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Game theoretic model for lane changing: Incorporating conflict risks

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ABSTRACT

The study employs a Quantal Response Equilibrium framework to model lane changing manoeuvres. Prior game theoretic studies in lane changing have pre-eminently assumed Nash equilibrium solutions with deterministic payoffs for actions. The study method involves developing expected utility models for drivers' merge and give-way decisions. These utility models incorporate explanatory variables representing driver trajectories during a lane changing manoeuvre. The model parameters are estimated using maximum likelihood on lane changing data at a freeway on-ramp using the NGSIM dataset. Based on the estimated parameters it was concluded that longer acceleration lanes and reduction of speed limits on on-ramps could help significantly reduce likelihood of conflict. To demonstrate the robustness of the model, predictions of lane changing on an out-of-sample data were found to be reasonably accurate.

1. Introduction

In recent years, there has been an increasing focus towards using game theory to model the interdependence of manoeuvres between conflicting drivers in traffic (Barmpounakis et al., 2016; Chatterjee and Davis, 2013; Elvik, 2014; Kita, 1999; Liu et al., 2007; Luo et al., 2015; Meng et al., 2016; Talebpour et al., 2015; Wang et al., 2015). The game-theoretic approach assigns a utility to each combination of driver decisions instead of only their disparate individual decisions. This focus on interaction ultimately leads to further insights in traffic safety and operations, in particular the quantification of behavioural norms and moral hazards of interaction.

The dominant assumption in game solutions of driver manoeuvres presented in prior literature is a mathematical Nash equilibrium of interactive behaviour. These studies include those models calibrated against data of observed field interactions (Kita, 1999; Liu et al., 2007; Talebpour et al., 2015), those models with arbitrarily specified incentives for choices (Chatterjee and Davis, 2013; Meng et al., 2016; Prentice, 1974) and purely theoretical models (Pedersen, 2003).

However Nash equilibrium solutions assume drivers have correct anticipations or beliefs of other drivers' decisions. A Quantal Response Equilibrium game solution on the other hand assumes drivers' beliefs are correct on average, however make errors according to a probability distribution (McKelvey and Palfrey, 1995). This formulation may greater reflect real driving behaviour, as it acknowledges errors in perception arising from mistakes in judgement or having imperfect vision of others. In particular, accounting for drivers' stochastic errors in perception may improve modelling of mean and variance in driver interactions. Dixit and Denant-Boemont (2014) showed that Strategic User Equilibrium (analogous to Quantal Response Equilibrium for route choice decisions) is able to accurately model mean and variability in strategic route choice decisions. Outside the driving context, McKelvey and Palfrey (1995) were able to produce Quantal Response Equilibrium estimates of strategies more accurate than Nash Equilibrium estimates.

A paper by Barmpounakis et al. (2016) presents a Quantal Response Equilibrium in a sequential game abstraction for overtaking manoeuvres. However the study in this paper adopts a different approach to Barmpounakis et al. by inter-relating the decision payoff functions of game players to explicitly account for interactions. Further, when calibrating decision payoffs against observed interactions, the parameters in payoff functions for decisions are calculated simultaneously as game solutions are arrived. The authors of this paper believe it is integral to calculate game payoffs simultaneously with game solutions in order for payoffs to explicitly reflect interactions and not individual decisions.

The study in this paper investigates the efficacy of a Quantal Response Equilibrium solution for lane changing manoeuvres, by first defining the game structure: a simultaneous two-player, non-cooperative, non-zero sum game. Expected utility decision models for merging and give-way behaviour are accordingly developed. A probability distribution is specified for drivers' anticipation for payoffs in the decision models; this anticipation is probabilistic in Quantal Response Equilibrium but deterministic in Nash Equilibrium.

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The model is calibrated and tested against a large trajectory dataset collected under the NGSIM program (Federal Highway Administration, 2006). 45 min of vehicle trajectory data is used, describing positional information along a section of the Interstate 80 in Emeryville, California. Moridpour et al. (2010) mentions the importance of using large trajectory datasets to improve development of lane changing models.

The study focuses on merging and give-way interactions at freeway on-ramps.

Once the theoretical model is developed and data sample is prepared, the study utilises a maximum likelihood procedure to estimate the Quantal Response Equilibrium model parameters against the observed field interactions. In particular, the study allows driver decision payoff parameters to be heterogeneous across vehicle class types and traffic conditions experienced. This allows for further interaction insights for these subgroups. A fixed point algorithm is used to converge to QRE game solutions. The calibrated QRE models in the study are cross-validated against separate test datasets.

2. Game Theoretic Representation

2.1. Type of game

The lane changing interaction is modelled as a simultaneous two-player, non-cooperative, non-zero sum game. The game-theoretic studies of Kita (1999), Liu et al. (2007), Meng et al. (2016) and Talebpour et al. (2015) likewise adopt this structure.

In this study, lane changing interactions between one mainline driver and one on-ramp driver is modelled. Each driver can make either one of two manoeuvres. The on-ramp driver can choose to either 'merge' or 'do not merge', whilst the mainline driver can choose to 'give-way' or 'do not give-way'. The interaction between these two drivers is modelled, as it is considered dominant over the interaction with any other surrounding vehicles (Kita, 1999).

The lane changing interaction at the on-ramp is represented as a simultaneous game. That is the on-ramp and the mainline players both decide their manoeuvre at the same time. The simultaneous representation is considered because there is a limited amount of time for each player to make their decision given the stimuli provided by each other and surrounding vehicles (Fig. 1).

2.2. Decision timing

Every instance when the longitudinal coordinate of the rear of an on-ramp vehicle passes the longitudinal coordinate of the front of an adjacent mainline lane vehicle with respect to the direction of travel, is considered as an interaction in this paper (see Fig. 2). Hence the instance when the rear of the on-ramp player's vehicle passes the front of the mainline player's vehicle is taken as the decision time. At this time and vehicle positioning it is assumed that these conflicting drivers have already formulated anticipations of each other.

Having a definition for interactions allows for a consistent calibration dataset for the decision models. The traffic conditions at the decision time are input into the lane changing decision models.

2.3. Definition of manoeuvres

Mainline player 'give way' and 'do not give way' decisions are defined based on their acceleration behaviour at the time and positioning in Fig. 2; accelerations $\leq -0.25ms^{-2}$ are defined as 'give way'

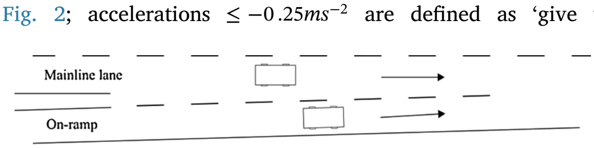

Fig. 1. Schematic representation of the Powell Street on-ramp along Interstate 80 in Emeryville California; the location of the data collection.

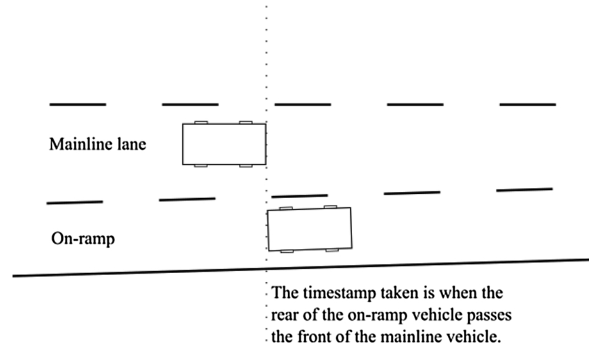

Fig. 2. Schematic representation of the interacting players.

manoeuvres, whereas accelerations $> -0.25ms^{-2}$ are defined as 'do not give way' manoeuvres. The threshold value $-0.25ms^{-2}$ is taken instead of $0ms^{-2}$ to hedge against noise in acceleration values in the model calibration and verification datasets.

On-ramp player 'merge' and 'do not merge' decisions are defined on whether they take the gap in front of the mainline player. This lane changing decision by on-ramp vehicles to merge or to not merge is latent in nature, hence taking the earliest interaction timing when the on-ramp vehicle can physically merge in front of the mainline vehicle (as in Section 2.2) allows to account for all potential merge maneuvers after passing the putative follower, whether they are done immediately, later, or not done.

The normal form of the game is presented in Table 1. The payoff equations are a function of vehicle trajectories, and are discussed in the section following.

3. Methodology

3.1. Payoff model formulation

In Quantal Response Equilibrium drivers make decisions with the lowest perceived costs. These perceptions are subject to error however, and hence drivers behave stochastically against rational expectations.

The models for merge and give-way decisions in this study follow Expected Utility Theory, whereby drivers have decision utilities dependent upon expected actions of conflicting drivers. In this way payoff functions between conflicting drivers are inter-related, and interactions are explicitly accounted for.

The expected utility decision models are shown in Eqs. (1)–(4).

The coefficients of the co-decision utilities are the anticipations or beliefs of the other drivers' decisions, p_{merge} and $p_{giveway}$. p_{merge} is the mainline player's anticipation that the on-ramp player will merge, and $p_{giveway}$ is the on-ramp player's anticipation that the mainline player will give way. The anticipations are probabilities that lie between 0 and 1 inclusive: $0 \leq p_{merge} \leq 1$, $0 \leq p_{giveway} \leq 1$.

Game theoretic decision models for on-ramp player utilities:

$$\mathrm{EU_{merge}} = (1 - \mathrm{p_{giveway}}) \times (\mathrm{a_1}.\ \mathrm{v^2_{onramp}}) \tag{1}$$

$$\mathrm{EU_{donotmerge}} = \mathrm{a_0} + \mathrm{a_2}.\ \mathrm{d_{onramp}} \tag{2}$$

Game theoretic decision models for mainline player utilities:

$$\mathrm{EU_{giveway}} = \mathrm{b_0} + \mathrm{b_2}.\ \mathrm{d_{mainline}} + \mathrm{b_3}.\ \Delta\mathrm{V_{lm}} + \mathrm{b_4}.\ \Delta\mathrm{V_{om}} \tag{3}$$

$$\mathrm{EU_{donotgiveway}} = \mathrm{p_{merge}} \times (\mathrm{b_1}.\ \mathrm{v^2_{mainline}}) \tag{4}$$

Where,

v_{onramp}: Velocity of the on-ramp player at decision time

$v_{mainline}$: Velocity of the mainline lane player at decision time

ΔV_{lm}: Velocity difference between the leading vehicle on the mainline and the mainline lane player that needs to decide whether to give a gap or not, at decision time

Table 1
Payoff matrix for the two players.
[On-ramp driver payoff], {Mainline driver payoff}

		Mainline driver action	
		Give-way	Do not give-way
On-ramp driver action	Merge	[0], $\{b_0 + b_2.\ d_{mainline} + b_3.\ \Delta V_{lm} + b_4.\ \Delta V_{om}\}$	$[a_1.\ v^2_{onramp}]$, $\{b_1.\ v^2_{mainline}\}$
	Do not merge	$[a_0 + a_2.\ d_{onramp}]$, $\{b_0 + b_2.\ d_{mainline} + b_3.\ \Delta V_{lm} + b_4.\ \Delta V_{om}\}$	$[a_0 + a_2.\ d_{onramp}]$, {0}

ΔV_{om}: Velocity difference between the on-ramp player and the mainline player, at decision time

d_{onramp}: remaining distance to the end of the acceleration lane for the on-ramp player, at decision time

$d_{mainline}$: remaining distance to the end of the acceleration lane for the mainline lane player, at decision time

$a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,\ b_3,\ b_4$: parameters to be estimated

The above explanatory variables for player decisions have a Pearson correlation between them of less than 0.5 in the dataset, ensuring a level of independence amongst model variables.

Conflict risk and magnitude are characterised by the decision probabilities $p_{giveway}$ and p_{merge}, displacement variable $d_{mainline}$ and the kinetic energy variables $v^2_{mainline}$, v^2_{onramp}. $d_{mainline}$ indicates the risk mitigation employed by the mainline lane driver to cautiously give way to the on-ramp driver in the case they merge earlier than expected. The kinetic energy variables are used to explain the 'merge' and 'do not give way' co-decision (Eqs. (1) and (4)). Kinetic energy is thus used to represent the magnitude of crash consequences.

The payoff functions assume that interacting drivers are also motivated by time savings in addition to minimising conflict risks and consequences with each other. Motivation for time savings amongst mainline players is captured by the velocity differential variable ΔV_{lm}, describing the mainline lane vehicle's desire to achieve a suitable car following velocity for its current leader. It is also accounted amongst on-ramp players through d_{onramp}, whereby on-ramp drivers may choose to merge later to reach the front of mainline queues. Therefore as the interacting drivers are motivated beyond conflict risks and consequences which affect both players, the game equilibrium is non-trivial (Liu et al., 2007).

The decision models use remaining distance to the end of the acceleration lane as an explanatory variable, similar to Kita (1999) and Liu et al. (2007) who use remaining time. Remaining distance was chosen instead for this study as a large proportion of interaction observations had invalid remaining time values. That is, vehicles had deceleration values at decision time such that a complete stop would be achieved before reaching the end of the acceleration lane, and hence had infinite remaining time to reach the end of the acceleration lane.

3.2. Model estimation and cross validation

The expected utilities for decisions are used as arguments in logit functions to calculate the probability for choices (Eqs. (5) and (6)).

$$p_{merge} = \frac{e^{E[u_{merge}(1-p_{giveway})]}}{e^{E[u_{donotmerge}]} + e^{E[u_{merge}(1-p_{giveway})]}} \quad (5)$$

$$p_{giveway} = \frac{e^{E[u_{giveway}]}}{e^{E[u_{donotgiveway}(p_{merge})]} + e^{E[u_{giveway}]}} \quad (6)$$

The anticipations p_{merge} and $p_{giveway}$ in the expected costs (Eqs. (1) and (4)) are the same as the choice probabilities estimated by the logit models (Eqs. (5) and (6)). This is the premise behind quantal response equilibrium; the perceived probability of other drivers' choices are equal probability of drivers' choices on average, however are subject to some error.

Statistically computing p_{merge} and $p_{giveway}$ are thus fixed point problems of the type $p_{merge} = \mathrm{F}(p_{giveway})$ and $p_{giveway} = \mathrm{H}(p_{merge})$, where F and H are functions. The probabilities p_{merge} and $p_{giveway}$ are solved iteratively, with seed values of p_{merge} and $p_{giveway}$ used in the expected utility functions to generate the first set of parameter estimates. The first set of parameter estimates are used as inputs for the logit models to generate new values of p_{merge} and $p_{giveway}$. These generated values of p_{merge} and $p_{giveway}$ are then used to update estimates of the parameters, and this method is iterated until the values of p_{merge} and $p_{giveway}$ are converged.

The convergence in this case is a Logit QRE (McKelvey and Palfrey, 1995). Errors in driver perception against choices follow an extreme value distribution. In a Nash Equilibrium, the driver anticipations are correct and equal driver choices with no error.

This logit fixed point QRE convergence method is also displayed in McKelvey and Palfrey (1995); Offerman et al. (1998) and Rogers et al. (2009).

Maximum likelihood estimation is used to solve Eqs. (5) and (6). The most likely parameter values of $a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,\ b_3$ and b_4 in the EU models are estimated jointly by maximising their fit against observed decisions in the calibration dataset.

The maximum likelihood estimation procedure involved constructing expected utility indices (∇ EU) from the above logit models. This index was the difference in expected utility between each of the binary choices.

Expected utility index for the on-ramp driver decisions:

$$\nabla EU_{merge} = (EU_{merge} - EU_{donotmerge}) \quad (7)$$

The log-likelihood to be maximised for the on-ramp driver decisions is thus:

$$\mathrm{LL}^{\mathrm{onramp}} = \mathrm{lnL}(\mathrm{a_0},\ \mathrm{a_1},\ \mathrm{a_2}\!:\ \mathrm{y},\ \mathrm{X}) \quad (8)$$

$$= \sum_i [\ln(\Phi(\nabla EU_{merge}) \times \boldsymbol{I}(y_i = 1)) + \ln((1 - \Phi(\nabla EU_{merge})) \times \boldsymbol{I}(y_i = 0))] \quad (9)$$

Expected utility index for the mainline driver decisions:

$$\nabla EU_{merge} = (EU_{merge} - EU_{donotmerge}) \quad (10)$$

The log-likelihood to be maximised for the mainline driver decisions is thus:

$$\mathrm{LL}^{\mathrm{mainline}} = \mathrm{lnL}(\mathrm{b_0},\ \mathrm{b_1},\ \mathrm{b_2},\ \mathrm{b_3},\ \mathrm{b_4}\!:\ \mathrm{y},\ \mathrm{X}) \quad (11)$$

$$= \sum_j [\ln(\Phi(\nabla \mathrm{EU_{giveway}}) \times \mathbf{I}(\mathrm{y_j} = 1)) + \ln((1 - \Phi(\nabla \mathrm{EU_{giveway}})) \times \mathbf{I}(\mathrm{y_j} = 0))] \quad (12)$$

Where y_i and y_j represent the binary choice of a player, $\boldsymbol{I}$ and $\boldsymbol{J}$ are indicator functions which take a value of 1 when the condition is satisfied and zero otherwise. $\boldsymbol{X}$ is a vector of traffic conditions during the lane changing interaction, derived from the NGSIM trajectory data.

The parameters $a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,\ b_3$ and b_4 were all jointly estimated. The end result is a final log-likelihood to be maximised:

$$\mathrm{LL} = \mathrm{LL}^{\mathrm{onramp}} + \mathrm{LL}^{\mathrm{mainline}} \quad (13)$$

To estimate the impact of surrounding traffic conditions upon the model parameters, the ML analysis was generalised to allow the core parameters $a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,\ b_3$ and b_4 to be a linear function of

Table 2
Descriptive statistics of the total sample; n = 397 interactions, 794 players.

Variable	Description	Mean	Standard deviation
d_{onramp}	The remaining distance between the front of the on-ramp player to the end of the acceleration lane (m)	45.35	22.96
$d_{mainline}$	The remaining longitudinal distance between the front of the mainline player to the of the acceleration lane (m)	50.08	23.02
d_{lm}	Distance between the leading vehicle on the mainline and the mainline player (m)	10.30	7.03
v_{onramp}	Velocity of the on-ramp player (km/hr)	31.38	12.07
$v_{mainline}$	Velocity of the mainline player (km/hr)	16.65	6.57
v_{leader}	Velocity of the leading vehicle on the mainline (km/hr)	17.18	7.05
ΔV_{lm}	Velocity difference between the leading vehicle on the mainline and the mainline player	0.52	4.07
ΔV_{om}	Velocity difference between the on-ramp player and the mainline player (km/hr)	14.73	9.56
ΔV_{lo}	Velocity difference between the leading vehicle on the mainline and the on-ramp player (km/hr)	−14.20	10.87
a_{onramp}	Acceleration of the on-ramp player (m/s/s)	−0.61	1.29
$a_{mainline}$	Acceleration of the mainline player (m/s/s)	−0.13	0.88
Vehicle_length	The length of a vehicle (m)	4.73	1.67
motorcycle	A binary variable taking the value 1 if the vehicle is a motorcycle; 0 otherwise	0.001	0.04
car	A binary variable taking the value 1 if the vehicle is a car; 0 otherwise	0.97	0.16
truck	A binary variable taking the value 1 if the vehicle is a truck; 0 otherwise	0.02	0.15
merge	A binary variable taking the value 1 if the on-ramp player merged in the interaction; 0 otherwise	0.390	0.488
giveway	A binary variable taking the value 1 if the mainline player gave way in the interaction; 0 otherwise	0.539	0.499

them. The models are extended to be $a_n = a_{n0} + \beta_n \boldsymbol{X}$ and $b_n = b_{n0} + \alpha_n \boldsymbol{X}$, where $a_{n0}$and $b_{n0}$are fixed parameters, β_n and α_n are vectors of effects associated the traffic condition variables being represented by $\boldsymbol{X}$ (Table 2), and $n = [0,1,2]$ for the on-ramp player and $n = [0,1,2,3,4]$ for the mainline player.

Cross-validation was performed to test the efficacy of the QRE framework to model the interactive decisions. The sample was split 70/30; 70% of observations were used for model calibration whilst 30% were used for verification testing.

The fixed-point problem was first solved for the calibration sample. Once the probabilities p_{merge} and $p_{giveway}$ converged, the parameter values a_0, a_1, a_2, b_0, b_1, b_2, b_3, b_4 were used as inputs to the verification test sample. In the verification testing the a_0, a_1, a_2, b_0, b_1, b_2, b_3, b_4 values were fixed, and the p_{merge} and $p_{giveway}$ values were evaluated using a fixed point till convergence of the two probabilities.

Cross-validation testing was performed 10 times (the sample was split randomly 70/30 ten times). It is important to note that an equal number of mainline players and on-ramp players were included in each calibration and verification dataset.

4. Data

The decision models were calibrated against empirical trajectory data of vehicles travelling along a section of the Interstate 80 in Emeryville, California. This data was collected under the Next Generation simulation (NGSIM) program in April 2005. 45 min of trajectory was collected, and the whole of this dataset was utilised for the study in this paper.

Fig. 3 illustrates the data collection site, and provides a representation of the lane geometry. The on-ramp tapers to join the adjacent mainline lane, forming one lane. Adjacent to the on-ramp and outer mainline lane (not shown in Fig. 3) is a shoulder lane.

It is important to note the trajectory data was smoothed according to Thiemann et al. (2008) to address noise in the positional information. In particular, the NGSIM I-80 trajectory data exhibits unrealistic velocity and acceleration distributions with spikes present. The smoothing post-processing was performed before any data analysis.

First, displacement values were differentiated to velocities and accelerations using symmetric difference quotients, then a symmetric exponential moving average filter was applied to these displacement, velocity and acceleration values. The smoothing times for displacement, velocity and acceleration were respectively $T_x = 0.5s$, $T_v = 1s$ and $T_a = 4s$ akin to Thiemann et al. (2008).

There were 735 instances where an on-ramp vehicle passed a mainline vehicle on the adjacent lane. All instances where at least one

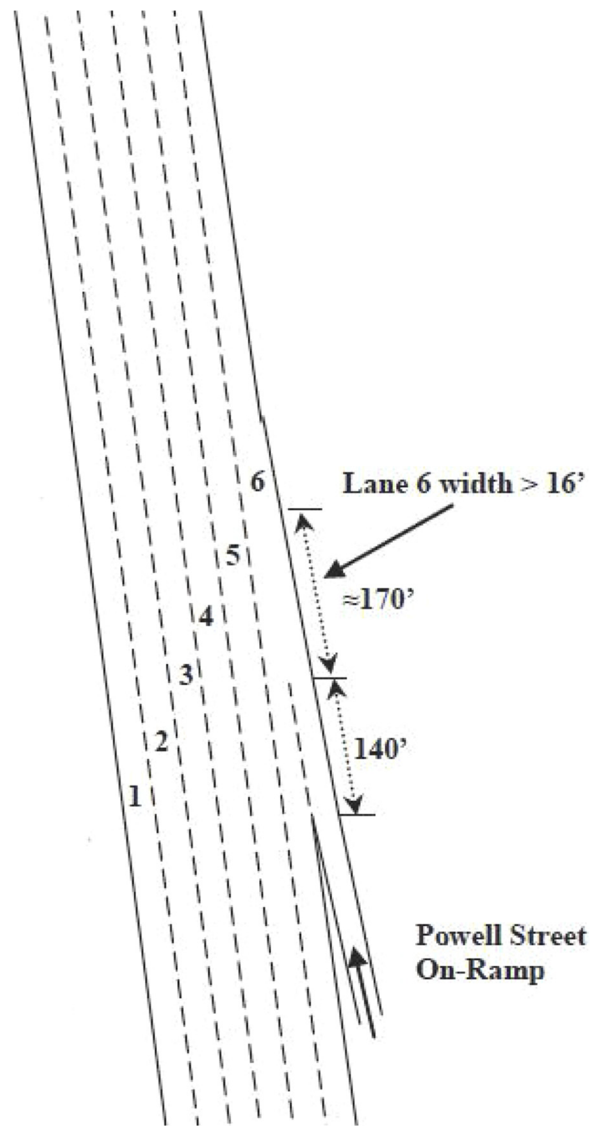

Fig. 3. Schematic representation of the on-ramp section, with lanes 1–6 marked. Measurements are in feet. Source: US DOT FHWA.

Table 3
Parameter estimates of the full dataset.

	Coefficient	Robust Std. Err	z	P > |z|
a_0				
constant	−1.530955	0.40353	−3.79	0.000
a_1				
ΔV_{lo}	0.00010	0.00003	4.13	0.000
constant	0.00157	0.00063	2.47	0.014
a_2				
constant	0.04316	0.00649	6.65	0.000
b_0				
constant	−4.93997	0.70767	−6.98	0.000
b_1				
d_{lm}	0.00027	0.00008	3.46	0.001
constant	−0.00998	0.00236	−4.23	0.000
b_2				
constant	0.03893	0.00894	4.35	0.000
b_3				
constant	−0.17256	0.03903	−4.42	0.000
b_4				
constant	0.04088	0.01602	2.55	0.011

of these vehicles was travelling less than 10 km per hour were removed, as the model was to be calibrated by interactions at speed. Any driver interactions under this 10 km/hr threshold speed were assumed irrelevant to unsafe throughput or significant give-way behaviour. Thus a total sample of 397 defined interactions was ultimately used. Descriptive statistics of the dataset are presented in Table 2. The variable values are collected at the time of interaction.

5. Results and discussion

The estimation results for the full sample are shown in Table 3. The seed values of p_{merge} and $p_{giveway}$ were chosen as 0.4 and 0.5 respectively. These values were closest to the observed mean decision probabilities in the full sample to the nearest 0.1. Given these initial values the QRE fixed point problem was iterated through till the probabilities p_{merge} and $p_{giveway}$ converged. This usually took 10 iterations.

In addition, the parameter estimates for each of the 10 calibration samples (i.e. randomly chosen 70% dataset) were consistent with those estimated using the full 100% sample. These are displayed in the Appendix A. This demonstrates the robustness of the estimation process, and for the purpose of interpreting the coefficients the model estimated on the full data set is discussed. The ten calibration models are used to study the robustness of the prediction in the out of sample datasets.

As observed in Table 3, the kinetic energy parameter a_1 for the ramp player was found to be affected by ΔV_{lo}, while the kinetic energy parameter b_1 for the mainline player affected by d_{lm}. A positive coefficient on ΔV_{lo} suggests that greater relative velocity of the mainline leaders provides further utility to merge. This aligns with the intuition that a higher speed of the lead vehicle on the mainline reduces the likelihood of the on-ramp vehicle causing a rear-end crash with the vehicle in front, and hence increasing the likelihood of merging. Due to congestion on the mainline the speed of the on-ramp vehicle higher than the speed of the mainline lead vehicle, this resulted in ΔV_{lo} being negative (see Table 2). Therefore, the overall estimate of a_1 was negative which represents the disutility arising from crashing.

A positive coefficient for coefficient for d_{lm} in the equation for b_1, suggests that a greater gap distance to the leading vehicles lowers the probability to give-way. Again this is intuitive given that with larger gap to the vehicle in front on the mainline, a vehicle from the on-ramp can reasonably safely enter the mainline without requiring the mainline vehicle to give it a gap. Furthermore, the negative constant in the equation for b_1 ensures that the overall estimates for b_1 are negative. This demonstrates the disutility arising from the kinetic energy in the event of a crash.

Table 4
Comparison of observed equilibrium with QRE. The averaged results derived from ten verification test datasets are presented below.

	On-ramp player merge decisions (# of interactions = 119)		Mainline player give way decisions (# of interactions = 119)	
Equilibrium	Average expected	Average stdev	Average expected	Average stdev
Observed	45.30	5.27	23.00	4.30
QRE	44.67	5.27	25.44	4.47

The parameter estimates for a_2 and b_2 associated with the remaining distance to the end of the acceleration lane for the on-ramp and mainline player respectively were both positive. On-ramp vehicles do not want to merge earlier and prefer to merge closer to the end of the acceleration lane, whereas mainline players look to give-way earlier.

A negative coefficient for b_3 indicates mainline players are less likely to give-way if the relative velocity to their lead vehicle is higher. With increasing relative velocity of the lead vehicle, the distance between the lead vehicle and the mainline player increases and therefore the mainline vehicle might not require to give a gap for an on-ramp vehicle to enter.

A positive coefficient for b_4 indicates that if the relative speed between the on-ramp vehicle and the mainline player is high, the mainline player is more likely to give a gap. This implies that if the on-ramp vehicle is faster than the mainline player, then the mainline player is more likely to give a gap.

5.1. Out-of-sample predictive performance

The 30% verification datasets were used to evaluate the out-of-sample predictive power of the QRE estimated models to the observed data (Table 4). First, parameter values were determined using a 70% training dataset. These parameter values were applied to the corresponding 30% testing dataset, whereby the logit functions of Eqs. (5) and (6) were used to estimate the probabilities p_{merge} and $p_{giveway}$ 'merge' and 'give-way', such that they converged. These estimated probabilities were used to determine the expected number and variance in merging and give-way decisions in the verification datasets. The expected number was the sum of the decision probability over all the decisions in the verification data ($\sum p_i$). The standard deviation in number of decisions was calculated as $\sqrt{\sum p_i(1-p_i)}$. The expected output was compared to those observed in Table 4.

The QRE was able to accurately estimate the expected number and standard deviation in interactive decisions. It is demonstrated that a QRE framework can be used effectively to model operational decisions in aggregate. Dixit and Denant-Boemont (2014) were able to show that Strategic User Equilibrium (analogous to Quantal Response Equilibrium for strategic decisions) is able to likewise model mean and variability in strategic route choice decisions.

5.2. Parameter sensitivity on conflict

A lane changing conflict is defined as a case when an on-ramp vehicle merges by the mainline vehicle does not give way. Therefore, the likelihood of a conflict ($p_{conflict}$) is given by:

$$p_{conflict} = p_{merge} * p_{do\ not\ giveway}$$

$$p_{conflict} = p_{merge}(1 - p_{giveway})$$

Understanding the sensitivity of the different factors to reduce the likelihood of conflict would provide important insights for improve safety. We conduct a sensitivity analysis for each of the variables and show the distribution of the sensitivity of the conflict. This is done by

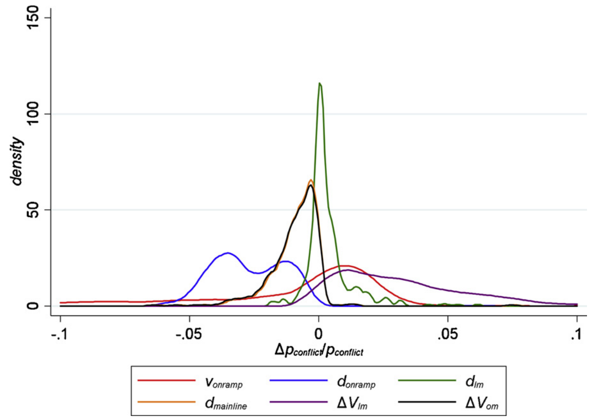

Fig. 4. Sensitivity of conflict probability to the variables of interest.

predicting the change in conflict probability for each interaction if there is a unit change to each of the variables of interest are shown in Fig. 4.

Increased speed on on-ramp (v_{onramp}) as well as relative speed between the lead vehicle and vehicle giving the gap on the mainline (ΔV_{lm}) increases the likelihood of lane changing conflict. However increases in, the remaining distance between the vehicle on the on-ramp and the end of the acceleration lane (d_{onramp}), the remaining distance between the mainline vehicle giving the gap and the acceleration lane ($d_{mainline}$) and the relative speed between the on-ramp vehicle and mainline vehicle (ΔV_{om}) reduces the likelihood of conflict. While the distance gap on the mainline (d_{lm}) has almost no effect on the conflict likelihood.

Parameters such relative speeds (ΔV_{lm}, ΔV_{om}) and distance gap on the mainline (d_{lm}) are traffic properties which are hard to control. However having longer acceleration lanes could help increase $d_{mainline}$ and d_{onramp} which would then reduce likelihood of conflict. While reducing speed limits on on-ramps would reduce v_{onramp} and therefore likelihood of conflict as well.

6. Conclusion

The aim of this paper was to assess Quantal Response Equilibrium as a game solution for interactions in lane changing manoeuvres. Prior studies of interaction in manoeuvring decisions have pre-eminently assumed Nash equilibrium solutions to behaviour, with one study testing Quantal Response Equilibrium (Barmpounakis et al., 2016). The Quantal Response Equilibrium approach adopted in the study in this paper inter-relates the utilities of player decisions, and calculates game payoff functions simultaneously as game solutions are arrived. In this way, values for payoff functions explicitly reflect interactions instead of only individual decisions.

QRE assumes drivers have stochastic instead of deterministic perceptions of competing players' decisions. This differentiates QRE from Nash Equilibrium. In particular, the equilibrium modelled in the study was that of merging and give way decisions at a freeway on-ramp. The calibration and verification data used against the proposed model was the NGSIM trajectory dataset collected in April 2005 (Federal Highway Administration, 2006). The lane changing interactions were identified in the trajectory dataset using an automated manner.

The decision models developed in the study incorporate incentives for time savings, and conflict avoidance. Therefore as players are motivated beyond conflict risks, the QRE equilibrium game solution achieved is non-trivial (Liu et al., 2007).

The parameter estimation approach allowed for payoff function parameter estimates to be heterogeneous across trajectory variable effects, allowing for interaction insights across these effects. In particular, the study found that interactions were affected by velocities and gap distances in the mainline. Rogers et al. (2009) adopt a similar approach where behavioural attributes across agents are allowed to be heterogeneous, finding improved QRE model estimation.

The study finds through cross-validation testing that QRE is able to accurately model not only means but also variance in choices. It demonstrates QRE as a suitable theoretical framework to model operational decision making. QRE takes into account errors in perception of other drivers' payoffs, whether they are caused by mistakes in judgement or lack of vision.

We find that longer acceleration lanes and reduction of speed limits on on-ramps could help significantly reduce likelihood of conflict. We do suggest replicating this study with other datasets.

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Appendix A

Table A1

Table A1
Parameter estimates of the 10 training datasets.

1	−1.075221**	0.0028097*	0.0001513*	0.0398763*	−4.272658*	−0.0087288*	0.0002435*	0.0319793*	−0.13841*	0.036291**
2	−0.9705581**	0.0024342*	0.0001215*	0.0407284*	−5.519255*	−0.0140168*	0.0006422*	0.0473344*	−0.1371338*	0.0449081**
3	−1.633824*	0.0015744**	0.0001108*	0.0496235*	−5.092362*	−0.010291*	0.0002719*	0.0424085*	−0.1933731*	0.0339948***
4	−1.293333*	0.0018427**	0.0001175*	0.0364012*	−5.061979*	−0.0099956*	0.0002349*	0.0385667*	−0.1679707*	0.0431448**
5	−1.305078*	0.0017756**	0.0001041*	0.0396278*	−4.204278*	−0.0086193*	0.0002676*	0.03176*	−0.1655011*	0.02888
6	−1.962715*	0.001185***	0.0000918*	0.0492981*	−5.160431*	−0.0099157*	0.0003275**	0.0365974*	−0.21866*	0.0609013*
7	−1.781591*	0.0014233**	0.0000855*	0.048948*	−4.452004*	−0.0104964*	0.0004486**	0.0323207*	−0.1682229*	0.0406523**
8	−1.524949*	0.0015176***	0.0000966*	0.0484909*	−4.658318*	−0.0073638**	0.0001868***	0.0386258*	−0.1567176*	0.0382055**
9	−1.323212*	0.0021935*	0.0001274*	0.04126*	−4.909822*	−0.0125107*	0.0004136*	0.0345682*	−0.2114646*	0.043576**
10	−1.908911*	0.00120	0.0000916*	0.0514509*	−4.743899*	−0.0099046*	0.0002643**	0.0392787*	−0.172561*	0.0319741***

*** 0.10 significance level.
** 0.05 significance level.
* 0.01 significance level.

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