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Evaluating the predictive power of an SPF for two-lane rural roads with random parameters on out-of-sample observations

Houjun Tang, Vikash V. Gayah*, Eric T. Donnell

Department of Civil and Environmental Engineering, The Pennsylvania State University, 231 Sackett Building, University Park, PA 16802, United States

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ABSTRACT

Negative binomial (NB) regression is among the most common statistical modeling methods used to model crash frequencies due to its simple functional form and ability to handle over-dispersion commonly found in crash data. However, a drawback of this approach is that regression parameters are assumed to be the same across observations, which could contribute to biased parameter estimates. To alleviate this concern, the random parameters negative binomial (RPNB) model was recently proposed, which allows regression parameters to differ across observations following some known distribution. The resulting coefficients should be less biased, and thus the RPNB approach is believed to provide a more accurate relationship between independent variables and expected crash frequency. However, the prediction accuracy of the RPNB model relative to the standard NB model has not been thoroughly evaluated, particularly with respect to out-of-sample observations for which unique random parameters cannot be estimated. In this paper, the predictive power of the RPNB and NB models are examined using two-lane rural highway data from three engineering Districts in Pennsylvania. Multiple evaluation metrics are applied—root-mean-square error (RMSE) and mean absolute error (MAE), coefficients from calibration functions and cumulative residual (CURE) plots—to assess each model type. The results show that the RPNB model outperforms the NB model when applied to within sample observations (i.e., those used to estimate the model) by making use of the observation-specific coefficients. However, the predictive power of the RPNB model appears to be similar to or slightly less precise than the traditional NB model when applied to out-of-sample observations. Since the RPNB model is estimated using a simulation-based approach, sensitivity tests were also performed to see how the parameter estimates change with the number of Halton draws used to perform the simulation. For the sample sizes used in this paper, the estimates were fairly insensitive when more than 50 Halton draws were used. The findings suggest that the RPNB model is more reliable when applied to the same set of sites that were used to estimate the model but might not be as robust as the traditional NB model when applied to other sites.

1. Introduction

Safety performance functions (SPFs) are statistical models that relate the number of crashes that occur at a roadway segment, intersection, or interchange over a predefined time period (usually annually) to location-specific data, such as traffic volume, roadway characteristics, as well as roadside features. Significant research has been completed to understand the statistical association between crash frequency and site-specific features to develop reliable SPFs for practical use. The negative binomial (NB) regression model is one of the most popular statistical approaches used to develop SPFs due to its simple functional form and ability to handle over-dispersion commonly found in crash data. Past studies have demonstrated the methodological appropriateness of the NB model and identified key variables contributing to crash frequencies (Poch and Mannering, 1996; Shankar et al., 1995; Milton and Mannering, 1998; Hadi et al., 1995), such as traffic volume, horizontal curve density and shoulder width. The NB model has also been used to estimate many of the SPFs provided in the first edition of the AASHTO Highway Safety Manual (HSM) (American Association of State Highway Transportation Officials, 2010).

Recently, several studies have shown that NB regression models can suffer from unobserved heterogeneity when influential factors related to the dependent variable—in this case, crash frequency—are not included in the model (Lord and Mannering, 2010; Anastasopoulos and Mannering, 2009; Mannering et al., 2016). This unobserved heterogeneity can lead to biased parameter estimates in NB regression models. One solution to overcome this deficiency is to use a random parameters modeling framework. In this approach, unique coefficients are assigned to each observation for one or more of the independent variables

* Corresponding author.
E-mail addresses: hut26@psu.edu (H. Tang), gayah@engr.psu.edu (V.V. Gayah), edonnell@engr.psu.edu (E.T. Donnell).

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included in the model to better capture unobserved effects (Lord and Mannering, 2010). These random coefficients are assumed to follow some predefined distribution. Anastasopoulos and Mannering (2009) evaluated the appropriateness of the random parameters negative binomial (RPNB) model as a methodological alternative in crash frequency analysis. El-Basyouny and Sayed (2009) developed random parameters Poisson lognormal models in an urban arterial corridor crash frequency analysis. Both studies noted an improvement in goodness of fit metrics applied to the samples used to estimate the model—specifically, the log-likelihood and Deviance Information Criteria (DIC), respectively—when random parameters were introduced into model estimation. The researchers concluded that the RP framework can provide a more comprehensive understanding of variable effects by using observation-specific model coefficients to account for heterogeneity in the data. Since then, several studies have applied the RPNB approach to estimate expected crash frequencies, with a focus on different crash types, site types, roadway functional classifications, or other factors (Venkataraman et al., 2013, 2011; Ukkusuri et al., 2011; Dinu and Veeraragavan, 2011; Dong et al., 2014; Kamla et al., 2016; Chen and Tarko, 2014).

Although previous studies have demonstrated the advantages of the RPNB model in improving the ability of a model to fit observed data, several unanswered research questions remain with respect to the application of SPFs using random parameters methods. First, a thorough evaluation of the predictive power of the RPNB model relative to the traditional NB model has not been completed, particularly with respect to “out-of-sample” observations that are not used in model estimation. While observations (i.e., sites) used to develop the model can be assigned a unique coefficient for each parameter considered as “random” in the model, only mean values of these coefficients can be applied to sites that were not included in the model estimation for predictive purposes. Thus, for out-of-sample observations, the prediction process proceeds similar to the standard NB model. Such lack of transferability of random parameters models to out-of-sample observations was previously discussed in a qualitative manner, but the impacts have yet to be quantified in a meaningful way (Mannering et al., 2016). A second research question relates to the impact of sample size on the coefficients and standard deviations for the independent variables that are specified as random in model estimation. Previous studies apply the RPNB framework to relatively small datasets. A sensitivity analysis is needed to test how robust the RPNB is to larger datasets or data with low sample means. Third, models with random parameters must be estimated using simulation-based approaches. This is typically done by randomly generating model coefficients and adjusting the outcomes over several iterations. In many empirical settings, 200 Halton draws are employed to estimate RPNB models (Anastasopoulos and Mannering, 2009; Milton et al., 2008; Bhat, 2003), but each draw requires significant computational time. The sensitivity of the RPNB model to the number of Halton draws should be evaluated with respect to larger datasets.

In light of these issues, the objective of this paper is to evaluate and compare the results obtained from the RPNB and NB models developed using three different datasets from Pennsylvania, with a focus on prediction accuracy. Furthermore, regression parameter sensitivity is assessed in the current study based on the number of Halton draws used to estimate the random parameters in the RPNB model.

The remainder of this paper is organized into the following sections. Section 2 presents the methodology used in the current study. Section 3 provides a description of the data used in the research. Section 4 summarizes the modeling results. Section 5 includes a discussion of the implications of this work for the development of SPFs. Section 6 summarizes key results and identifies directions for future work.

2. Methodology

The methodology described in this section starts with brief introductions of NB and RPNB model development, and then describes the metrics that were used to assess the predictive power of the models: root-mean-square error (RMSE) and mean absolute error (MAE), cumulative residual (CURE) plots, and coefficients from calibration functions, respectively.

2.1. Negative binomial regression model

Negative binomial regression is a count regression modeling approach that addresses over-dispersion commonly found in reported crash data in which the variance exceeds the mean in the crash frequency distribution. The NB model takes the following general functional form:

$$\ln \lambda_i = X_i \beta + \varepsilon_i, \tag{1}$$

where λ_i is the expected number of crashes for observation i; X_i is a vector of independent variables for observation i; β is a vector of regression coefficients; and, ε_i is an error term that follows a gamma distribution.

The mean-variance relationship for observations obtained from the NB model is provided by:

$$Var\,(y_i) = E\,(y_i) + \alpha E\,(y_i)^2, \tag{2}$$

where y_i is the reported number of crashes for observation i, $E\,(y_i)$ is the expected number of crashes for observation i and α is an over-dispersion parameter estimated from the model. The observations in this paper are roadway segments on two-lane rural highways.

Note from Eq. (1) that the regression coefficients (β) used to estimate the expected value of any observation i are the same for all observations. These regression coefficients are estimated using the maximum likelihood method, which selects the coefficients that maximize the following log-likelihood function:

$$L(\lambda_i) = \prod_{i=1}^{N} \frac{\Gamma(\theta + y_i)}{\Gamma(\theta) y_i!} \left(\frac{\theta}{\theta + \lambda_i}\right)^{\theta} \left(\frac{\lambda_i}{\theta + \lambda_i}\right)^{y_i}, \tag{3}$$

where θ is the inverse of the over-dispersion parameter, Γ is the gamma function, and N is the total number of observations in the sample.

2.2. Random parameters negative binomial model

The traditional NB model assumes that the impacts of each parameter (i.e., the β values in Eq. (1)) are the same for each observation i. However, this may not adequately capture unobserved differences across observations in the sample (Anastasopoulos and Mannering, 2009; El-Basyouny and Sayed, 2009; Venkataraman et al., 2013, 2011; Ukkusuri et al., 2011; Dinu and Veeraragavan, 2011; Dong et al., 2014; Kamla et al., 2016). To account for such heterogeneity in the data, the random parameters negative binomial model (RPNB) model allows the coefficients to vary across individual observations (i.e., parameters are random across observations), while the same basic functional form of the NB model in Eq. (1) is still applied. The parameters in this statistical approach can be described using both a fixed and random component (Anastasopoulos and Mannering, 2009):

$$\beta_{i,j} = \bar{\beta}_j + \varphi_{i,j}, \tag{4}$$

where $\beta_{i,j}$ is a unique regression coefficient for the j th independent variable for observation i ; $\bar{\beta}_j$ is the mean value for this j th coefficient across all observations; and, $\varphi_{i,j}$ is a randomly distributed error term applied to the j th coefficient for observation i following some known distribution.

Note that not all parameters must be treated as random. The parameters that are identified in the modeling process as not having significant variation across segments may be modeled as having fixed-effects, similar to the traditional NB approach in Eq. (1). The RPNB model is equivalent to the random effects NB model if only the constant term is assumed to be random across segments (Shankar et al., 1998).

Based on the above assumptions, the expected crash frequency λ_i in the RPNB model is conditioned on the randomly distributed term and can be written as $\lambda_i|\varphi_i = \exp(\beta X_i + \varepsilon_i)$. The log-likelihood of the RPNB model is (Anastasopoulos and Mannering, 2009):

$$LL = \sum_{\forall i} ln \int_{\varphi_i} g(\varphi_i) P(n_i|\varphi_i) d\varphi_i, \tag{5}$$

where $g(\bullet)$ is the probability density function of φ_i and $P(n_i|\varphi_i)$ is the Poisson probability of segment i having n_i crashes conditioned on φ_i.

Because the random terms in the coefficients shown in Eq. (5) do not have a closed form expression, the standard maximum likelihood estimation process is too computationally intensive to perform. As a result, these models are estimated using a simulation-based maximum likelihood approach using some number of Halton draws. Each draw is generated from a quasi-random sequence that follows some pre-defined distribution (e.g., normal or uniform) over a unit interval and used to calculate the likelihood of the sample. The final estimated likelihood of each sample is the expected value over all Halton draws (Venkataraman et al., 2013, 2011).

2.3. Evaluation metrics

The root-mean-square error (RMSE), mean absolute error (MAE) and calibration function coefficients are used to assess the predicted power of the RPNB and NB models when applied to individual observations. Cumulative residual (CURE) plots are used to assess general trends with respect to the model specification as a function of an individual independent variable or the predictive value.

2.3.1. RMSE and MAE

RMSE and MAE quantify the average difference between reported crash frequencies and predicted crash frequencies. MAE is easier to interpret than RMSE as the metrics directly reflect the absolute difference in the sample. Meanwhile, RMSE is more sensitive to large errors and outliers, which may exist in segments with high reported crash counts. A smaller RMSE or MAE value indicates a more accurate prediction. The RMSE and MAE are calculated as shown in Eqs. (6) and (7), respectively.

$$RMSE = \sqrt{\frac{\sum_i (y_i - \hat{y}_i)^2}{N}} \tag{6}$$

and

$$MAE = \frac{\sum_i |y_i - \hat{y}_i|}{N}, \tag{7}$$

where y_i is the reported crash frequency for observation i and $\hat{y}_i$ is the predicted crash frequency for observation i.

2.3.2. Calibration functions

The second set of evaluation metrics applied in this paper are the coefficients derived from calibration functions that were recently proposed by Srinivasan et al. (Srinivasan et al., 2016) to assess the closeness of predicted values to reported crash frequencies. The calibration function is expressed using the following equation:

$$y_i = a \times (\hat{y}_i)^b, \tag{8}$$

NB regression is used to estimate the parameters a and b for each developed model. Parameters closer to one represent SPFs that provide better predictions of the observed data.

2.3.3. Cumulative residual plots

Cumulative Residual (CURE) plots are another method used to assess the overall model fit, appropriateness of the assumed functional form and independent variables included in an SPF (Bahar and Hauer, 2014; Srinivasan et al., 2013). These plots illustrate the relationship between cumulative residuals—defined as the difference between the predicted and reported crash frequencies—as a function of various metrics, such as predicted crash frequency or any of the independent variables included in the SPF. Cumulative residuals that trend upward indicate that the crash frequencies are under-estimated by the model for that range of observations, while cumulative residuals that trend downward suggest that crash frequencies are over-estimated by the model prediction. In general, a CURE curve that oscillates around and close to zero represents better overall model fitness. Specifically, CURE plots for a good-fitting model should be consistent with a random walk process around 0. The random walk process can be used to estimate confidence intervals that reflect the range of cumulative residual values that do not reflect a systematic bias in model estimates.

3. Model estimation

This section describes the data that were used to estimate the SPFs in this paper and the results of the NB and RPNB regression models.

3.1. Data

The data applied in this study were obtained from a previous study to estimate SPFs for two-lane rural roadways in Pennsylvania (Donnell et al., 2016). The data consisted of two parts: roadway and crash data. Roadway data were collected from the Pennsylvania Department of Transportation (PennDOT) Roadway Management System (RMS) database, and supplemented with manual data collection using PennDOT's online video photolog system and Google Earth. The RMS database provided information such as traffic volume, segment length, cross-section widths, and posted speed limit. PennDOT's online vehicle photolog system and Google Earth provided supplemental data including roadside hazard rating, horizontal curvature, presence of passing zones, presence of low-cost safety improvements (e.g., shoulder and centerline rumble strips), and access density. A total of eight years of crash data (2005 through 2012) were included in the analysis data files. All crashes were reported on state-owned two-lane rural highways in Pennsylvania and were merged with roadway data based on the crash locations. Additional details concerning the data used in the analysis can be found in Donnell et al. (Donnell et al., 2016).

To examine the transferability and generalization of results, three datasets from PennDOT Engineering Districts 2, 3, and 8, which have the highest two-lane rural roadway mileage over all engineering Districts in Pennsylvania, were extracted from the statewide database. The District 2 dataset contained 25,952 observations from 3,244 unique roadway segments, the District 8 dataset included 22,896 observations from 2,862 unique roadway segments, and the District 3 dataset had 22,488 observations from 2,811 unique roadway segments.

The functional form of the NB and RPNB models developed here were assumed to be the same as those for SPFs previously developed for PennDOT using these data (Donnell et al., 2016); this functional form is provided in Eq. (9). The list of dependent and independent variables considered for the models in all three Districts, along with their definitions, are provided in Tables 1 and 2. These variables represented that factors that have known relationships with safety performance and were available for use in this study. Basic summary statistics of these variables are presented in Tables 3 and 4. Continuous variables were entered into the model without modification, with the exception of traffic volume and segment length. For these two variables, a logarithmic transformation was considered. Furthermore, segment length was treated as an offset variable so that it is directly proportional to the predicted crash frequency as in Eq. (9). All of the categorical variables with binary classes were considered as indicator variables in the modeling process. The RHR categorical variable was converted into multiple binary indicators for the purposes of model estimation. Various groupings of RHR were tested to determine which provided the best model fit and that grouping was used in the final model. Models

Table 1
Continuous Variables Descriptions.

Variable	Description	Units
Total_crash	Total number of crashes	Crashes per year
AADT	Average annual daily traffic	Vehicles per day
Length_mi	Segment length	Miles
Speed	Posted speed limit on roadway segment	Miles per hour
Ls_pave	Width of paved shoulder on the left-hand side of the roadway segment	Feet
Rs_pave	Width of paved shoulder on the right-hand side of the roadway segment	Feet
Curve_den	Horizontal curve density within segment	Curves per mile
D_seg_mi	Degree of horizontal curvature within segment	Degrees per 100 ft
Acc_den	Access density	Access points per mile

Table 2
Categorical Variables Descriptions.

Variable	Description	Categories
RHR	Roadside hazard rating	Estimated on the 1 to 7 scale, 1 represents least hazardous
Pass_zone	Passing zone indicator	1 = yes and 0 = no
Cl_rs	Centerline rumble strip indicator	1 = yes and 0 = no
Sh_rs	Shoulder rumble strip indicator	1 = yes and 0 = no
Curve_warn	Advanced curve warning indicator	1 = yes and 0 = no
Int_warn	Intersection warning indicator	1 = yes and 0 = no
Agg_dots	Aggressive driving dots indicator	1 = yes and 0 = no
County17	County 17 indicator (Dist. 2 only)	1 = yes and 0 = no
County4452	County 44 or 52 indicator (Dist. 2 only)	1 = yes and 0 = no
County0136	County 1 or 36 indicator (Dist. 8 only)	1 = yes and 0 = no
County2250	County 22 or 50 indicator (Dist. 8 only)	1 = yes and 0 = no
County66	County 66 indicator (Dist. 8 only)	1 = yes and 0 = no
County8	County 8 indicator (Dist. 3 only)	1 = yes and 0 = no
County4147	County 41 or 47 indicator (Dist. 3 only)	1 = yes and 0 = no
County5659	County 56 or 59 indicator (Dist. 3 only)	1 = yes and 0 = no

were then estimated using the following functional form:

$$\lambda_i = e^{\beta_0} \times L \times AADT^{\beta_1} \times e^{(\beta_2 X_2 + \ldots + \beta_n X_n)} \quad (9)$$

where:
λ_i = expected number of crashes per year on roadway segment i;
L = roadway segment length (miles);
$AADT$ = average annual daily traffic (veh/day);
β_0 = regression coefficient for the constant;
β_1 = regression coefficient for AADT;
$\beta_2, \ldots, \beta_n$ = regression coefficients for explanatory variables, i = 2, …, n; and,
$X_2, \ldots, X_n$ = site-specific explanatory variables.

The summary statistics in Table 3 indicate that the mean annual crash frequency per segment is lower in Districts 2 and 3 than in District 8. This was expected because District 8 is a more urban area relative to the other districts. The average traffic volume in District 8 was higher than in Districts 2 and 3. The access density was higher in District 8 than in Districts 2 and 3, while posted speed limits were higher in Districts 2 and 3 relative to District 8.

The categorical variables are summarized in Table 4. Because of the rural nature of Districts 2 and 3, there are more roadway segments with higher RHR ratings in these areas than in the more urbanized District 8. Similarly, there are more passing zones in Districts 2 and 3 due to the lower traffic volumes on roadways in these areas. The county indicator variables were created for adjacent counties within a district that exhibited similar safety performance.

Table 3
Statistics Summary of Continuous Variables.

Variable	District 2 (N = 25952)				District 8 (N = 22896)				District 3 (N = 22488)			
	Mean	Std. Dev	Min	Max	Mean	Std. Dev	Min	Max	Mean	Std. Dev	Min	Max
Total_crash	0.409	0.779	0	10	0.951	1.455	0	20	0.481	0.866	0	16
AADT	2408	2276	100	17481	4376	3671	206	23542	2756	2395	118	18981
Length_mi	0.469	0.105	0.026	0.751	0.474	0.127	0.026	0.754	0.445	0.120	0.035	0.754
Speed	48.8	7.7	20	55	45.6	6.8	25	55	49.3	7.3	15	55
Ls_pave	2.3	2.2	0	12	3.6	2.1	0	19	3.4	2.5	0	22
Rs_pave	2.3	2.2	0	11	3.6	2.1	0	19	3.4	2.5	0	14
Curve_den	2.1	2.5	0	42.6	2.2	2.5	0	28.2	2.7	2.6	0	19.6
D_seg_mi	16.5	45.6	0	1217.3	18.9	37.3	0	439.7	20.0	46.9	0	1263.5
Acc_den	12.5	13.8	0	120	17.4	12.6	0	84.4	13.9	12.7	0	96.4

N – number of observations (segment-year combinations) within each district.

Table 4
Statistics Summary of Categorical Variables.

Variable	District 2 (N = 25932)		District 8 (N = 22896)		District 3 (N = 22488)	
	Category	Proportion (%)	Category	Proportion (%)	Category	Proportion (%)
RHR	1	0	1	0.1	1	0.0
	2	0.3	2	0.6	2	0.1
	3	4.7	3	7.0	3	2.1
	4	17.1	4	29.6	4	16.9
	5	56.8	5	44.9	5	48.5
	6	20.9	6	17.9	6	31.9
	7	0.2	7	0.1	7	0.5
Pass_zone	0 (No)	63.7	0 (No)	80.0	0 (No)	61.4
	1 (Yes)	36.3	1 (Yes)	20.0	1 (Yes)	38.6
Cl_rs	0 (No)	81.5	0 (No)	77.0	0 (No)	80.6
	1 (Yes)	18.5	1 (Yes)	23.0	1 (Yes)	19.4
Sh_rs	0 (No)	84.2	0 (No)	97.6	0 (No)	96.2
	1 (Yes)	15.8	1 (Yes)	2.4	1 (Yes)	3.8
Curve_warn	0 (No)	99.4	0 (No)	97.2	0 (No)	99.6
	1 (Yes)	0.6	1 (Yes)	2.8	1 (Yes)	0.4
Int_warn	0 (No)	99.3	0 (No)	99.4	0 (No)	99.8
	1 (Yes)	0.7	1 (Yes)	0.6	1 (Yes)	0.2
Agg_dots	0 (No)	99.8	0 (No)	100.0	0 (No)	99.9
	1 (Yes)	0.2	1 (Yes)	0.0	1 (Yes)	0.1
County17	0 (No)	82.3	—	—	—	—
	1 (Yes)	17.7	—	—	—	—
County4452	0 (No)	81.6	—	—	—	—
	1 (Yes)	18.4	—	—	—	—
County0136	—	—	0 (No)	75.6	—	—
	—	—	1 (Yes)	24.4	—	—
County2250	—	—	0 (No)	75.2	—	—
	—	—	1 (Yes)	24.8	—	—
County66	—	—	0 (No)	85.1	—	—
	—	—	1 (Yes)	14.9	—	—
County8	—	—	—	—	0 (No)	81.8
	—	—	—	—	1 (Yes)	18.2
County4147	—	—	—	—	0 (No)	77.4
	—	—	—	—	1 (Yes)	22.6
County5659	—	—	—	—	0 (No)	86.0
	—	—	—	—	1 (Yes)	14.0

N – number of observations (segment-year combinations) within each district.

Each of three District-level full datasets were randomly split into two subsets: a training set and a validation set. The training set consisted of 70 percent of the observations from the full dataset and was used to estimate the NB and RPNB models. The validation dataset comprised of the remaining 30 percent of the observations and was used to evaluate the predictive power of the model on out-of-sample observations. In total, the training set for the three districts contained 18,167 observations from District 2, 16,027 observations from District 8, and 15,742 observations from District 3. The validation set contained 7,765 observations, 6,869 observations, and 6,746 observations, respectively, from Districts 2, 8, and 3.

3.2. Estimation results

The models were estimated considering all potential explanatory variables shown in Table 1 and Table 2. Variables that were marginally significant ($p < 0.3$) were retained in the model. Once a final model specification (i.e., set of variables included in the model) was developed using NB regression, the same specification was used for the RPNB model. Random parameters were specified using a normal distribution with zero mean.[1] For the results in this section, 200 Halton draws are used to estimate the model; however, the sensitivity of the random parameters is assessed in Section 4.4 when the number of Halton draws is modified. All models were estimated using the latest version of the LIMDEP software. Due to the restriction of LIMDEP software in recording estimates for individual coefficients, only a limited number of random parameters can be included in the final model. Thus, only the random parameters with smallest p-values in the preliminary estimation (i.e., same random parameter model estimation without recording individual coefficients) were selected as random in the final model. Table 5 presents the estimation results of the RPNB and NB models for all three engineering districts.

The final models are all consistent with engineering expectations. The NB models reveal that expected crash frequency increase with traffic volume, access density, curve density, horizontal curvature within the segment, and for segments that have poorer roadside hazard ratings. Expected crash frequency decreases with the presence of a passing zone or shoulder rumble strips within the segment. There are also some regional differences within individual counties within each of the three engineering districts.

In the final RPNB models, both traffic volume and curve density were deemed random parameters based on their p-values in the preliminary estimation as described above. Both variables had the smallest p-values relative to the other random parameters tested in the model. In the District 2 RPNB model, traffic volume is normally distributed with a mean value of 0.645 and standard deviation 0.009. This suggests that traffic volume always has a positive effect on the expected crash frequency, while the magnitude varies from 0.627 to 0.663 (based on 95 percent confidence interval). In the District 8 RPNB model, traffic

[1] Since the distribution of the random parameters are unknown a priori, several distributions were tested in this work. However, the normal distribution appeared to provide the most reasonable results for the RPNB models.

Table 5
Estimation Results of Models.

Variable	District 2		District 8		District 3	
	RPNB	NB	RPNB	NB	RPNB	NB
Intercept	−5.352(0.128)***	−5.228(0.136)***	−5.528(0.102)***	−5.326(0.118)***	−5.337(0.137)***	−5.203 (0.152)***
Lnaadt	0.645 (0.014)***	0.646 (0.016)***	0.704 (0.012)***	0.700 (0.014)***	0.646 (0.017)***	0.647 (0.019)***
Acc_den	0.010 (0.001)***	0.010 (0.001)***	0.005 (0.001)***	0.005 (0.001)***	0.012 (0.001)***	0.012 (0.001)***
Curve_den	0.018 (0.007)***	0.018 (0.007)***	0.033 (0.006)***	0.035 (0.006)***	0.031 (0.006)***	0.041 (0.007)***
D_seg_mi	0.001 (0.000)***	0.001 (0.000)***	0.002 (0.000)***	0.002 (0.000)***	0.002 (0.000)***	0.002 (0.000)***
Pass_zone	−0.278(0.027)***	−0.280(0.029)***	−0.254(0.025)***	−0.260(0.028)***	−0.155(0.027)***	−0.158 (0.029)***
RHR4567	0.075 (0.058)*	0.080 (0.059)*	—	—		
SH_rs			—	—	−0.105(0.057)**	−0.108 (0.063)**
County17	0.126 (0.030)***	0.128 (0.034)***	—	—	—	—
County4452	−0.322(0.038)***	−0.322(0.040)***	—	—	—	—
County0136	—	—	0.228 (0.023)***	0.229 (0.026)***	—	—
County2250	—	—	−0.124(0.027)***	−0.122(0.031)***	—	—
County66	—	—	0.102 (0.028)***	0.091 (0.032)***	—	—
County8	—	—	—	—	0.117 (0.031)***	0.108 (0.034)***
County4147	—	—	—	—	0.069 (0.034)***	0.069 (0.037)**
County5659	—	—	—	—	−0.161(0.042)***	−0.163 (0.046)***
Standard Deviation of Random Parameters						
Intercept	0.513 (0.012)***	—	0.584 (0.009)***	—	0.545 (0.012)***	—
Lnaadt	0.009 (0.002)***	—	0.006 (0.001)***	—	0.004 (0.001)***	—
Curve_den	—	—	0.034 (0.003)***	—	0.048 (0.003)***	—
Other model outputs						
Over-dispersion	0.080***	0.380***	0.123***	0.511***	0.103***	0.476***
LL (Null)	−15354.82	−15354.82	−21531.07	−21531.07	−14739.14	−14739.14
LL (Final)	−14037.25	−14042.74	−19708.79	−19736.09	−13745.05	−13756.44
McFadden's R^2	0.086	0.085	0.085	0.083	0.067	0.067
LR Test	10.98 (5.99)***			54.60 (7.81)***	22.78 (7.81)***	
Training Set Size	18167			16027	15742	

Notes: 1. *** – significant on 95 percent level; ** – significant on 90 percent level; * – marginally significant;
2. Value in parentheses of LR Test row represents critical value following Chi-square distribution.
3. All other values in parentheses represent standard errors.

volume is normally distributed with a mean of 0.704 and standard deviation of 0.006, and curve density is normally distributed with a mean of 0.033 and standard deviation of 0.034. This indicates that effect of traffic volume in District 8 is always positive, ranging from 0.692 to 0.716, while the effect of curve density varies from -0.034 to 0.100. Note that this range includes zero even though *t*-test for the mean and standard deviation of curve density shows that the parameter estimates are significantly different from zero. The first suggests that the average coefficient across all segments is positive and statistically different from zero. The second suggests that the variable effects differ among the roadway segments included in the training dataset (i.e., heterogeneity) and thus the random parameters assumption is valid in this case. However, the combination of these two suggests that the range of coefficients for individual segments does include zero, and thus the expected crash frequency among some of the segments in the sample may not be associated with curve density, based on the normal distribution of the random parameter. Based on these values, the distribution reveals that about 83.41 percent of samples are associated with a positive curve density effect (associated with higher expected crash frequencies) and 16.59 percent of samples are associated with a negative effect (associated with fewer expected crash frequencies), when holding other variables constant. Similarly, in District 3, the effect of traffic volume is always positive over all observations. The curve density parameters show that 74.08 percent of estimated distribution are greater than zero and 25.92 percent of samples are less the zero. Interestingly, for any given district, all other coefficients are nearly identical when comparing the RPNB and NB models across all three engineering districts.

In terms of model fitness, the log-likelihood of the final RPNB and NB models are significantly improved from the null log-likelihood (i.e., with intercept only), suggesting the appropriateness of final functional forms. Comparing the NB and RPNB models, in all three districts the RPNB models provide a better fit to the data used to estimate the model than the NB model. This is evidenced by the higher log-likelihood and McFadden's R-squared values. This finding was expected because RPNB models can capture unobserved heterogeneity using multiple random parameters, rather than a single over-dispersion parameter. However, these improvements appear to be rather modest in magnitude. To assess the statistical improvement, a likelihood ratio test was performed to

Table 6
Summary Table of RMSE.

Dataset	District 2			District 8			District 3		
	RPNB1	RPNB2	NB	RPNB1	RPNB2	NB	RPNB1	RPNB2	NB
Training	0.615	0.715	0.714	0.922	1.287	1.278	0.671	0.811	0.808
Validation	—	0.742	0.740	—	1.347	1.333	—	0.807	0.804

compare the RPNB and NB model within each district (Anastasopoulos and Mannering, 2009; Washington et al., 2011). These tests all revealed a statistically significant improvement in model fit of the RPNB model over the NB model. In addition, the over-dispersion parameters in the RPNB models are smaller (closer to zero) relative to the standard NB models.

4. Comparison of predictive ability

This section examines the performance of the NB and RPNB models using the goodness-of-fit metrics described in Section 2.3.

4.1. RMSE and MAE comparison

The RMSE and MAE values were calculated for each model in each district using the training and validation dataset, respectively. The calculation for the RPNB model is based on the following two sets of coefficients: 1) the observation-specific coefficients estimated through the modeling process; and, 2) the mean values in the model outputs. These two options are labeled RPNB1 and RPNB2, respectively, in Tables 6 and 7. Note, however, that since observation-specific coefficients are not available for the out-of-sample observations, prediction for the validation dataset using the RPNB model can only utilize the mean coefficients from the model shown in Table 5. The RMSE and MAE results are summarized in Tables 6 and 7, respectively.

The results show that the RPNB model with observation-specific coefficients reduces estimation error based on both RMSE and MAE metrics for each engineering district when applied to the training sample. This suggests that the RPNB models can provide better prediction accuracy when applied to the data used to estimate the model. However, the performance of the RPNB reduces when individual coefficients are not used (RPNB2 applied to the training dataset) or are not available (out-of-sample predictions performed on the validation dataset). Interestingly, in these cases, the predictive power of the RPNB model over the NB model is no longer superior as before. In all three engineering districts, the NB model outperforms the RPNB model when the mean model coefficients are applied to the out-of-sample validation dataset based on RMSE values in Table 6. In the MAE comparisons shown in Table 7, the RPNB model offers nominally better prediction accuracy over the NB model when the mean coefficients from the model are used on a training dataset from all three engineering districts. In summary, the comparison reveals that the RPNB model has its own advantage in making predictions when observation-specific coefficients are considered, which supports the assumption that variable effects are not fixed over segments. However, when applying the RPNB and NB models to predict the expected number of total crashes on two-lane rural highways in three engineering districts of Pennsylvania, the two models produce similar results when applied to an out-of-sample dataset (i.e., hold-out sample). This finding is particularly relevant when applying the models for new road construction projects or on roadway segments that were not used to estimate the models.

Table 8
Summary Table of Coefficients of Calibration Functions, Training Set.

	District 2		District 8		District 3	
	a	***b***	***a***	***b***	***a***	***b***
RPNB1	1.433	1.385	1.040	1.371	1.461	1.486
RPNB2	1.094	0.948	1.175	0.922	1.127	0.944
NB	0.962	0.947	1.002	0.926	0.969	0.949

Table 9
Summary Table of Coefficients of Calibration Functions, Validation Set.

	District 2		District 8		District 3	
	a	***b***	***a***	***b***	***a***	***b***
RPNB2	1.114	0.948	1.209	0.939	1.141	0.979
NB	0.979	0.947	1.027	0.944	0.976	0.984

4.2. Calibration function comparison

Tables 8 and 9 summarize the coefficients of the calibration function proposed in Eq. (8) for all three prediction approaches, with respect to the training and validation sets. The results show that both calibration coefficient (i.e., a and b) for the NB model predictions are closer to one (which represents perfect data fitness) than the other two approaches in each district. Furthermore, the RPNB1 prediction, which applies observation-specific coefficients to the validation dataset, produces coefficients that are furthest from 1.0 in this analysis. This coincides with the CURE plot findings in the next section, where the NB prediction shows the best overall data fitness, and RPNB curves tend to create monotonic increasing sections, resulting in large cumulative residuals at the end of the curves. In addition, the trends identified by the calibration functions are similar across the three district datasets, which suggests that similar findings may be obtained when applying standard NB models in SPF estimation and crash frequency prediction.

4.3. CURE plots comparison

While the RMSE, MAE and calibration coefficients evaluate the predictive power for individual segments, CURE plots offer the opportunity to examine the overall model fitness as a function of predicted

Table 7
Summary Table of MAE.

Dataset	District 2			District 8			District 3		
	RPNB1	RPNB2	NB	RPNB1	RPNB2	NB	RPNB1	RPNB2	NB
Training	0.426	0.480	0.495	0.644	0.844	0.871	0.476	0.553	0.571
Validation	—	0.491	0.506	—	0.878	0.903	—	0.546	0.564

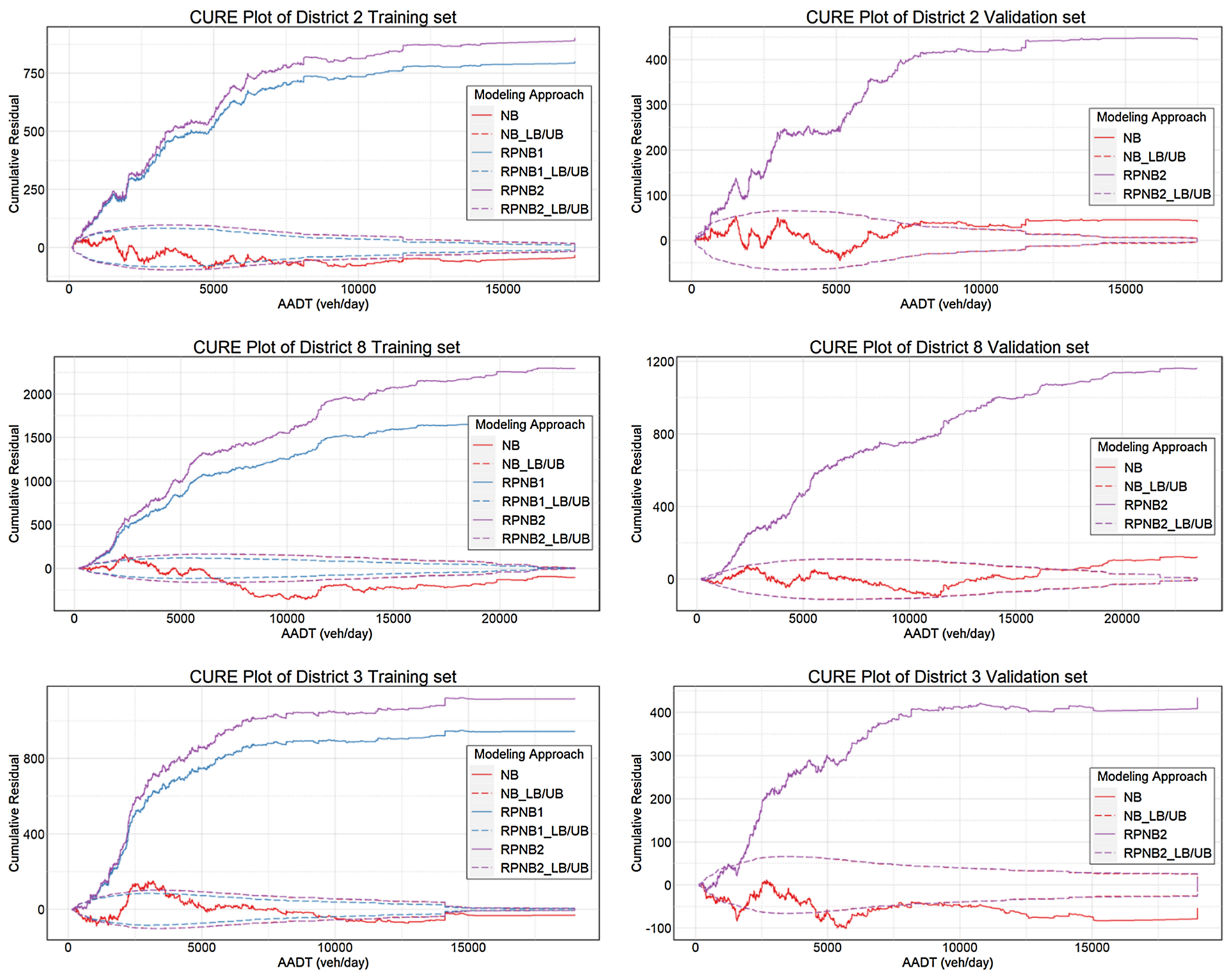

Fig. 1. CURE plots of different model prediction vs. AADT.

value or individual explanatory variables. In this paper, CURE plots were created for each model prediction approach (i.e., RPNB1, RPNB2 and NB) as a function of traffic volume and predicted crash frequency. Fig. 1 depicts the CURE plots as a function of AADT, and Fig. 2 describes the CURE plots as a function of predicted crash frequency. In both cases, the CURE plots are generated on both the training set and validation set for each of the three engineering districts. 95% confidence intervals for the cumulative residuals are also provided as in Hauer (Hauer, 2015). Note that the confidence intervals for the RPNB2 overlap and cover the 95% confidence interval for the NB model.

Fig. 1 reveals that even though the cumulative residual plot for the NB model does not always lie within the 95% confidence interval, the cumulative residual curves obtained for the NB model are always closest to zero. Furthermore, the cumulative residual curves for the RPNB model predictions result in a large gap from zero at the end of all curves. The phenomenon is validated through the mean error provided by predictions made with each of the models. For example, in the District 2 training set, the mean error produced by RPNB1, RPNB2 and NB are 0.044, 0.050, and -0.002, respectively; due to the large number of observations, small differences in the mean error can lead to large differences in cumulative residuals. Since the magnitude of the error is larger for the RPNB models than the NB models, the cumulative residual curves were expected to end further away from zero for the RPNB model than the NB model. Also, curves of RPNB model predictions generally show an increasing trend with short decreasing sections in between, which continuously travel beyond the 95 percent confidence interval, while curves of NB model predictions oscillate better around the x-axis and fall between the confidence bounds. The findings suggest that RPNB models tend to under-estimate the expected crash frequencies from a global perspective, and NB models provide better overall data fitness over the range of observed values and traffic volumes.

Similarly, from Fig. 2, we can identify the differences among the cumulative residual curves. Curves generated with RPNB1 predictions, which apply observation-specific coefficients, show a decreasing trend at first followed by an increasing segment until plateauing, whereas curves developed using mean coefficients from RPNB models perform a monotonic increasing trend. However, cumulative residual curves obtained from the NB model oscillate around zero. The results again show that the NB model performs better than RPNB model in terms of overall data fitness.

The CURE plots do not necessarily mean that RPNB models have no advantage over the NB models when used for prediction. First, for roadway segments with low predicted crash frequencies (e.g., curve segments with predicted crash frequencies less than one in the District 2 training set), the cumulative residuals obtained using the RPNB1 and NB models are similar, which indicates a similar performance level. Second, as the expected crash frequencies increase, each cumulative residual plot reaches a stable state, while RPNB1 prediction produces a longer tail before reaching the end of the cumulative curve. This suggests that RPNB1 prediction can provide a more precise value for cases with very high reported crash counts. For example, the predictions of three approaches are around 3.8, 5.3, and 3.3, while the maximum reported crash frequency is 10 in the District 2 training set.

In summary, the RPNB model offers unique advantages over the NB model when using it to make predictions on specific subgroups within

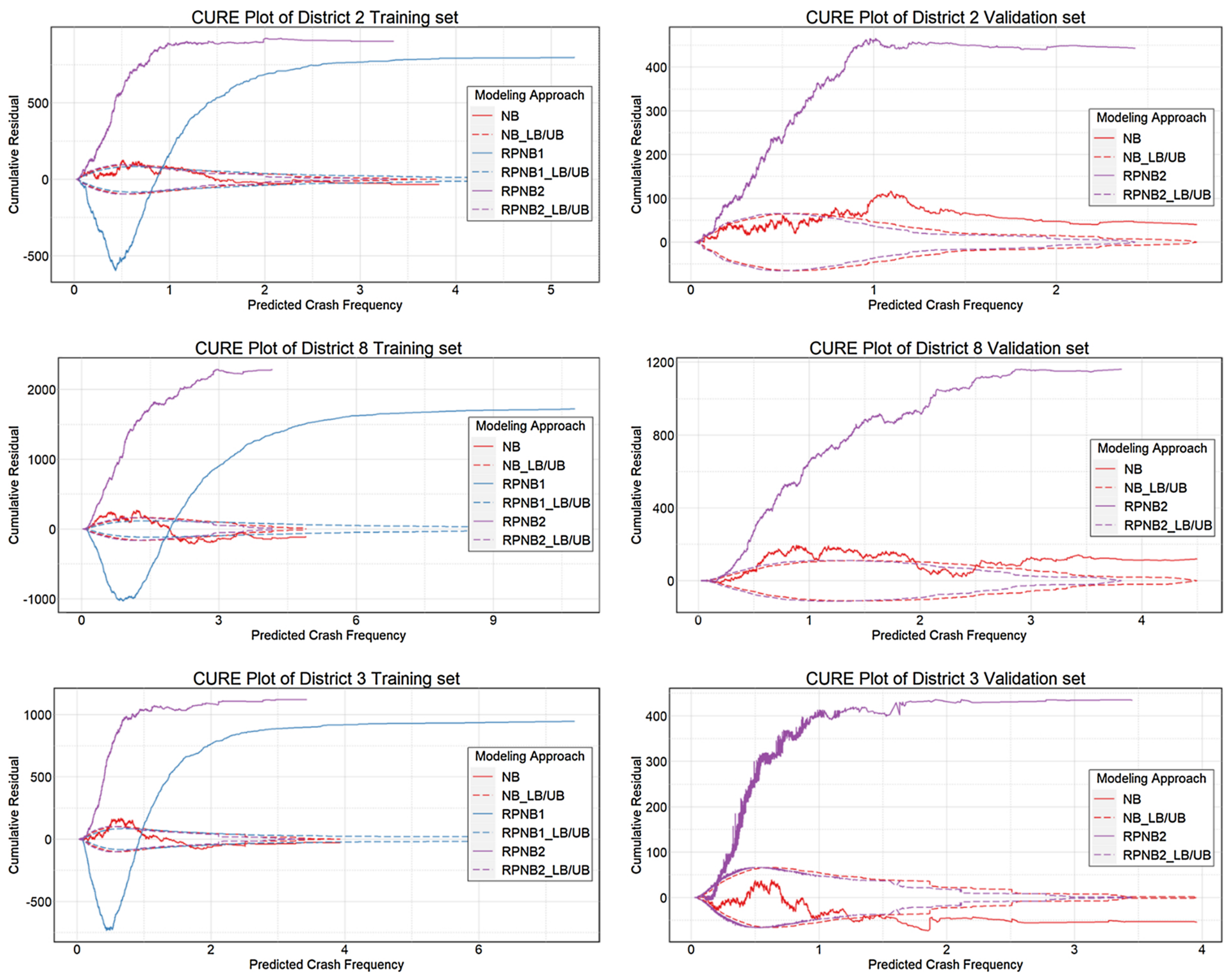

Fig. 2. CURE plots of different model prediction vs. predicted crash frequency.

the data. Based on the CURE plots in the current study, the NB appears to offer improvements over the RPNB model when used for the purposes of prediction.

4.4. Sensitivity analysis on Halton draws

To further evaluate the parameters from the RPNB model, the number of Halton draws is varied. This affords an opportunity to examine the relationship between the number of Halton draws and the number of iterations before model convergence, model estimation time, model fitness (i.e., log-likelihood), and predictive power (i.e., RMSE and MAE) of the model. The RPNB model specification that is shown in Table 5 was used for this sensitivity analysis. The evaluation trends are the same for all three districts, and for brevity, only the results from District 2 are shown in Table 10.

The results show that the number of iterations required before model convergence initial increases with the number of Halton draws, and then reaches a steady state of approximately 45. Meanwhile, increasing the number of Halton draws results in proportional increases in the estimation time required to fit the model, but does not appear to offer improved overall goodness-of-fit. In fact, the log-likelihood declines slightly when the number of draws increases. Also, the predictive power of the model does not improve significantly as the number of Halton draws increases. The predictive power was again assessed using both within-sample and out-of-sample observations, as described earlier. Both RMSE and MAE on the training and validation sets are relatively stable. From a practical point of view, the findings reveal that 50 Halton draws may be suitable for datasets of similar size as the ones used in this paper if the sole objective is to predict crash frequency using the RPNB model, since such settings could produce satisfactory

Table 10
Sensitivity of RPNB Model on Halton Draws, District 2.

Halton draws	Convergence iteration	Time (min)	LL	RMSE training		RMSE validation	MAE training		MAE validation
				RPNB1	RPNB2	RPNB2	RPNB1	RPNB2	RPNB2
50	39	10:00	−14035.51	0.621	0.715	0.742	0.431	0.482	0.492
100	46	18:24	−14034.35	0.600	0.716	0.743	0.422	0.481	0.492
150	45	26:25	−14037.72	0.614	0.715	0.742	0.428	0.481	0.492
200	45	34:08	−14037.25	0.615	0.715	0.742	0.426	0.480	0.491
250	45	41:12	−14037.96	0.610	0.716	0.742	0.423	0.480	0.490
300	42	46:59	−14038.06	0.618	0.715	0.742	0.429	0.481	0.491

predictive power while requiring less computational time to produce model output. However, it should be noted that this finding should be validated for models developed using different variable list or sample sizes.

5. Discussion

This study confirms that RPNB models provide a better fit to observed data than standard NB models. This finding is expected because the RPNB model accommodates unobserved heterogeneity among a sample of roadway segments. The likelihood ratio test in all three engineering districts suggests that the RPNB model is statistically superior to the NB model with regards to goodness-of-fit. Also, the RPNB model provides better prediction accuracy when applied to the sample of observations used to fit the model. In addition, the CURE plots show that the RPNB model performs well when the expected crash frequency is high or within a low range, typically less than one or two crashes per year. If used to make policy or safety decisions on locations that were used to estimate the model, the RPNB model offers clear advantages over the NB regression model.

However, the RPNB model also has its disadvantages. The primary drawback of applying the RPNB model is that the model loses its predictive power when applied to observations that were not used to estimate the model, which is a common traffic safety evaluation scenario. Under such circumstances, the NB model is a viable alternative based on the results of the present study. When the mean coefficients from the RPNB models are directly applied, the predictive power is similar to the NB model based on MAE and RMSE values. The CURE plots suggest that the standard NB model provides smaller cumulative residuals over the entire sample used in the present study, while the RPNB model tends to under-estimate the expected crash frequencies when applied to roadway segments that were not used to estimate the model. The coefficients from calibration functions also suggest that the NB modeling approach can provide the best overall prediction accuracy when the model is applied to out-of-sample observations.

Based on the results of the present study, there is a tradeoff when considering RPNB versus NB models. When collecting a sample of data that will then be used to estimate models and make safety decisions using the sites included in model development, the RPNB model offers benefits over NB models. When used for out-of-sample prediction, the NB models appear superior to RPNB models based on the present study. This finding was expected because the RPNB model offers a distribution associated with each segment in the sample – the distributional benefits are not available for application to an out-of-sample dataset.

For practical purposes, note that the RPNB modeling approach requires that agencies collect sufficient crash and roadway data to develop their own SPFs for use within their jurisdiction and thus might be better-suited for agencies that have the resources for SPF development. This approach might be cost-prohibitive to agencies that either do not have the resources to develop their own SPFs or those that are smaller and do not have a sufficient sample size for SPF development. Jurisdiction-specific SPFs developed using the RPNB framework might also not be applicable to new roadway segments that have not yet been constructed. In addition, the RPNB approach might not be practically useful for the development of national level SPFs, which are typically estimated using a sample of roadway miles (and corresponding crash data) from a few states if the models are intended to be applied outside the estimation sample.

6. Conclusions

In this paper, the predictive power of random parameters negative binomial regression models was thoroughly investigated using three large datasets, each developed from two-lane rural highway data from three engineering Districts in Pennsylvania. The same model specifications (i.e., set of variables and functional forms included in the models) were used to compare the NB and RPNB models using three different metrics: RMSE and MAE, coefficients from calibration functions and CURE plots. In addition, the sensitivity of the RPNB model performance based on the number of Halton draws was tested and analyzed.

The results show that both models provided the expected relationships between expected crash frequency and explanatory variables. Specifically, expected crash frequency increases with traffic volume, access density, curve density, horizontal curvature within the segment, and for segments that have poorer roadside hazard ratings. Expected crash frequency decreases with the presence of a passing zone or shoulder rumble strips within the segment. For the RPNB models, traffic volume and curve density were found to have different effects among the individual observations used to estimate the models. Likelihood ratio tests showed that the RPNB model fits the training data better than the NB model for all three districts included in the present study. In addition, the RPNB model provides superior prediction accuracy when applying the model to sites included in model estimation and using the observation-specific coefficients estimated in the model to predict crash frequency. However, the RPNB loses its superior accuracy when applied to observations not used to estimate the model or when observation-specific coefficients are not used. In these cases, the NB often model outperformed the RPNB model.

Based on the large, regional samples included in the present study, it appears that 50 Halton draws may be adequate to produce stable model coefficients. Since previous studies recommend a much larger number (approximately 200), the results are promising for the development of RPNB-based SPFs with less computational effort. However, this result was obtained for SPFs developed using very large datasets with over 15,000 observations from approximately 3,000 segments used to train the model. Additional sensitivity analyses are recommended on smaller samples to further assess this finding.

Because the study considered a sample of data from two-lane rural highways in Pennsylvania, similar comparisons are recommended for at-grade intersections, freeways and interchanges, rural multi-lane highways, urban and suburban arterials, and other roadway types.

Disclaimer

The contents of this paper reflect the views of the authors who are responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration or the Commonwealth of Pennsylvania at the time of publication. This paper does not constitute a standard, specification or regulation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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