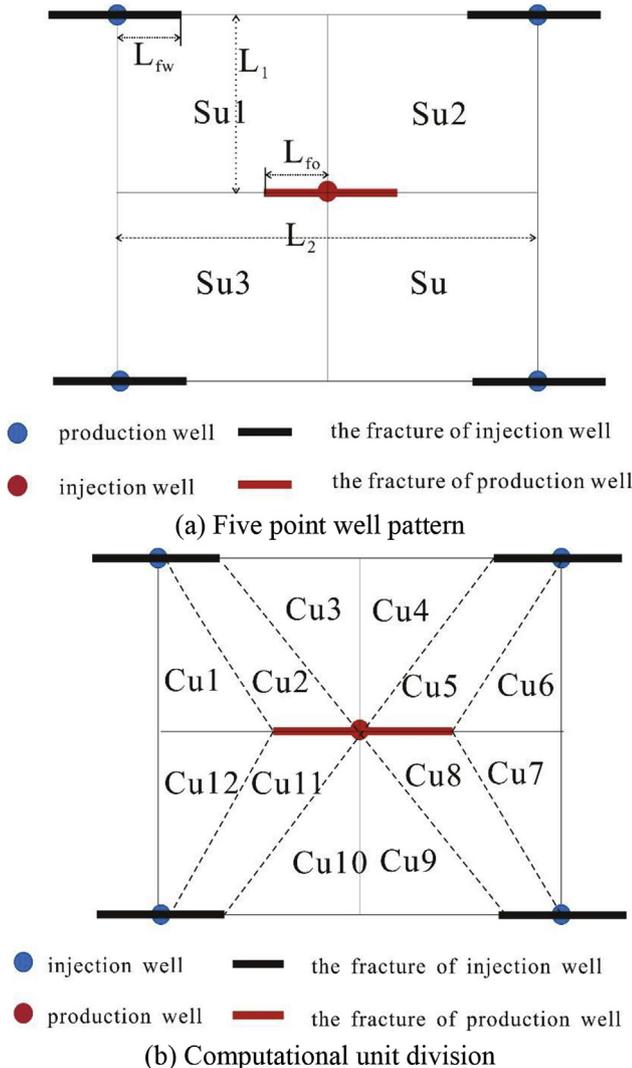


Fig. 1. The Physical model of five point well pattern.

Fig. 1a.

Through elemental analysis, the five point well pattern unit is divided into four sub-units (SU), and each unit is divided into three small calculation units (CUs). Therefore, an injection-production unit is divided into 12 small calculation units (CUs) totally, which is shown in Fig. 1b.

In order to facilitate mathematical modeling, the following assumptions are made in this paper.

- ① After the reservoir is at high water cut stage, the saturation distribution in each injection-production unit is not uniform, but the saturation distribution in the same injection-production unit is consistent.
- ② The effects of gravity and capillary forces are not taken into consideration.
- ③ The fractures in production and injection wells have infinite conductivity.
- ④ Multiphase unstable seepage occurs in the fluid of the reservoir.

According to the principle of conservation of mass, the instantaneous production of the total five point well pattern is equal to the sum of the production of all the calculation units (CUs). The calculation of the production of the calculation unit (CU) is performed in the next section.

3. Methods

There are many mathematical models to describe non-Darcy flow [14, 15, 16, 17, 18, 19, 20], which is shown in Table 1. Table 1 shows that the model of non-Darcy flow can be divided into three types, namely starting pressure gradient model, segmentation model and parametric model.

Table 1 shows that the two-parameter continuous model really reflects the flow of underground fluids, so it is chose as the basic seepage model to establish the production model for CU.

3.1. Flow tube production

The flow tube schematic is shown in Fig. 2. Taking the oil-water saturation into consideration, the production of a cross-sectional of the flow tube can be written as [9]:

$$\Delta q_o = c \frac{k_o(\bar{s}_w)}{\mu_o} A(\xi) \left(1 - \frac{1}{a + b \frac{dp_e}{d\xi}} \right) \tag{1}$$

Integrate over ξ , the Eq. (1) can be rewritten as:

Table 1
Nonlinear mathematical models.

types of models	equation	model description
proposed starting pressure model	$v = \begin{cases} \frac{k}{\mu}(\nabla p - G) & \nabla p \geq G \\ 0 & \nabla p < G \end{cases}$	When the displacement pressure is less than G, the seepage velocity cannot be calculated
power exponential form	$v^n = \frac{k}{\mu} \nabla p, n \geq 1$	There is high simulation error of simulating the linear segment
full description	$v = \frac{k}{\mu} \nabla p^{1+n_d}, n_d = 1 + \frac{2}{1 + \frac{\Delta p/L}{G}}$	It is very difficult to solve the connection position between the linear area and nonlinear
three-parameter continuous model	$v(a_1 + \frac{a_2}{1+bv}) = -\nabla p$	It does not reflect the actual seepage
two-parameter continuous model	$v = -\frac{k}{\mu} \nabla p \left(1 - \frac{1}{a + b \nabla p } \right)$	It can reflect the seepage flow of underground fluid very well

$$\Delta q_o = c \frac{k_o(\bar{s}_w) (P_{in} - P_{pro} - Ml)}{\mu_o} \frac{1}{\int_l \frac{d\xi}{A(\xi)}} \frac{1}{B_o} \tag{2}$$

$$\text{Where } M = \frac{2}{\int_0^l A(\varepsilon)d\varepsilon + \int_{0.5l}^l A(\varepsilon)d\varepsilon} \frac{\mu_o}{k_o(\bar{s}_w)}$$

3.2. Triangular CU production

The triangular CU schematic is shown is shown in Fig. 3. By rewriting

$$Q_{o2} = \int_0^\alpha q_{o2} d\alpha$$

$$= c \frac{k_o(\bar{s}_w)}{\mu_o} \left[\int_{r_w}^{L_{fo}} \frac{\sqrt{\left(a + 1 + \frac{\Delta q}{\Delta a} \frac{\mu b}{kh\xi_1}\right)^2 - 4a}}{2b} d\xi_1 + \int_{r_w}^{L_{fw}} \frac{\sqrt{\left(a + 1 + \frac{\Delta q}{\Delta a} \frac{\mu b}{m kh\xi_2}\right)^2 - 4a}}{2b} d\xi_2 \right] \tag{6}$$

the Eq. (2), the single flow tube production of CU can be written as:

$$q_{o1} = c \frac{k_o(\bar{s}_w)}{\mu_o} \frac{\left(P_{in} - P_{pro} - \lambda \left(\sqrt{L_1^2 + \left(\frac{L_2}{2} - l_1\right)^2} \frac{(\sin\alpha_m + \sin\beta_m)}{\sin(\alpha_m + \beta_m)} - r_w \right) \right)}{\frac{1}{2h \tan\left(\frac{\Delta\alpha}{2}\right)} \ln \left(\frac{\sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw}\right)^2} \sin\beta_m}{\sin(\alpha_m + \beta_m)M} \right) + \frac{1}{2h \tan\left(\frac{\Delta\beta}{2}\right)} \ln \left(\frac{\sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw}\right)^2} \sin\alpha_m}{\sin(\alpha_m + \beta_m)M} \right)} \tag{3}$$

Where

$$\alpha_1 = \angle CAB, \beta_1 = \angle CBA,$$

$$\alpha_{11} = \angle EAB, \beta_{11} = \angle EBA,$$

$$\Delta\alpha_m = \angle EAD, \Delta\beta_m = \angle EBD,$$

$$l_1 = \sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw}\right)^2} \frac{(\sin\alpha_{11} + \sin\beta_{11})}{\sin(\alpha_{11} + \beta_{11})} - r_w - w_f.$$

So the triangular CU production is:

$$Q_{o1} = \int_0^\alpha q_{o1} d\alpha$$

$$= c \int_0^\alpha \frac{k_o(\bar{s}_w) \left(P_{in} - P_{pro} - \lambda \left(\sqrt{L_1^2 + \left(\frac{L_2}{2} - x_i\right)^2} \frac{(\sin\alpha_m + \sin\beta_m)}{\sin(\alpha_m + \beta_m)} - r_w \right) \right)}{\mu_o} d\alpha$$

$$\frac{1}{2h \tan\left(\frac{\Delta\alpha}{2}\right)} \ln \left(\frac{\sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw}\right)^2} \sin\beta_m}{\sin(\alpha_m + \beta_m)M} \right) + \frac{1}{2h \tan\left(\frac{\Delta\beta}{2}\right)} \ln \left(\frac{\sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw}\right)^2} \sin\alpha_m}{\sin(\alpha_m + \beta_m)M} \right) \tag{4}$$

3.3. Quadrilateral CU production

The quadrilateral CU schematic is shown is shown in Fig. 4. By rewriting the Eq. (2), the single flow tube production of CU can be written as:

$$q_{o2} = c \frac{k_o(\bar{s}_w)}{\mu_o} \left[\frac{\Delta q}{\Delta a} \frac{1}{2mh} \ln \frac{l_2}{r_w} + \frac{\Delta q}{\Delta a} \frac{1}{2mh} \right] \tag{5}$$

$$\text{Where } l_2 = \sqrt{0.5(L_2 - L_{fw} - L_{fo})}, m = \frac{L_{fo}}{L_{fw}} = \frac{\Delta L_{fo}}{\Delta L_{fw}}.$$

So the quadrilateral CU production is:

Above these two situation, the production for a five point well pattern is

$$Q_o = \sum_{i=1}^{12} Q_{oi} \tag{7}$$

3.4. Transient productivity

Generally, at the high water cut stage, the process of oil-water two phase flow is instantaneously changed. However, it is known from the successive steady state method that the transient flow process is a superposition of many stable flow processes [21, 22, 23, 24, 25]. Therefore, the key to the successive steady state method is how to connect each stable flow process in each discrete time. According to the material conservation theory, it can be obtained that:

$$\bar{s}_{vj}^{n+1} = \bar{s}_{vj}^n + \frac{Q_{osj} \Delta t B_o}{V_{\phi j}} \tag{8}$$

Where $\Delta t = t_{n+1} - t_n$.

So the Q_{osj} (j = 1, 2, 3 or 4) is

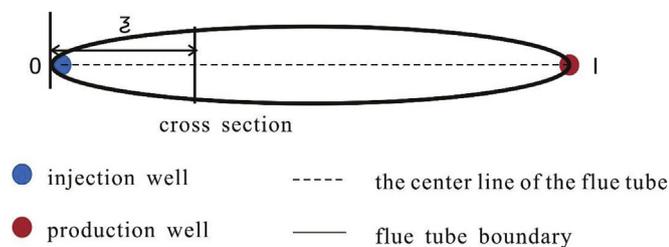


Fig. 2. The single flow tube schematic.

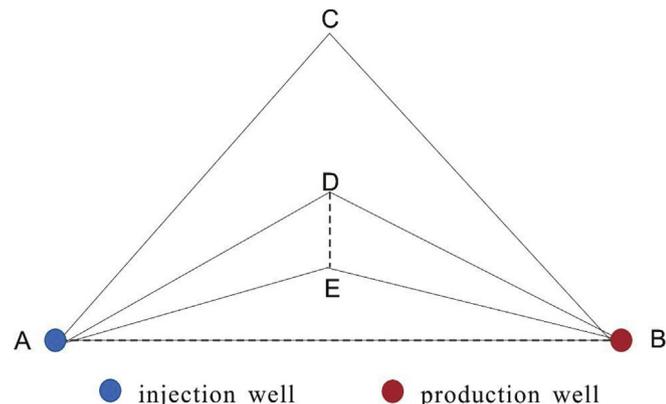


Fig. 3. The illustration of triangular CU.

$$\begin{cases} Q_{os1} = \sum_{j=1}^3 Q_{oj} \\ Q_{os2} = \sum_{j=4}^6 Q_{oj} \\ Q_{os3} = \sum_{j=7}^9 Q_{oj} \\ Q_{os4} = \sum_{j=10}^{12} Q_{oj} \end{cases} \quad (9)$$

Combining with the Eq. (7), the production of an injection-production unit of the five point well pattern is:

$$Q_o = \sum_{i=1}^4 Q_{osi} \quad (10)$$

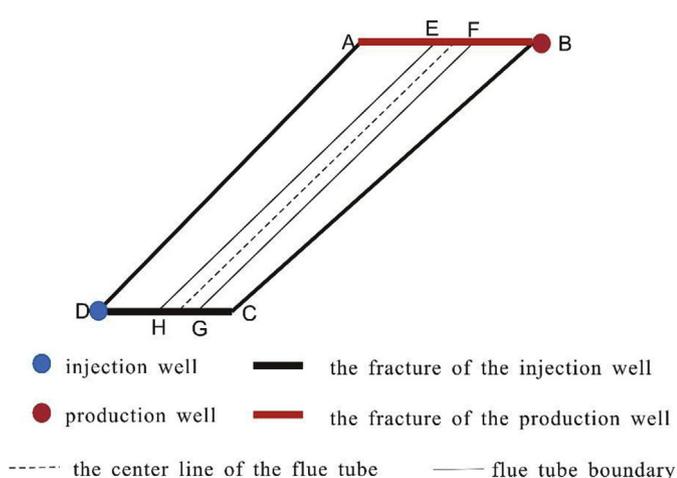


Fig. 4. The illustration of quadrilateral CU.

Table 2
The value of the input parameters.

Parameters	value	Parameters	value
oil viscosity, μ_o	7.01 mPa s	porosity, ϕ	0.192
injection pressure P_{in}	20.357 MPa	production pressure, P_{pro}	8.7 MPa
SU 1 permeability, k_1	37.35mD	SU 1water saturation, s_{w1}	0.64
SU 2 permeability, k_2	33.09mD	SU 2water saturation, s_{w2}	0.71
SU 3 permeability, k_3	31.04mD	SU 3water saturation, s_{w3}	0.69
SU 4 permeability, k_4	34.76mD	SU 4water saturation, s_{w4}	0.73
reservoir effective thickness, h	18.9 m	wellbore radius, r_w	0.1 m
injection well fracture half length, L_{fw}	140 m	well spacing, L_2	240 m
production well fracture half length of, L_{fo}	120 m	fracture width, w_f	0.012 m

Follow the operation of ‘six step’, the transient production of the five point well pattern can be obtained. The ‘six-step’ is present as below [26, 27, 28]:

- ① The basic reservoir parameters and initial saturation of each injection-production unit SU is given.
- ② Use the Eqs. (4) and (6) to calculate the productivity of each CU at the first time step.
- ③ Use the Eq. (9) to calculate the productivity of each SU at the first time step.
- ④ Use the Eq. (10) to calculate the productivity of the five point well pattern at the first time step.
- ⑤ Use the Eq. (8) to calculate the saturation of each SU at the second time step.
- ⑥ Update the saturation for each CU in the step ①, and then repeat steps ②-⑥, the productivity of the five point well pattern at the next time step can be obtained, and so on, finally, the transient productivity of the five point well pattern is obtained.

4. Discussion

The basic reservoir parameters of a low permeability block in Daqing oil field are shown in Table 2. At present, the average water cut of this low permeability block has reached 90%.

According to the literature [29, 30, 31], the calculation equation of relative permeability is:

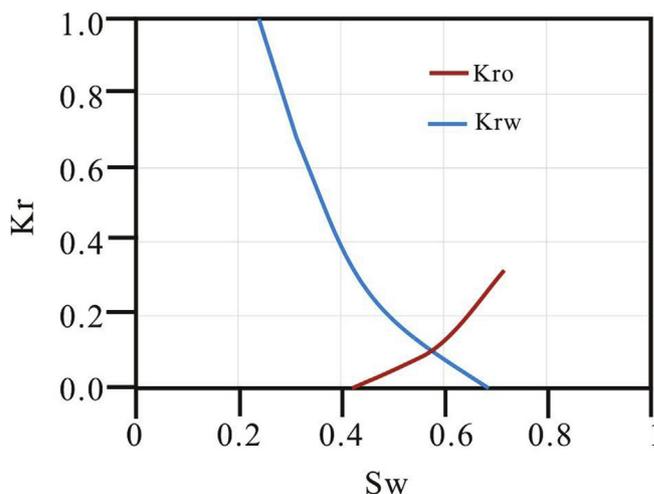


Fig. 5. Relative permeability curves.

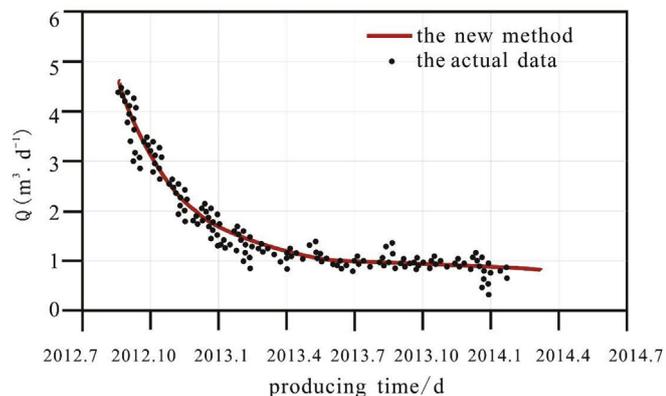


Fig. 6. Calculating results.

Table 3

Calculation results.

well space	reference solution (m ³ /d)	this paper solution (m ³ /d)	relative error (%)
200	1.6093	1.6018	0.4306
225	1.5826	1.5753	0.4261
250	1.5595	1.5521	0.4349
275	1.5391	1.5320	0.4280
300	1.5209	1.5139	0.4300
325	1.5047	1.4980	0.4082
350	1.4898	1.4834	0.4025
375	1.4764	1.4700	0.3987
400	1.4640	1.4581	0.3677

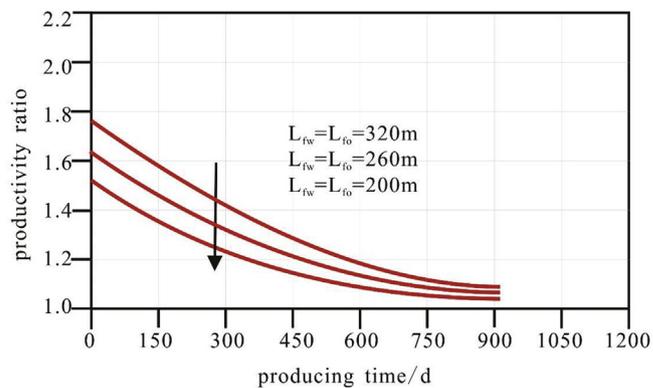


Fig. 9. Comparison of the effect of the fracture length on productivity (production well/injection well).

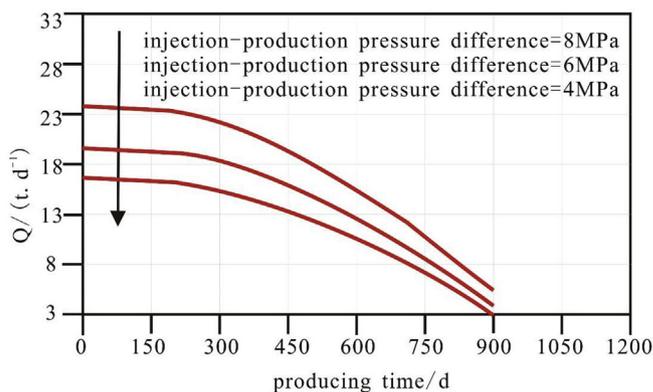


Fig. 10. Effect of injection-production pressure difference on productivity.

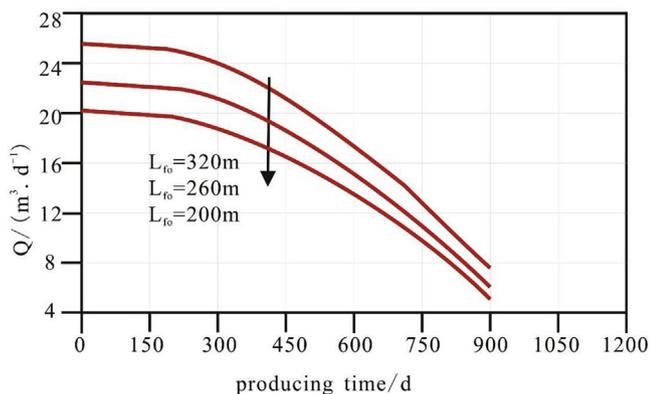


Fig. 7. Effect of the production well's fracture length on productivity.

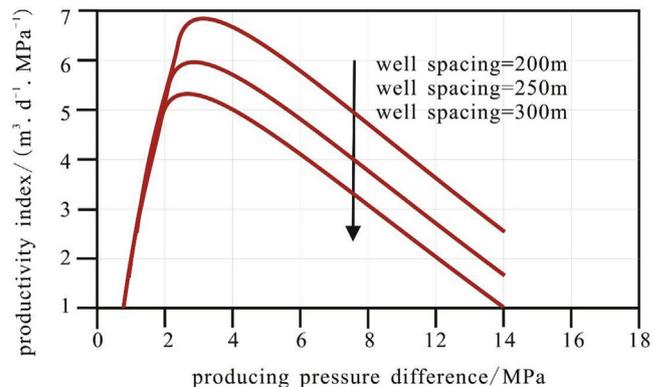


Fig. 11. Effect of injection-production pressure difference on production index.

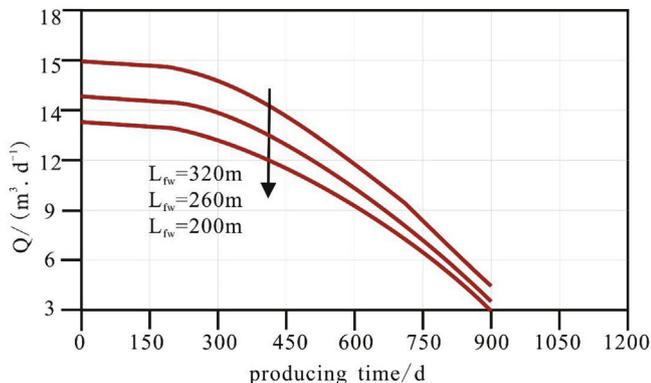


Fig. 8. Effect of the injection well's fracture length on productivity.

$$k_{rw} = (1 - s_o)^{0.5} \left\{ 1 - \left[1 - (1 - s_o)^{\frac{1}{m}} \right]^m \right\}^2 \tag{11}$$

$$k_{ro} = s_o^{0.5} \left[1 - (1 - s_o)^{\frac{1}{m}} \right]^{2m} \tag{12}$$

The relative permeability of this low permeability block is calculated, as shown in Fig. 5.

Use the data in Table 1 and this newly established method in this paper, the production of the five point well pattern was calculated and

compared with the actual data. The comparison results are shown in Fig. 6. Fig. 6 shows that the productivity calculated in this paper is in good agreement with the actual data, indicating that the method in this paper is feasible and scientific. Therefore, this method is suitable to predict the transient productivity of five point well patterns.

In addition, when the influence of the starting pressure gradient is not considered, that is, the parameter b is 0, then the corresponding seepage is Darcy seepage, and the results calculated in this paper (this paper solution) are compared with the classical Muskat [24] calculation (reference solution), as shown in Table 3. Table 3 shows that the relative error is controlled within 0.5%, indicating the method in this paper is available.

5. Analysis

Use the single factor variable method and the data in Table 1, the sensitivity factor analysis was carried out. The influence of fracture length (injection well and the production well) and the pressure difference on the instantaneous productivity of the five point well pattern were analyzed, as shown in Figs. 7, 8, 9, 10, and 11.

5.1. Fracture length of the production well

Fig. 7 shows that the larger the fracture length of the production well is, the higher the productivity of the five point well pattern is. According to the analysis of the flow states, the flow states in the CU 2, CU 4, CU 6 and CU 8 regions are linear flows, and the flow states of other CUs are radial flows, so when the fracture length of the production well is larger, the volume of the reformed reservoir is bigger, and the seepage area of CU 2, CU 4, CU 6 and CU 8 is larger [32]. The fluid seepage flow channel in the reservoir is more obvious, and more and more fluid flows to the wellbore. When the fracture is longer, the fracture conductivity is enhanced, and the corresponding seepage resistance of the five point well pattern is smaller, and all these lead to that the productivity is higher.

In the early stage of the production (producing time is less than 400 days), the fracture length only changes the level of productivity, and it does not affect the speed of productivity decline. Because the orientation and position of the fracture around the wellbore in the reservoir have not changed. The cross-sectional area of the fluid flowing to the wellbore is similar. The flow rate through the section of the fluid does not change much. When the producing time exceeds a certain value (1000 days), the influence of the fracture length on the productivity becomes smaller and smaller. Because the formation is not supplemented by external energy and the productivity is limited by other parameters, such as boundary layer conditions or energy replenishment methods.

5.2. Fracture length of the injection well

Fig. 8 shows that the larger the fracture length is, the higher the productivity of the five point well pattern is. When the fracture length increases, the injected fluid can flow relatively uniformly to the production well in the direction of the fracture extension, which reduces the probability of the turbulent layer or turbulence of the injected fluid, and improves the utilization of the injected fluid [33]. The water drive area is larger, which causes more crude oil to flow to the wellbore and improves the utilization of the reservoir. Therefore, the productivity increases with the increase of the fracture length.

Fig. 9 shows that the fracture length of the production well has a greater impact on the productivity. Because during the water flooding process, the leading edge of the injected water takes time to move in the formation, while the fluid near the wellbore or the fracture in the production well can flow to the wellbore directly and quickly. Relatively speaking, the amount of injected fluid is greater than the amount of crude oil produced, but when the dominant channel of fluid seepage in the reservoir is established, the injection-production balance is reached at this time. The degree of impact on productivity is basically the same.

Therefore, in the hydraulic fracturing design, in order to save costs and improve economic efficiency, the fractures near the production wellbore should be considered first.

5.3. Injection-production pressure difference

Fig. 10 shows that the greater the injection-production pressure difference is, the higher the instantaneous productivity of the five point well pattern is. It is mainly because when the injection-production pressure difference increases, the water flooding area is larger, and the water flooding efficiency is higher, and the reservoir utilization degree is greater, so instantaneous productivity of the five point well pattern is larger.

However, it is not true that the higher the injection-production pressure difference is, the more favorable it is to improve productivity. Fig. 11 shows that within a certain range, increasing the injection-production pressure difference is conducive to increasing productivity. When the pressure difference exceeds a certain value, the productivity decreases. Because when the injection pressure is too large, the injected fluid is pressed into the new fractures under high pressure, which reduces the effective sweeping area of the fluid displacement between the injection and production wells. When the bottom hole pressure of the production well is too small, the range of the fluid used around the production well is small and limited, especially when the bottom hole pressure is less than the starting pressure, the fluid flow is blocked, and the productivity is significantly reduced. Therefore, it is necessary to optimize the pressure difference in order to obtain high productivity and economic benefits [34]. It can be seen from Fig. 10 that the optimal pressure difference under this condition is about 6 MPa.

6. Conclusions

- (1) This paper presents a new method to predict the productivity of five point well pattern, and the remaining oil heterogeneity is taken into account. Although this new method has many drawbacks, such as breaking the conductivity of infinity and ignoring the flow disturbance during two-phase flow, it can accurately predict the productivity of the five point well pattern at high water cut stage.
- (2) The instantaneous productivity unit is divided into four sub-units, and each sub-unit is divided into three calculation units according to the streamline distribution characteristics. Then, the instantaneous productivity is equal to the sum of the productivity of each calculation unit. The key to solve the instantaneous productivity is how to calculate productivity of each calculation unit. This paper uses flow tube integration method to obtain the productivity of each calculation unit. When the oil-water two phase flow is considered, the successive steady state method and 'six step' are used to obtain the transient production of the five point well pattern.
- (3) This method solution agreed with the actual data very well. In addition, the new method was also validated available with the published analytical solution for a relative simple situation.
- (4) The fracture length and pressure difference between the injection well and the production well have a significant effect on the productivity of the five point well pattern at high water cut stage. The greater the length of the fracture is, the higher the productivity is. The greater the pressure difference is, the higher the productivity is.

Declarations

Author contribution statement

Long Liu: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents,

materials, analysis tools or data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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