Accident Analysis and Prevention 131 (2019) 225–233

Contents lists available at ScienceDirect

Accident Analysis and Prevention

journal homepage: www.elsevier.com/locate/aap

Explaining crash modification factors: Why it's needed and how it might be done

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ARTICLE INFO

Keywords:
Crash modification factor
Pedestrian hybrid beacon
Pedestrian injury
Micro-simulation
Abductive inference

ABSTRACT

Although the *Highway Safety Manual* (HSM) now provides empirical tools for predicting the safety consequences of highway engineering decisions, these tools represent the driver and vehicle conditions prevailing in the United States during the last few decades. As automated vehicles improve in capability and increase in market share these conditions will change, possibly reducing the accuracy of HSM predictions. Assessing the transferability of a crash modification factor to new situations almost certainly requires an explanation of how the modification achieves its effect, but at present there is little guidance on how such explanations might be posed and tested. This paper describes the use of micro-simulation to develop an explanation of how pedestrian hybrid beacons (PHB) modify pedestrian crash likelihood. Since the literature indicated that PHBs can affect both pedestrian and driver behavior it was necessary to include both possibilities in the model. To simulate injury severity distributions similar to those recorded in a crash database it was necessary to propose that almost all simulated drivers attempt to brake in pedestrian/vehicle encounters. Then changing the simulated fraction of careful pedestrians from between 0% and 30% to between 80% and 90% gave simulated crash modification factors similar to estimates reported in the literature. The resulting working hypothesis then is that PHBs achieve their crash reduction effect in large part by modifying pedestrian behavior. This is not so much a direct observation as it is an inference to the best explanation. That is, the support for the hypothesis comes from its ability to explain the data at hand. This hypothesis should be tested further, and additional tests are proposed.

> "If we have a forecasting-technique which not only works, but works for explicable reasons, that is of course, doubly satisfactory; this will be the case only where we can point to the natural connection or mechanism which accounts for our predictive success." (Toulmin 1961, p. 36)

1. Introduction

In 1988 Ezra Hauer issued a challenge: to base the design and operation of roads on "science-based" predictions of how engineering decisions affect road safety (Hauer, 1988). Hauer identified several practices that, although ostensibly based on safety concerns, appeared to have little solid evidence supporting their effectiveness. These included guidelines for selecting radii on horizontal curves, for providing sight distances on crest vertical curves, and for providing sight distances at intersections. In each case the original development of the guidelines followed a similar pattern: a crash mechanism was hypothesized and modeled using principles from basic physics, geometry, and human factors, and then constraints on designs were derived using the model and empirical values for model parameters. For example, guidelines for selecting curve radii were (and are) developed from the geometry of circular arcs, the physics of objects undergoing uniform circular motion, and empirical values for the ranges of centripetal acceleration that drivers find comfortable. Hauer's argument was not that the resulting guidelines were *prima facie* unreasonable but that (1) empirical evidence for their relation to crash occurrence was at best limited and often non-existent, and (2) the guidelines provided little help for the sorts of decisions faced by engineers, such as determining how safety might change when increasing a horizontal curve's radius from 1500 feet (457 m) to 2000 feet (610 m).

With the first edition of the *Highway Safety Manual* (HSM) (AASHTO 2010) Hauer's challenge was partially met. The core methodology that supports the HSM uses empirically-fitted regression models, called safety performance functions (SPF), to predict crash frequencies under specified base conditions and uses empirically-developed crash modification factors (CMF) to predict how crash frequencies would change when conditions change. Since geometry, basic physics, and human factors remain part of the scientific canon a better characterization of Hauer's original call might be for "evidence-based" methods, similar to

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https://doi.org/10.1016/j.aap.2019.06.015
Received 18 March 2019; Received in revised form 19 June 2019; Accepted 29 June 2019
Available online 17 July 2019

the call for evidence-based decision-making in medicine, and elsewhere. At the center of evidence-based medicine (EBM) is a ranking of the quality of evidence used to guide recommendations for patients' treatments. Roughly, the strongest evidence is provided by randomized controlled trials (RCTs), with observational studies, such as case-control and cohort studies, forming a second rank. Evidence of disease mechanisms, such as might be identified in laboratory studies, is given a yet lower rank and this is justified by citing examples where mechanism-based treatments were found to be ineffective when evaluated using RCTs (Howick, 2011). In road safety, Hauer (2013) has made a similar argument, pointing out that the mechanism-based guidelines regarding sight distances on crest vertical curves appear to have no detectable effect on safety (Urbanik et al., 1989). Since discoveries of disease mechanisms have occasionally led to more effective treatments, EBM has sparked a vigorous, and continuing, debate about the relative value of statistical versus mechanism-based research, both for understanding disease causation and for guiding treatment decisions. A plausible synthesis is the Russo-Williamson thesis (Russo and Williamson, 2007) which can be stated as follows:

> **RWT**. In order to establish that A causes B in medicine one normally needs to establish two things. First, that A and B are suitably correlated–typically, that A and B are probabilistically dependent, conditional on B's other known causes. Second, that there is some underlying mechanism linking A and B that can account for the difference that A makes to B. (Clark et al. 2014, p. 343.)

Here, statistical associations and mechanisms are not seen as competitors but as allies; the strongest evidence comes from mutually-supporting statistical and mechanism-based research. Knowledge of a plausible mechanism supports decisions regarding external validity and inferences from populations to individuals (Clark et al., 2014), and can also guide the design of future research (Bluhm, 2010). Returning to road safety, Elvik's (2007) application of Bradford-Hill's criteria identified both reliable associations and plausible mechanisms as important evidence for establishing causation, while Davis et al. (2017) guided selection of predictors for regression analyses using a simple causal mechanism that connected traffic density to rear-ending crash risk. Currently, however, road safety research in the United States focuses primarily on extending the SPF/CMF paradigm, although interest in developing causal models has been expressed (Bonneson and Ivan, 2013).

The statistical estimates that support the HSM are summaries describing the times and places providing data, but are also presented as potentially applicable to other situations. The HSM offers guidance on re-calibrating SPFs to reflect local conditions, but rigorous justifications for "transferring" CMFs to new times/places are, at best, incomplete (Hauer et al., 2012). This is not surprising, since determining the transferability of a CMF is essentially a special case of assessing the external validity of an empirical result (Campbell and Stanley, 1966), which can require additional knowledge going "far beyond the statistical study itself" (Cartwright, 2011). What is this additional knowledge? The original work by Campbell and Stanley identified several "threats" to external validity and rated different research designs as to their vulnerability to these threats. Avoidance of the threats then provided a necessary, but not sufficient, condition for external validity. More recently, Cartwright and Hardie (2012) have argued that "…facts about the causal role the policy plays and facts about the support factors that must be in place for the policy to work" are needed to conclude "it will work here" from evidence that "it works somewhere" (p. 6). They described several modeling formats, such as signed graphs and "causal pancakes," with which a modeler could express qualitative background knowledge about a system of interest. Parkkinen et al. (2018) argue convincingly that to establish the external validity of a claim that A causes B it is necessary to establish that "the mechanism responsible for B in the target population is sufficiently similar to that responsible for B in the study population" (p. 16). Bareinboim and Pearl (2013, 2014) have identified sufficient conditions, or warrants, that would justify "transporting" an estimated causal effect from one situation to another. They were able to show that if (1) the operation of a modification could be represented using what they call a probabilistic causal model and, (2) it was possible to identify which model variables differed between two situations, then the transportability of a causal effect could be assessed and, when feasible, formulas for computing the transported effect could be derived.

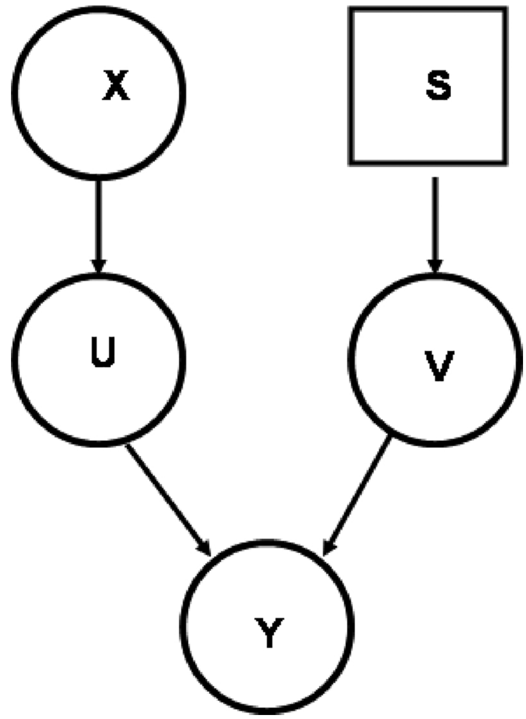

Fig. 1. Graphical model illustrating how a crash outcome Y responds to a safety-related modification X and to situational differences S, through variables U and V.

Regarding CMFs, the issue of transferability is likely to become even more important as automated vehicles improve in capability and increase in market share. Current CMF estimates are essentially statistical summaries of the driver and vehicle characteristics prevailing during the last 10–20 years, and as these conditions change there is no reason to expect that current CMF estimates will remain valid. However, exploratory results using the Bareinboim and Pearl theory have shown how a CMF characterizing one situation might be re-calibrated to reflect a different situation (Davis and Gao, 2019). To illustrate Bareinboim's and Pearl's approach, Fig. 1 represents a hypothetical crash mechanism as a directed acyclic graph (DAG). Node Y denotes a crash outcome and is a function of two inputs U and V. X denotes the presence/absence of a safety-related modification while S indicates which features might differ between different situations. Specification of how the probability distributions for U, V, and Y vary as functions of their parent nodes then gives a probabilistic causal model. In Fig. 1 X affects the outcome through the variable U while V reflects how situations differ. The problem is to transport knowledge of X's impact, learned in situation $S = s$, to a new situation $S = s^*$. For models with Fig. 1's structure Pearl's and Bareinboim's results can be used to show that (1) the V-specific causal effect of manipulating X is potentially "transportable" between the two situations, and (2) a formula for computing the transport can be derived. Using Pearl's (2009) notation, where do$(X = x)$ represents the action of experimentally setting the modification to $X = x$, the structure of the graph in Fig. 1 implies that the outcome Y is conditionally independent of the situational differences given values of the input V. That is

$$P[Y = y|do(X = x), S = s, V = v] = P[Y = y|do(X = x), S = s^*, V = v] \quad (1)$$

Pearl's and Bareinboim's Corollary 1 then implies that a formula for transporting experimental findings from situation S = s to situation S = s* is

$$P[Y = y|do(X = x), S = s^*] = \sum_{v} P[Y = y|do(X = x), S = s, V = v]P[V = v|S = s^*] \quad (2)$$

That is, when the mechanism relating Y to X and S has a structure similar to that in Fig. 1, knowledge of how V varies in a new situation S = s*, together with experimental knowledge of how Y responds to X = x at different levels of V, learned in the original situation, can be combined to predict how Y will respond to the causal intervention do (X = x) in the new situation.

In short, possession of a probabilistic causal model that essentially *explains* how a modification affects an outcome is a prerequisite for applying Bareinboim's and Pearl's transportability analysis. But while reliable predictions can often be had by identifying and exploiting useful empirical regularities, even when the processes generating those regularities are poorly understood (and an impressive array of statistical tools supports this) explanatory reasoning has proven more difficult to routinize (Psillos, 2002). The ultimate nature of scientific explanation remains a topic of debate among philosophers (e.g. Woodward, 2008) but fortunately, as the history of science shows, it is not necessary to resolve this debate in order to develop good explanations. For our purposes, we can take an explanation of a fact to be a model of a process that, together with additional facts and hypotheses, in some sense entails the fact to be explained. For example, a simple model of the relationship between runny noses and colds could be the generalization that colds tend to cause runny noses. This together with the hypothesis that I have a cold would then explain the fact that I have a runny nose. Note that other explanations might be possible; I might have eaten very spicy food for lunch, and additional research might be needed to decide which possibility is best. If I have a cold then I should also have a fever, which can be checked with a thermometer, while lunch fare might be revealed by spots on my tie. Using probabilistic causal models, Halpern and Pearl (2005) defined an explanation for an individual fact as an assignment of values to a subset of the model's variables that, subject to several technical conditions, provided a cause for the fact to be explained. Development and evaluation of explanations has also been a topic of research in both artificial intelligence and cognitive science and a process, sometimes called abductive inference, offers a framework for this (e.g. Josephson and Josephson, 1994; Thagard, 2012; Douven, 2017). Attributed sometimes to the philosopher Charles Sanders Peirce, in abductive inference we propose (guess) a hypothesis that could explain a fact. We then deduce observable consequences from this hypothesis and test the hypothesis by comparing these predicted consequences to additional observations (Psillos, 2011). If we wish to apply abductive inference to explain a crash modification effect we will require a framework that allows us to (a) explicitly represent our hypotheses about how the modification might work and (b) derive testable consequences from our hypotheses. In road safety, methodologies that support explanatory research are less well-developed than those supporting prediction-oriented modeling but, as a step towards a more comprehensive treatment, what follows is a case study where a probabilistic causal model, coupled with Monte Carlo simulation, is used to develop an explanation for a CMF.

2. Pedestrian hybrid beacons and pedestrian crashes

A pedestrian hybrid beacon (PHB) is "a special type of hybrid beacon used to warn and control traffic at an unsignalized location to assist pedestrians in crossing a street or highway…" (FHWA, 2011), and there is reasonable statistical evidence indicating that installing PHBs at uncontrolled crossings reduces the frequency of pedestrian crashes. In an empirical Bayes before/after study of PHBs installed at 21 sites in Tucson, AZ, Fitzpatrick and Park (2010) reported an estimated CMF for pedestrian crashes of 0.308 (69.2% reduction), with an associated standard error of 0.155. In another empirical Bayes before/after study that used 27 PHB sites, some of which were also in Tucson, Zegeer et al (2017) estimated a CMF of 0.244 (75.6% reduction) for pedestrian crashes, with an associated standard error 0.128.

Many jurisdictions give right-of-way to pedestrians once they have entered a marked or unmarked crosswalk, as long as drivers are given sufficient distances to stop and as long as right-of-way is not controlled by a traffic signal. Studies using staged crossing attempts, however, have found locations where substantial numbers of approaching drivers failed to yield to pedestrians (e.g. Fitzpatrick et al., 2014; Bertulis and Dulaski, 2014). Also, reviews of vehicle/pedestrian crash reports have shown that drivers' "Failure to yield right-of-way" is frequently cited as a contributing factor (e.g. Shankar, 2003; Kimley-Horn, 2017). Finally, high rates of driver yielding have been observed at PHBs, in both before/after and in cross-sectional studies, and high rates of PHB use by pedestrians have also been reported (Fitzpatrick et al., 2014, 2016). This leads to a first hypothesis about how PHBs reduce pedestrian crash frequency: At uncontrolled crossings crashes tend to occur when pedestrians attempt to cross but drivers fail to slow or stop, despite having adequate stopping distances. After the PHB is installed pedestrians tend to use it, drivers tend to stop as required, and crashes are prevented. If this is the case then it should be possible to generate CMFs similar to those reported in the literature by simulating vehicle/pedestrian encounters consistent with this hypothesis.

If the scenario proposed by the first hypothesis, that crashes tend to occur when careful pedestrians encounter non-braking drivers, is typical then this should be seen when crashes are investigated in detail. In particular, there should be no evidence of pre-impact braking on the part of drivers and the collision speeds of vehicles should tend to mirror the running speeds on the roads where crashes occurred. Regarding whether or not drivers brake when encountering pedestrians, a review of the crash reports from an in-depth investigation of fatal pedestrian crashes in Adelaide, Australia showed pre-impact skid marks in approximately 24% of the cases, indicating that, even in very serious collisions, at least some drivers braked prior to collision (McLean et al., 1994). Regarding the relationship between impact speeds and running speeds, Fig. 2 plots estimated impact speed versus speed limit for vehicle/pedestrian crashes investigated in NHTSA's Pedestrian Crash Data Study (PCDS) (Chidester and Isenberg, 2001). The data shown in Fig. 2 are for pedestrians between ages 15 and 60 and for vehicles going straight (not turning). A fitted linear relationship between speed limit and impact speed, also shown in Fig. 2, gave an estimated slope of 0.67

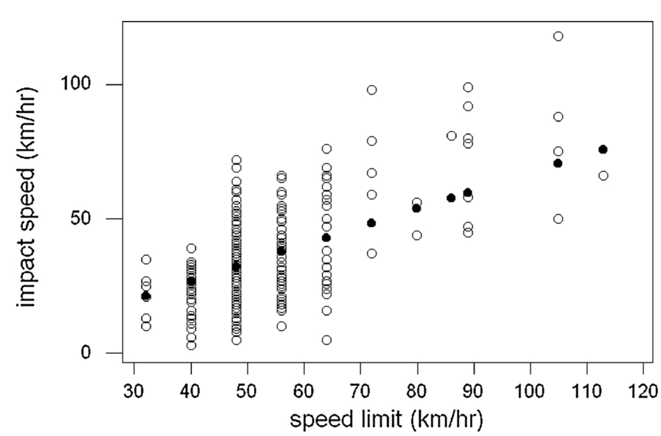

Fig. 2. Impact speed versus speed limit for pedestrians crashes in the PCDS. The dark circles show the fitted linear relationship.

which indicates that the estimated impact speeds tended to be lower than the associated speed limits, and raises the possibility that some drivers might have decelerated before collision. Finally, Randles et al. (2001) were able to estimate impact speeds from video recordings of 13 vehicle/pedestrian crashes which occurred on a busy arterial in Helsinki. The speed limit on this road was 50 km/hour but in all cases the estimated impact speeds were less than the speed limit. Overall then, these findings suggest that we should be careful about equating drivers' failure to yield to pedestrians, as seen in field studies, with a failure to brake when a collision appears imminent.

Turning to pedestrian behavior, in the field studies staged crossings were initiated only when drivers had adequate stopping distance. The video study of pedestrian crashes in Helsinki, however, found that the crashes tended to result when pedestrians entered the roadway more or less independently of vehicle position, and that the collisions involved freely-moving vehicles, as opposed to vehicles in platoons (Pasanen and Salmivaara, 1993). This suggests that a reasonable simulation model should allow for "heedless" pedestrians as well as for evasive action on the part of drivers, and should employ a traffic model that allows for a mixture of platooned and freely-moving vehicles.

3. Simulating vehicle/pedestrian encounters

A collisions between a vehicle and a pedestrian can occur when a pedestrian is walking or running along the edge of a road, when a pedestrian is crossing at an intersection and is hit by a vehicle turning left or right, or when a vehicle leaves the roadway and strikes a pedestrian on the roadside (Stutts et al., 1996). Crashes also occur when pedestrians attempt to cross roads and are struck by vehicles following straight paths and, arguably, these are the crashes a PHB should prevent. Fig. 3 shows a scene diagram for a crash investigated by the PCDS, where an adult pedestrian, running or jogging across a road in a crosswalk, was struck by an SUV. The relevant speed limit was 30 mph (48.4 km/hr) and the NHTSA investigators estimated the impact speed at about 19 mph (30.6 km/hr). This particular crash occurred at a signalized intersection, but it has kinematic similarities with the type of encounters a PHB ought to address.

Fig. 4 depicts the type of event captured by our simulation model. A car, initially traveling at speed v_1, is a distance d_1 from the conflict zone when a pedestrian initiates crossing. After a reaction time r_2 the pedestrian enters the road, traveling at speed v_2. The pedestrian then enters the conflict zone after traveling a distance d_2 from the pavement edge and exits the conflict zone after traveling a distance d_2+w from the pavement edge. If the driver takes no action then the vehicle continues at its initial speed v_1. If the driver attempts to slow or stop then,

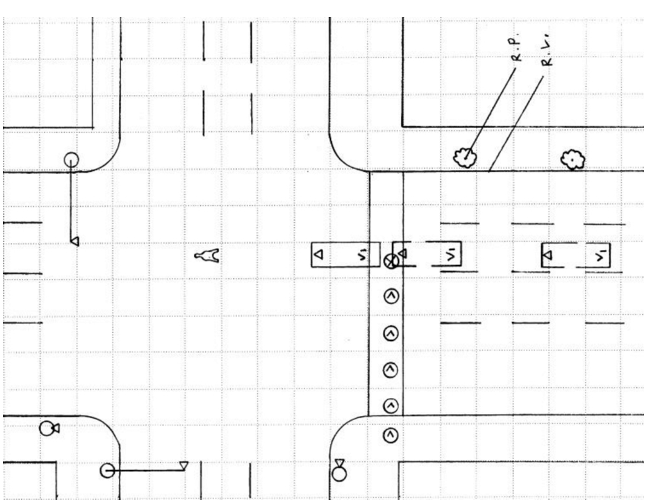

Fig. 3. Post-crash scene diagram from PCDS case 72639p97.

after a reaction time r_1, the vehicle begins decelerating at constant rate a_1. A crash occurs if the vehicle arrives at the conflict zone while the pedestrian is in this zone. Otherwise, if the vehicle stops before reaching the conflict zone, or arrives either before the pedestrian enters or after the pedestrian exits the zone, a crash does not occur. If a crash occurs then v_i denotes the vehicle's speed at the point of impact. Formally:

Time pedestrian arrives in the conflict zone: $tped_1 = r_2 + d_2/v_2$
Time pedestrian exits conflict zone: $tped_2 = r_2 + (d_2+w)/v_2$
Vehicle's arrival time:

$$t_0 = \begin{cases} d_1/v_1,\ if\ \ no\ braking\ or\ d_1 < r_1 v_1 \\ r_1 + \left(\dfrac{v_1 - \sqrt{v_1^2 - 2a_1(d_1 - v_1 r_1)}}{a_1}\right),\ if\ braking\ and\ r_1 v_1 \le d_1 \\ \le r_1 v_1 + v_1^2/2a_1 \\ \infty,\ if\ braking\ and\ d_1 > r_1 v_1 + v_1^2/2a_1 \end{cases} \tag{3}$$

A crash occurs if $tped_1 < t_0 < tped_2$. The impact speed is then given by

$$v_i = \begin{cases} v_1,\ if\ \ no\ braking\ or\ d_1 < r_1 v_1 \\ \left(\sqrt{v_1^2 - 2a_1(d_1 - v_1 r_1)}\right),\ if\ braking\ and\ r_1 v_1 \le d_1 \le r_1 v_1 + v_1^2/2a_1 \\ 0,\ if\ braking\ and\ d_1 > r_1 v_1 + v_1^2/2a_1 \end{cases} \tag{4}$$

If values for the variables d_1, v_1, r_1, a_1, d_2, v_2, r_2, w, and the driver's braking decision are known, then whether or not a collision occurs, and the resulting impact speed, can be computed using Eqs. (3) and (4).

Eq. (4) predicts the impact speed in a vehicle/pedestrian collision. To link the impact speed to pedestrian injury severity a logit model, developed for pedestrians ages 15–60, is used (Davis and Cheong, 2019). Using the KABCN injury coding system, a Possible injury corresponds to injury codes C or N, a Probable injury corresponds to codes A or B, and a Fatal injury corresponds to code K. Using injury versus impact speed data from the PCDS, supplemented by an exogenous sample of pedestrian injury severities occurring in the Twin Cities of Minnesota, Davis and Cheong fit several logit models, with different assumptions about measurement error and using both frequentist and Bayesian methods. The following model, representative of those fitted models, was used in the simulations:

$$P[probable \vee fatal\ injury|v_i] = \frac{\exp(0.071v_i - 1.89)}{1 + \exp(0.071v_i - 1.89)}$$
$$P[possible\ injury|v_i] = 1 - P[probable \vee fatal\ injury|v_i] \tag{5}$$

In Eq. (5) v_i is the impact speed in kilometers/hour.

Fig. 5 represents the simulation model as a DAG; the symbols appearing in Fig. 5 are defined as follows:

Driver/vehicle variables:
π_1 – fraction of braking drivers
d_1 – vehicle's initial distance from conflict zone
v_1 – vehicle's initial speed
r_1 – driver's reaction time
a_1 – driver's braking deceleration
Pedestrian variables:
π_2- fraction of careful pedestrians
d_2 – pedestrian's initial distance from conflict zone
v_2 – pedestrian's speed
r_2 – pedestrian's reaction time
Other variables:
w – conflict zone width
v_i – impact speed

The model was coded to be run by WinBUGS (Lunn et al., 2013), a program for simulating conditional probability distributions using Markov Chain Monte Carlo. A standard Monte Carlo simulation would generate a random sample of vehicle arrivals (i.e gaps), a subset of

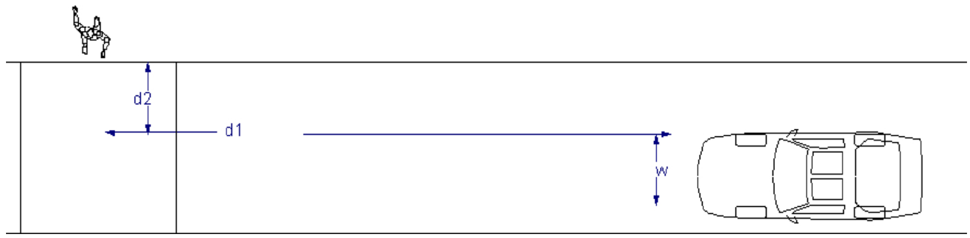

Fig. 4. Vehicle/pedestrian encounter at an uncontrolled crosswalk.

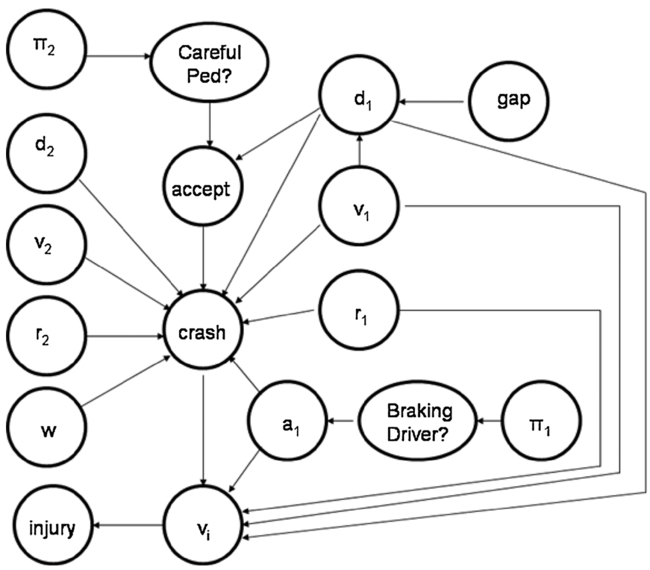

Fig. 5. Pedestrian crash simulation model represented as a directed acyclic graph.

these would be gaps accepted by pedestrians, and a subset of those would result in collisions. By using WinBUGS ability to condition on specific outcomes, such as a gap being accepted, it is possible to simulate only vehicle-pedestrian encounters and so estimate the probability that a pedestrian is involved in a collision rather than the probability that a traffic gap results in one. It is also possible, if desired, to condition on the occurrence of a collision and generate a Monte Carlo sample consisting only of crash events. The model was set up to simulate vehicle/pedestrian encounters on a hypothetical two-lane road, with both lanes being 12-feet (3.6 m) wide. Originally, encounters in both lanes were simulated but since almost all simulated collisions occurred in the far lane interest was focused there. Traffic headways followed a shifted exponential distribution, with a minimum headway of 2 s and the flow of freely-moving (non-platooned) traffic was 200 vehicles/lane/hour. The nominal speed limit was 30 mph (48.4 km/hr) and vehicle speeds were treated as normal random outcomes with a mean speed of 35 mph (56.5 km/hr) and a standard deviation of 5 mph (8.1 km/hr). Driver reaction times were treated as lognormal random outcomes with a mean of 1.07 s and a standard deviation of 0.248 s, consistent with findings reported by Koppa et al (1996) for tests of surprised emergency braking. Driver braking decelerations were treated as lognormal outcomes with a mean of 0.63 g and a standard deviation of 0.08 g, again consistent with statistics from Koppa et al (1996). Pedestrian walking speeds were taken to be normal random outcomes with a mean of 5.0 feet/second (1.52 m/second) and a standard deviation of 0.9 feet/second (0.27 m/second), roughly consistent with the data for "non-elderly" pedestrians reported by Fugger et al (2001). The pedestrian reaction time r_2 was set to zero, meaning that a driver's reaction phase began when the pedestrian entered the roadway.

4. Developing explanations

As noted above, the initial hypothesis is that the staged pedestrian crossings used in field studies are representative of pedestrian behavior, and that failures by drivers to brake when encountering pedestrians is a main cause of collisions. This hypothesis then suggests that PHBs reduce the frequency of collisions by increasing the fraction of braking drivers. If this hypothesis is accurate then it should be possible to simulate observed CMFs by changing the fraction of braking drivers. In the simulation model a careful pedestrian was defined as being similar to the staged pedestrians in Brewer et al (2015), who rejected a gap if the vehicle's initial distance (d_1) was less than the AASHTO design stopping distance, 200 feet (61 m) for a 30 mph (48.6 km/hr) speed limit. A simulated careless pedestrian accepted the first gap greater than the following headway for platooned vehicles, 2.0 s. A braking driver was defined as one who, after reaction time r_1, decelerated at rate a_1, while a non-braking driver maintained a constant speed v_1. Again letting π_1 denote the fraction of braking drivers and π_2 denote the fraction of careful pedestrians, WinBUGS was used to simulate crash occurrences for different combinations of π_1 and π_2. For each combination, 500,000 accepted gaps were simulated and the number that resulted in crashes according to Eq. (3) recorded. The simulated crash probabilities are displayed in Table 1.

Since a crash modification factor can be interpreted as the ratio of crash probabilities from two different situations (Davis, 2014) the collision probabilities listed in Table 1 can be used to compute the simulated CMFs that would result from changes in driver or pedestrian behavior. For example, if before a PHB is installed 100% of pedestrians are careful but no drivers brake, while after the PHB is installed all pedestrians are still careful but now 80% of drivers brake, the associated CMF would be

$$\begin{aligned} \text{CMF} &= (\text{Collision Probability After})/(\text{Collision Probability Before}) \\ &= (.00585)/(.0287) = 0.204 \end{aligned}$$

If we accept simulated CMFs between 0.2 and 0.35 as being roughly consistent with the CMFs estimated in the before/after studies then the initial hypothesis, that pedestrians are careful and that PHBs achieve their effect by increasing the fraction of braking drivers, provides an explanation of the observed CMFs. Unfortunately though, other changes in π_1 and π_2 also lead to simulated CMFs consistent with those from the before/after studies. For example, if 80% of drivers brake both before

Table 1
Simulated vehicle/pedestrian collision probability as a function of proportions of braking drivers and careful pedestrians.

		Fraction of careful pedestrians (π_2)					
		0	.2	.4	.6	.8	1
Fraction of braking drivers (π_1)	0	.0627	.0563	.0497	.0431	.0362	.0287
	.2	.0509	.0461	.0408	.0353	.0295	.0232
	.4	.0399	.0359	.0317	.0271	.0226	.0173
	.6	.0291	.0254	.0225	.0190	.0154	.0116
	.8	.0177	.0152	.0132	.0109	.00845	.00585
	1	.00639	.00543	.0042	.00272	.00149	.000086

and after installation of a PHB but the fraction of careful pedestrians changes from 0% to 100%, the simulated CMF would be

$$\text{CMF} = (\text{Collision Probability After})/(\text{Collision Probability Before}) = (.00585)/(.0177) = 0.331$$

which is again roughly consistent with the estimated CMFs. One can verify that other changes in driver or pedestrian behavior also lead to plausible CMFs. Clearly, the estimated CMFs do not by themselves provide enough information of identify a best explanation.

As noted earlier, developing an explanation by abductive inference involves a cycle of hypothesis formation, prediction, and testing of predictions. If, in the absence of a PHB, most drivers fail to brake for pedestrians then this should be reflected in collision impact speeds, and a viable explanation should also predict impact speeds similar to those which actually occur. Lacking detailed data on impact speeds it is still possible to use pedestrian injury severity as a proxy. That is, the initial (before PHB) values for π_1 and π_2 should reproduce observed distributions of pedestrian injury severity. Toward this end, 2764 police-reported collisions between adult (ages 15–60) pedestrians and sedans, SUVs, pickups, or small vans were identified using Minnesota's Crash Mapping tool (MNCMAT). These collisions all occurred in the Twin Cities metropolitan region during the years 2008–2015. The crash records included estimates of injury severity made by the investigating officers using the KABCN system, as shown in Table 2. In Table 2 the 'All' column shows the injury distribution for all crashes in the sample, the "Straight-Ahead" column shows the distribution for only those crashes where the vehicles were moving forward in straight lines, and the "Straight-Ahead 30–35 mph Limit" column shows the Straight-Ahead distribution restricted to roads where the speed limit was 30 mph (48.4 km/hr) or 35 mph (56.5 km/hr).

Let n denote the number of collisions observed (the bottom row in Table 2) and Y denote the number of those collisions resulting in Possible injuries. If collisions are independent of each other then Y is a binomial random variable with probability parameter

$$p(\pi_1, \pi_2) = P[Possible|\pi_1, \pi_2] = \int P[Possible|v_i] f(v_i|\pi_1, \pi_2) dv_i \quad (6)$$

and the log likelihood function for the observed number of possible injuries is proportional to

$$y \ln(P[Possible|\pi_1, \pi_2]) + (n - y)\ln(1 - P[Possible|\pi_1, \pi_2]) \quad (7)$$

Using WinBUGS it is also possible to condition on the occurrence of a collision and so, via Eq. (5), numerically evaluate the integral on the right-hand side of Eq. (6). The hypothesis that no drivers brake and all pedestrians are careful leads to a probability of Possible injury of p ($\pi_1 = 0, \pi_2 = 1$) = 0.09, i.e. only 9% of collisions result in Possible injury, which is inconsistent with the Possible injury rows of Table 2. Assuming that the observed injury distributions are due to mixes of braking drivers and careful pedestrians, one can search for combinations of values for π_1 and π_2 that best fit an observed injury distribution. Using the R2WinBUGS interface (Lunn et al., 2013) the simulation model was embedded in an R function that evaluated Eq. (7) for given values of π_1 and π_2. Using the Nelder-Mead algorithm implemented in the R function optim (Teetor, 2011) approximate maximum likelihood (ML) estimates for π_1 and π_2 were computed. When using the rightmost column of Table 2 (n = 940 and y = 481) ML estimates of $\pi_1 = 0.98$ and $\pi_2 = 0.32$ were found. The log-likelihood function was then evaluated on a grid of values surrounding the ML estimates, and Table 3 shows the variation in log-likelihood when the fraction of braking drivers ranges between 0.95 and 1.0 while the fraction of careful pedestrians ranges from 0 to 1 in increments of 0.1.

Table 2
Distribution of pedestrian injury severities: Adults in MnDOT's Metro district.

Injury Category	KABCN Range	Vehicle Movement/Speed Limit		
		All	Straight-Ahead	Straight-Ahead 30-35 mph Limit
Possible	N-C	1560 (56.4%)	573 (49.1%)	481 (51.2%)
Probable	B-A	1141 (41.3%)	551 (47.2%)	445 (47.3%)
Fatal	K	63 (2.3%)	43 (3.7%)	14 (1.5%)
Total		2764	1167	940

Table 3 shows a ridge in the likelihood function corresponding to 98% of drivers braking and between 0% and 30% of pedestrians being careful. Outside this range the associated likelihoods tend to be one or more orders of magnitude lower which implies, for non-informative priors on π_1 and π_2, that the corresponding posterior probabilities would also be at least an order of magnitude lower. The best explanation then of the injury distribution given in the rightmost column of Table 2 would be that it results when almost all drivers brake and when the fraction of careful pedestrians ranges between roughly 0% and 30%.

If almost all drivers brake in response to immanent collisions both before and after installation of a PHB then the crash modification effect would be due to changes in pedestrian behavior. Fixing the fraction of braking drivers at 0.98, Table 4 shows how simulated collision probabilities change as the fraction of careful pedestrians ranges from 0 to 1.0, while Table 5 shows simulated CMFs associated with different changes in the fraction of careful pedestrians. Again using simulated CMFs in the interval (0.2, 0.35) as being roughly consistent with the estimates from the before/after studies, it appears that the hypothesis where 98% of drivers attempt to avoid collision by braking and the fraction of careful pedestrians changes from between 0% and 30% to between 80% and 90% can explain both the observed injury distribution and the observed CMFs.

5. Summary and conclusion

To summarize, although research following the SPF/CMF paradigm can lead to empirically-based predictions that support short-term decision-making, assessing the transferability (i.e. external validity) of CMFs requires knowledge of how CMFs work. In this paper a process for developing such explanations was illustrated, using a relatively simple probabilistic causal model to explain how pedestrian hybrid beacons might achieve their reported crash modification effects. Since the literature indicated that PHBs can affect both pedestrian and driver behavior, it was necessary to include both possibilities in the model. The hypothesis that 98% of simulated drivers attempt to brake, and that between 0% and 30% of simulated pedestrians are careful, best explained an observed injury distribution. Simulations where 98% of drivers attempted to brake, and where the simulated fraction of careful pedestrians changed from between 0% and 30% to between 80% and 90%, gave simulated crash modification factors similar to estimates reported in the literature.

A new working hypothesis would then be that most drivers attempt to brake when a pedestrian collision appears immanent and that PHBs achieve their crash reduction effects in large part by modifying pedestrian behavior. This is not so much a direct observation as it is an inference to the best explanation (Lipton, 2000; Psillos, 2002); the support for the hypothesis comes from its ability to explain the data at hand. The explanation, if valid, has implications for the transferability of the associated CMFs. Sites/locations having significant fractions of heedless pedestrians would be good candidates for PHBs; if most pedestrians are already careful then the safety impact of a PHB would be less. (Note though that a PHB could still improve pedestrian level of service even where most pedestrians are careful.) Like any scientific hypothesis the new working hypothesis should be subjected to additional rounds of testing via prediction and observation. Ideally, direct observation of driver and pedestrian behavior, in a sufficiently representative sample of crashes, would provide confirmation or

Table 3
Variation in the log likelihood function near its maximum.

		Fraction Careful Pedestrians (π_2)										
		0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
Fraction Braking Drivers (π_1)	.95	−662	−663	−667	−668	−673	−678	−683	−696	−731	−796	−1086
	.96	−656	−656	−660	−660	−664	−667	−670	−680	−709	−765	−1065
	.97	−653	−653	−653	−654	−656	−658	−660	−667	−682	−731	−1061
	.98	−651	−651	−651	−651	−652	−652	−653	−654	−664	−691	−974
	.99	−655	−655	−654	−654	−661	−652	−653	−652	−652	−660	−862
	1.0	−665	−665	−666	−666	−666	−664	−669	−670	−667	−663	−652

Table 4
Variation in collision probabilities and injury severities with respect to changes in pedestrian behavior when 98% drivers attempt to brake.

Fraction Careful Pedestrians	Collision Probability	Proportion Possible Injury
0	.0077	0.52
.1	.0069	0.52
.2	.0064	0.515
.3	.0058	0.51
.4	.0051	0.50
.5	.0043	0.49
.6	.0035	0.485
.7	.0028	0.47
.8	.0021	0.43
.9	.0014	0.37
1.0	.00062	0.155

Table 5
Simulated crash modification factors resulting from increases in percentage of careful pedestrians when 98% drivers attempt to brake.

		Fraction Careful Pedestrians After										
		0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
Fraction Careful Pedestrians Before	0	1	.90	.83	.75	.66	.56	.45	.36	.27	.18	.08
	.1	–	1	.93	.84	.74	.62	.51	.41	.30	.20	.09
	.2	–	–	1	.91	.80	.67	.55	.44	.33	.22	.10
	.3	–	–	–	1	.88	.74	.60	.48	.36	.24	.11
	.4	–	–	–	–	1	.84	.69	.55	.41	.27	.12
	.5	–	–	–	–	–	1	.81	.65	.49	.33	.14
	.6	–	–	–	–	–	–	1	.80	.60	.40	.18
	.7	–	–	–	–		–	–	1	.75	.50	.22
	.8	–	–	–	–	–	–	–	–	1	.67	.295
	.9	–	–	–	–	–	–	–	–	–	1	.44
	1.0	–	–	–	–	–	–	–	–	–	–	1

disconfirmation. For example, using WinBUGS' ability to generate conditional distributions, a simulation of 50,000 collisions from a population where the original fraction of braking drivers was 0.98 and the original fraction of careful pedestrians was 0.30 predicted that only about 3% of the collisions would involve careful pedestrians, while 83% would involve braking drivers. Observations of actual collisions that showed most drivers not attempting to brake or most pedestrians accepting gaps exceeding design stopping distances, or observations that pedestrian behavior is the same with and without PHBs, would then tend to falsify the new working hypothesis. Confirmation/disconfirmation via indirect observations is also possible. Fig. 6 shows the distribution of impact speeds in a set of simulated crashes as a mixture of two distributions, one centered near the mean initial speed (56 km/hr) and one centered at about 17 km/hr. Also shown are estimated impact speeds from 13 actual crashes occurring on a Helsinki street where the speed limit was 50 km/hr (Randles et al., 2001). In this case the Helsinki estimated speeds are roughly consistent with those from the braking region of the simulated crashes, with a tendency to come from the higher end of this region. Testing via prediction of other crash

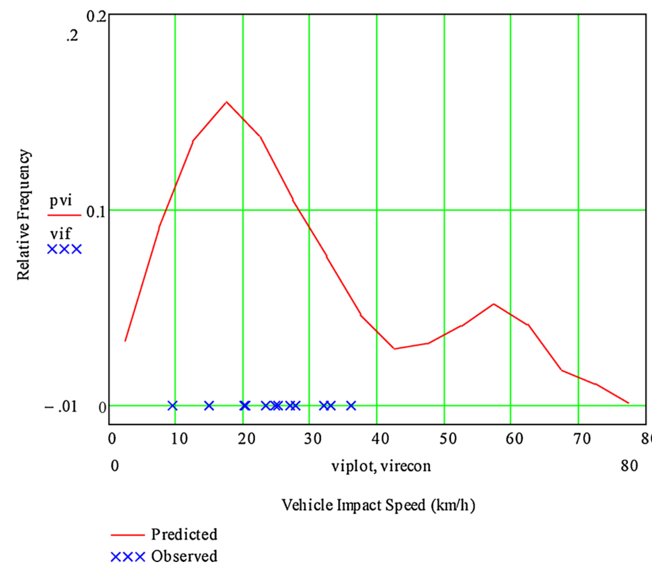

Fig. 6. Distribution of impact speeds predicted by simulation model, where fraction of braking drivers was 0.98 and fraction of careful pedestrians was 0.30. Also shown are estimated impact speeds from 13 actual crashes, observed in Helsinki.

features, such as the gaps accepted by pedestrians, or vehicle distances and speeds when pedestrians initiate crossings, is also possible when and if data collection methods make this feasible.

In conclusion, what does this example have to say to those wanting to explain a crash modification factor? (1) The starting point was the availability of roughly consistent estimates for the target CMF from higher-quality studies. If the current state of knowledge gives a poor idea of a CMF's magnitude there is little point in trying to explain it. In the PHB example the starting point was similar estimates from two (possibly overlapping) before/after empirical Bayes studies. (2) The second step was to gain insight into the relevant crash events by reviewing crashes that had been investigated and reconstructed in detail, the goal being to ensure that the proposed explanatory mechanism described actual crashes. In this case, the Adelaide study of fatal pedestrian crashes (Mclean et al., 1994), NHTSA's PCDS (Chidester and Isenberg, 2001), and the Helsinki video study (Pasanen and Salmivaara, 1993; Randles et al., 2001) were used. (3) Next, the structural equation describing crash occurrence was formulated, together with its proximal inputs, in this case the variables d_1, v_1, r_1, a_1, d_2, v_2, r_2, w, and the driver's braking decision. (4) The proximal variables were then related to traffic conditions and representations of driver behavior, leading to the graphical model shown in Fig. 5. This step included searching the literature for plausible probability distributions for model inputs. (5) Finally, the variables π_1 and π_2 were added to represent explicit hypotheses about how PHBs might affect crash occurrence. Coding the model to run in WinBUGS then supported making predictions about crash rates and injury severities that could be compared to data. To some extent the final form of the explanatory model depended on the hypotheses being tested. Model formulation was actually less difficult than was (is) finding quality data with which to test the model.

Declaration of Competing Interest

None.

Acknowledgements

Robyn Bluhm's presentation on mechanisms and evidence-based medicine, at the Minnesota Center for Philosophy of Science, was an important stimulus for this work. This research was supported in part by the Minnesota Dept. of Transportation and in part by the US Dept. of Transportation through the Roadway Safety Institute at the University of Minnesota. However, the contents of this paper are strictly the responsibility of the author.

Appendix A. Summary Statistics Related to Pedestrian Crash Simulation Model

See Table A1, Table A2, Table A3, Table A4

Table A1
Summary statistics from Koppa et al. (1996).

Driver Reaction Time (seconds)				
Condition	Car	Sample Size	Mean	St. Dev
Surprise	Own	10	1.04	0.27
On Road	Own	11	1.10	0.21
Driver Braking Rate (g-units)				
Condition	Car	Sample Size	Mean	St. Dev
Surprise	TTI	38	−0.63	0.08

Table A2
Example summary statistics from simulations: 80% careful pedestrians, 20% braking drivers.

Variable	All Gaps		Accepted Gaps		Crashes Only	
	Mean	St. Dev.	Mean	St. Dev	Mean	St. Dev
P[accept]	.92	–	–	–	–	–
P[crash]	.027	–	.0295	–	–	–
P[careful ped]	.80	–	.783	–	.62	–
P[braking driver]	.20	–	.20	–	.011	–
P[possible injury]	.0029	–	.0032	–	.107	–
P[probable injury]	.024	–	.026	–	.893	–
d_1 (feet)	1028	944	1105	945	227	54.5
a_1 (g-units)	.63	.08	.63	.08	.63	.08
r_1 (seconds)	1.07	.248	1.07	.248	1.07	.249
v_1 (ft/sec)	51.4	7.32	51.6	7.3	53.8	7.1
v_2 (ft/sec)	5.0	0.9	5.0	0.9	4.5	.86

Table A3
Summary statistics for Minnesota Twin Cities crashes: Pedestrians age 15–16, speed limits 30 mph-35 mph.

Junction Type	%	Speed Limit	%	Traffic Control	%	Vehicle Type	%
Not Intersection	41	30 mph	88	Signal	33	Sedan	64
T	9	35 mph	12	Stop sign	17	Pickup	6
4-Leg	37			None	47	SUV	19
Related	8			Other	3	Van	10
Other	5					Other	1

Lighting	%	Road Surface	%	Road Design	%
Day	47	Dry	69	Divided	3
Dawn	2	Wet	18	1-way	8
Dusk	3	Snow	6	4,6 lane	38
Dark	43	Slush	1	3 lane	1
Other	5	Ice	5	2 lane	43
		Other	1	Other	7

Table A4
Driver and pedestrian characteristics for Minnesota Twin Cities crashes: Pedestrians age 15–60, speed limits 30 mph-35 mph.

	Mean Age	Age St. Dev.	Percent Male	Percent Female
Drivers	39.7	17.1	63	37
Pedestrians	32.7	13.5	57	43

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