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Factors influencing the process of learning mathematics among visually impaired and blind people

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A R T I C L E I N F O

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Distance learning

A B S T R A C T

Effective instruction and comprehension of mathematics are important for achieving academic and professional success but are especially difficult for visually impaired individuals because of the inherent difficulty in managing structural information included in math formulae. An evaluation of an alternative for computer-aided math instruction and comprehension among visually impaired students was developed, and the evaluation included seven detailed categories of factors: behavioral, emotional, cognitive, social, distracting, motivational, and modeling factors. Then, the proposed method was used to compare the alternative teaching method, including problem decomposition and vector knowledge, to the classical teaching method with a teacher. The assessment of the impact of the developed approach on improving the process of teaching mathematics in a group of blind and visually impaired students was carried out by the completion of a questionnaire prepared by a psychologist.

The alternative teaching method achieved significantly better results in six of the seven proposed assessment categories. These experiments extend the knowledge base on the limitations and challenges associated with teaching and learning mathematics among blind people.

1. Introduction

Effective instruction and comprehension of mathematics, as shown by other authors [1,2], are important for achieving academic and professional success and are obligatory exclusively in technical professions. In the case of blind and visually impaired people, difficult access to mathematics instruction and appropriate computer aids is an additional cognitive barrier affecting various areas of life [3,4]. The difficulties in math accessibility encountered by blind people often lead to those individuals having more problems with science than sighted people. Sight disability affects social performance and academic achievement to similar extents. Almost 85% of what is learned socially is mediated through vision [5]. Additionally, Bogacka believes that deficits in the field of vision cause three types of difficulties: difficulties in cognitive processes, difficulties in physical and emotional activity, and additional difficulties associated with reduced self-esteem [6]. Analysis of the relevant literature indicates that blind and sighted children have the same intellectual ability. However, their methods of learning and assimilating learned information differ; blind children mainly learn through polisensory approaches and surrogate and analogous images [7]. These limitations can sometimes lead to faster discouragement and tiredness in blind and visually impaired people than in sighted people [8]. In summary, the education and cognitive development of blind and visually impaired students requires more effort, commitment and motivation.

In the past, the aim of the authors' research was to extend the Digital Accessible Information SYstem (DAISY) standard (books for visually impaired people) to allow visually impaired people to read math formulae [9], and this goal was partially achieved in the mathematics extension of the DAISY standard (ANSI/NISO Z39.86–2005 R2012 Specifications for the Digital Talking Book) [10,11]. Considering the significance of these issues, as discussed in the literature, one can find a distinct research effort aimed at qualitatively describing the phenomena accompanying the science of mathematics.

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The authors' motivation to begin research work related to the evaluation of computer-aided math instruction and comprehension was this previous experience working with blind students (9, 12–14) and the conclusions reached by the authors, which are described below.

Bouck and Weng [15] evaluated a single-subject, alternating treatment design to understand how the performance of three secondary students with visual impairments was impacted by accessing algebra via a digital textbook compared to accessing it via a traditional textbook. The results influenced the implementation of digital text in mathematics for visually impaired students and suggested that further research is needed. Regec [16] studied the ability of teachers in 11 counseling centers to use assistive technologies for blind and visually impaired people. He showed that the use of self-help tools was of the most benefit for blind people. Other authors [17] examined the application of assistive technologies for various topics in mathematics in a group of 82 visually impaired students. Personal computer and speaking systems were found to be the most effective for approximately 70% of the participants. Janu and Hari [18] developed a method of preparing materials for learning mathematics involving the use of an existing audio tactile device, which they examined in a group of 8 students. An increase in learning motivation was indicated by the high students' will to study, perseverance, enjoyment, and curiosity. In summary, the literature concerning assistive technologies used by visually impaired students demonstrates the effectiveness of using personal computers in combination with spoken mathematics, the increased motivation and the greatest benefit obtained by the use of self-help tools, and the need for further research in this area.

A separate group has been conducting research on aspects related to the learning of mathematics. These authors [2] discussed the problem of engagement in math and science, and their research was divided into two phases. In the first phase, the authors used qualitative methods to examine how students and teachers conceptualize math and science engagement and disengagement. In the second phase, they used qualitative information obtained from interviews to develop a new self-reported measure of engagement. This analysis supported the multidimensional construct of engagement and focused on behavioral, emotional/affective, cognitive and social engagement (four dimensions). Their research results show that social indicators are also key aspects of the new instructional emphases in both math and science classrooms on small group work. The question is whether social engagement is indeed a distinct dimension of engagement and whether it can be conceptualized as social, behavioral, emotional and cognitive engagement. The final conclusion from their research is that developing valid and reliable measures of engagement is especially important in math and science, which are critical for academic and professional achievement.

Other scholars have identified variables that can affect students' intrinsic motivation for learning mathematics [1]. The main point of their research was to examine the effects of the controlling strategies used by the mathematics teachers on students' intrinsic interest in mathematics. They divided their research into two studies: a cross-sectional study and a longitudinal path analysis of panel data collected from school-age children. The results clearly show that perceived autonomy had a positive impact on the perceived competence, intrinsic motivation and conceptual learning of the school-age children. Moreover, students with more motivation also perceived themselves as more competent in mathematics and achieved better results. Najafi et al. [19] analyzed the cooperative learning of mathematics and evaluated the method in a group of 40 visually impaired students. The results of t-tests indicated that cooperative learning could have a positive effect on the solving capacity of blind students for basic mathematical concepts.

The aim of the current paper was to identify factors affecting the process of learning mathematics in visually impaired and blind people. The main goal of the research was to determine whether the developed method providing an alternative description of mathematical expressions is better than the traditional method of learning mathematics for blind people. Moreover, this work examined whether the new approach to the structural description of mathematical expressions has a positive effect on blind and visually impaired students in terms of improving behavioral, cognitive, emotional, motivational and social aspects.

The article is organized in the following way. The Materials and Methods section includes assumptions, characteristics of the developed teaching, learning, and evaluation methods, and a description of the conducted experiment. The Results section presents the outcomes numerically and graphically. The Discussion section describes the significance of the results obtained, and the Conclusion section provides a summary.

2. Materials and methods

As mentioned in the Introduction, the main goal of this research was to determine the factors affecting the learning of mathematics by visually impaired and blind people. After reviewing the literature, 7 categories of factors were defined, and these categories are described below (section 2.2). For each of these categories, detailed research theses have been defined. Then, criteria for the research and control groups were defined, and experiments were conducted.

2.1. Characteristic challenges of teaching and learning mathematics among blind people

One of the features characterizing blind people is a reduced sense of security resulting from deprivation of a basic sense, i.e., vision, used in cognition of the world [20]. Often, blind people have different experiences with a complicated process of adaptation to their disability, which also reduces self-esteem. This, in turn, may be the basis for the faster resignation from attempts to solve an exercise that the blind person considers difficult. Thus, blind people may require more motivation and a stronger sense of security while solving such tasks. These phenomena led to the introduction of two additional evaluation categories regarding motivation and imitation.

As mentioned in the Introduction, the relevant literature indicates that blind and sighted children have the same intellectual ability. However, their methods of learning and assimilating learned information vary; blind children mainly learn through polisensory approaches and are taught with surrogate images and analogies [7]. These limitations may accelerate overload of the nervous system in blind people rather than in sighted people [8]. In turn, this rapid overload may result in a lower tolerance for distractors present while the student is solving a task. Therefore, additional evaluation categories related to distractors were also introduced. Moreover, neuroscience studies [21] have shown that blind people achieve better performance on all memory tasks than sighted people. The reason for this result is that blind people have trained themselves to apply a series of strategies to compensate for the absence of visual information. This superior ability is the result of the brain's adaptation to spatial, sequential, and verbal information. One major problem among blind and visually impaired people is maintaining the ability to concentrate and process sensory information. While blind people display a reliance on touch, smell and hearing for gaining information about the environment [22], people with visually impairments depend on both their visual sense and their other senses for such information [23]. Considering these results, to offer high-level math to blind students, it is necessary to optimize various aspects of the education process, including behavioral, cognitive, emotional, motivational, social, distracting and modeling factors.

2.2. Criteria for assessing the process of teaching mathematics to blind people

The assessment of the impact of the developed approach on improving the process of teaching mathematics to blind and visually impaired students was carried out by to completion of a questionnaire prepared by a psychologist. The experiment and the research groups are discussed in more detail in section 2.4. A previously reported questionnaire [2] was used for the assessment of the developed method. In the proposed approach, the assessment criteria were extended to those related to the nature of the method and specific challenges related to the learning of mathematics by blind people. Additionally, because the presented evaluation examined the alternative method for learning mathematics and not the teaching process itself, criteria for the teaching process were introduced. The groups of criteria together with their descriptions are presented below:

Categories based on an existing survey:

- Behavioral aspects – assessment of one's own process of learning mathematics. Examples of considered components: attention, participation, concentration and observance of rules in the classroom.
- Emotional aspects – assessment of the ability of a student to control their own states and control the influence of their behaviors and emotions on the assessment of the content presented in class.
- Cognitive aspects – assessment of the ability of a student to concentrate on information relevant to the goal. This category also includes the cognitive load, defined as the total number of cognitive processes triggered by short-term memory in a given period of time.
- Social aspects – assessment of the influence of the environment and relations with teachers and peers on learning mathematics.

Additional, newly introduced criteria are as follows:

- Distracting aspects – evaluation of the influence of external conditions on the acquisition of knowledge in mathematics.
- Motivational aspects – assessment of the impact of perception by others and reward for learning outcomes as a form of motivation to continue learning mathematics.
- Modeling aspects – evaluation of the impact of cooperation in a group of people with more experience in mathematics.

The questionnaire is presented in Table 1, and questions were assigned by the psychologist to specific categories. The survey was positively evaluated by a team of competent judges. Each question was rated on a 1–5 point scale. Then, the sum of points in each category was normalized (range, 1–100%). The summary of the results (Table 2) presents the medians of the standardized results obtained in each category.

During the research, the statistical analysis was performed by comparing the medians in each category (Wilcoxon test). The problem of verification of the null hypothesis H_0 that the distributions of certain variable X are the same in two dependent (related) populations can be solved e.g. by using the Wilcoxon signed-rank test proposed firstly in Ref. [24]. It can be treated as an alternative for the paired Student's t-test or the t-test for matched pairs. The procedure in the Wilcoxon signed-rank test can be described as follows. Having two dependent n-size samples $X = \{x_1, ..., x_n\}$ and $Y = \{y_1, ..., y_n\}$ we compute all the differences $d_i \underline{\underline{\text{def}}} x_i - y_i,\ i = 1, ..., n$ and exclude the ones being zeros. Next we rank absolute values $|d_i|,\ i = 1, ..., n_r$, starting from the smallest one, where n_r is the size of the reduced sample. Finally, we calculate the value of the test statistic (the so-called sum of the signed ranks) defined as (1):

$$W \underline{\underline{\text{def}}} \sum_{i=1}^{n_r} \text{sign}(d_i) \cdot R_i, \quad (1)$$

where R_i is the rank of $|d_i|$ and

$$\text{sign}(d_i) = \begin{cases} 1, & \text{if } d_i > 0, \\ -1, & \text{if } d_i < 0. \end{cases}$$

For small values of n_r the probability distribution of W has a complex form (see e.g. Ref. [25]) with mean $m_W = 0$ and variation

Table 1
A research questionnaire.

I	**Behavioral aspects**
	I remain focused in math classes.
	I answer questions in the classroom.
	I put my effort into learning.
	I continue despite the fact that something is difficult.
	I ask questions during class
	I do my homework on time.
	I talk about mathematical and scientific subjects outside the classroom.
	I try to learn more about the topics we're studying in the classroom.
	I do not take an active role in class activities (reverse scale).
	I do other things at the time when attention is expected from me (reverse scale).
	If I do not understand something, I stop it (reverse scale).
II	**Emotional aspects**
	I often like to solve difficult scientific and mathematical tasks.
	I wait impatiently for science and mathematics.
	I'm happy when I meet new scientific and mathematical topics.
	I try to understand what I'm learning during the lesson.
	I feel good during science and mathematical classes.
	I feel frustrated during science and mathematical classes (reverse scale).
	I think that strict/mathematics classes are boring (reverse scale).
	I do not want to participate in science/mathematics (reverse scale).
	I do not care about strict/mathematics (reverse scale).
	I'm irritated at science/math (reverse scale).
	I'm bad at science/math (reverse scale).
III	**Cognitive aspects**
	I try to make sure that what I do is correct.
	I think about different ways to solve a given problem.
	I try to combine new things I learn with the things that I've met before.
	I try to understand my mistake in case I do something wrong.
	When I study, I consider only those problems that I have previously done.
	I would rather ask for an answer than solve the task myself (reverse scale).
	I do not feel bad about it when I solve class tasks (reverse scale).
	When work is difficult, I only solve the easier part (reverse scale).
	I only do the necessary minimum (reverse scale).
IV	**Social aspects**
	I build on the ideas of other people.
	I try to understand the ideas of other people while studying mathematics and science.
	I try to work with other people who can help me learn math.
	I try to help other people who have problems learning math.
	I do not care about the ideas of other people (reverse scale).
	When I learn with others, I do not share my ideas (reverse scale).
	I do not like to cooperate with other people in the class (reverse scale).
V	**Distractors**
	It is more difficult for me to learn mathematical problems after a few hours of science subjects preceding mathematics on a given day
	Physical effort preceding math classes has a positive effect on my efficiency during classes.
	I need silence when doing math tasks.
	I am disturbed by the presence of more people in the group while performing mathematical tasks.
	When performing mathematical tasks, I need an independent work station.
	I am disturbed by the questions of other people asked to the teacher while performing mathematical tasks in the group.
VI	**Motivation**
	I like working with a person who has more knowledge of mathematics than me.
	The method of solving certain mathematical tasks is easier for me to remember when we come to them in a group together.
	I quickly remember my mathematical knowledge when it is given to me directly by the person conducting the classes.
	In the process of acquiring new mathematical skills, I need constant feedback on my progress.
	I am happy to use the methods of learning mathematics used by other people who I rate as capable in this field.
	Examples of persistence, diligence and ways to acquire knowledge, by outstanding mathematicians, are an incentive for me to continue working in this field.

(*continued on next page*)

Table 1 (*continued*)

VII	**Modeling**
	My professional future is tied to mathematics.
	I find math knowledge helpful in everyday life.
	Good grades from mathematics testify to my intelligence.
	I am frustrated by achieving the lowest grades in the group in mathematics.
	Recognition of a mathematics lecturer is as important to me as the grade in this subject.
	My relatives expect good grades in mathematics from me.

Table 2
Normalized numerical results in individual categories.

No.	Category	E1	E2	E3
I	Behavioral aspects	58	67	61
II	Emotional aspects	48	54	50
III	Cognitive aspects	58	71	67
IV	Social aspects	55	64	60
V	Distractors	60	68	70
VI	Motivation	58	73	65
VII	Modeling	67	75	73
	Median	**58**	**68**	**65**

$\sigma_W^2 = \frac{n_r(n_r+1)(2n_r+1)}{6}$. In the case of $n_r \geq 10$ the distribution of $\frac{W}{\sigma_W}$ can be approximated by $N(0,1)$. In consequence, for the fixed α we reject H_0 if the computed W falls into the critical region $K = \left(-\infty, -u_{1-\frac{\alpha}{2}}\right) \cup \left(u_{1-\frac{\alpha}{2}}, \infty\right)$, where u_p is a quantile of order p of the standard normal distribution.

Let us note that a well-known U Mann-Whitney test introduced in Ref. [26] that is also based, in fact, on the investigation of the median of all possible differences d_i, cannot be used here since it is dedicated for independent (not related) samples.

An alternative for Wilcoxon test is a Mood's (median) test being, in fact, a special case of Pearson's chi-square test, however (see Ref. [27]) the Wilcoxon signed rank test is usually more powerful than the Mood's one.

Other approaches used in the literature to solve both regression problems (multiple regression) and classification problems, the purpose of which is to find the values of output variables (dependent) on the basis of input (prediction) variables are methods such as MARS (Multivariate Adaptive Regression Splines) or CMARS [28,29]. MARS is a nonparametric regression technique used in the context of linear or cubic splines as basis functions. The authors of the [29–31] papers have obtained very satisfactory results using these methods, and their choice should be considered in the future in the context of the evaluation of the factors influencing the learning process of mathematic.

However, in this study the authors decided to use the Wilcoxon test for statistical analysis whether the new approach of using the structural description of mathematical expressions has a positive effect on blind and low vision students in terms of improving such aspects as: behavioral, cognitive, emotional, motivation and social. Static tests will be carried out by comparing individual mentioned categories using different techniques of an alternative description of mathematical expressions.

2.3. Detailed research theses concerning the developed factor categories

The answers to the questions (Table 1) in the survey were assessed on a normalized scale and ranged from 0 to 100%. Individual answers in a given category were summed. The final result obtained in a given experiment in a given category is the average number of points from the answers from individual questions in a given category.

2.3.1. Detailed research theses

2.3.1.1. Behavioral aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves attention, participation and concentration of the student.

2.3.1.2. Emotional aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves the ability of the student to influence their emotions.

2.3.1.3. Cognitive aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves the ability of the student to concentrate on the relevant information.

2.3.1.4. Social aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves the influence of the environment and the relations of the student with teachers and peers on their learning of mathematics.

2.3.1.5. Distracting aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves the student's knowledge of mathematics.

2.3.1.6. Motivational aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves the motivation of the student to learn mathematics.

2.3.1.7. Modeling aspects. The presentation of structural information contained in mathematical formulae in the form of sound alternative descriptions containing additional information about the structure of the pattern improves the impact of the science of mathematics on the student's learning.

2.4. The proposed alternative method for learning mathematics

The analysis of the problem started with a review of available tools supporting the learning of mathematics among the blind and surveys conducted among mathematics teachers for the blind. After reviewing the literature and surveys and taking into account many years of experience in teaching mathematics, a method for teaching mathematics was proposed at the Silesian University of Technology, Poland [3,12]. The proposed method is characterized by the following elements:

1. Dividing the exercise into elementary stages.
2. Checking the results at individual stages.
3. Entering user interactions.
4. Providing contextual support through the selection of theories and hints.
5. Providing alternative presentations with descriptions containing structural elements.

Based on the developed method, a platform was designed and implemented at the Silesian University of Technology; the platform was used by approximately 1000 students.

Studies on the comparability of verbal word associations between sighted and blind students indicate that it has increased from what it was in the 1960s and 1990s to 70% currently [32]. This fact motivated the authors of the method to adapt it to the needs of blind students.

Then, an alternative method for learning mathematics was

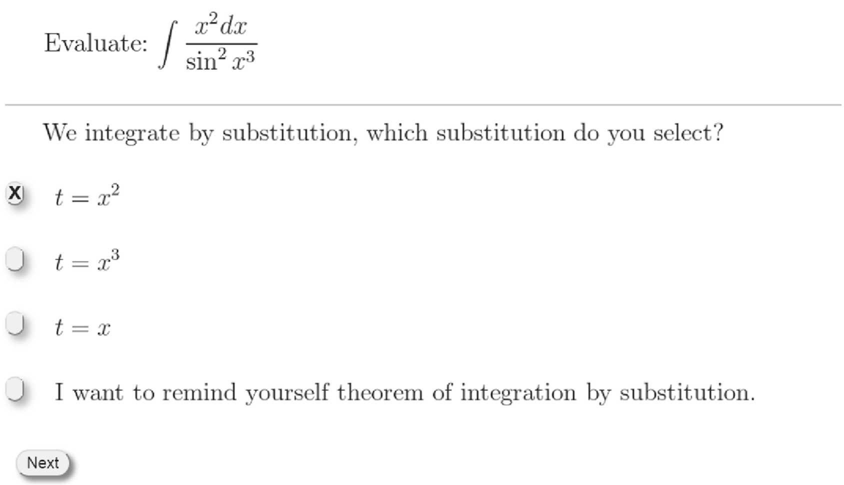

Fig. 1. Example of math exercise presented in the prepared platform.

developed for blind students by developing principles of alternative descriptions presenting structural information contained in mathematical formulae [13]. Next, the developed principles were implemented in the platform established for blind people [14]. An example of one exercise is presented in Fig. 1.

As a part of the proposed method, a method of evaluation based on the vector of knowledge was included [33]. Within the developed branch of mathematics, a graph of concepts was created that combines concepts ranging from the simplest to the most difficult concepts. With each term in the concept set, a set of several exercises was combined. Learning begins with exercises related to the simplest concepts and gradually progresses to exercises related to difficult concepts. Such an organization of the exercises ensures an organized learning path, avoids repetition of already established material and optimizes effort. The discussed methodology has been implemented in the form of an independent platform module adapted for use by blind people, in which the next exercise is selected based on the recently solved exercise.

The range of materials developed for blind and visually impaired students covers 100 exercises with corresponding alternative descriptions prepared in Polish. The exercises and alternative descriptions are organized to cover the following areas of mathematics: algebra, derivatives, integrals, limits and sequences.

Table 2 presents a summary of the characteristics of the proposed approach in relation to the classical method of blind students learning mathematics from a teacher, and the characteristics are broken down into the proposed assessment categories.

2.5. Research group, control group and conducted experiments

The research group consisted of 71 blind or visually impaired students who would solve math exercises at the level of a high school course or an academic beginner. People in the group did not have any other disabilities. All blind or visually impaired participants in the study had an acquired defect. The group consists of 42 women and 29 men, aged 19–24, with a median age of 20 and a standard deviation of 1.52. The following inclusion criteria were adopted in the studies:

1. The degree of visual impairment or blindness was significant.
2. The vision defect was acquired.
3. The student was in secondary school or the first year of technical school in Silesia.

3. There were no co-occurring emotional disorders

The following exclusion criteria were adopted in the research:

1. The vision defect was present from birth.
2. There was a coexisting emotional disorder.

A total of 75 people were found to meet the inclusion criterion, all of whom were invited to participate in the study. Among those invited, 71 people participated in the research. According to the sample size calculation, the sample should be 63 for a confidence level of 95, a fraction size of 0.5 and a maximum error of 0.05.

3.1. Research – stage 1

The students solved exercises without using the platform developed for assisting blind students in learning mathematics– (the first experiment, E1).

3.2. Research – stage 2

The students solved exercises using the platform developed for assisting blind students in learning mathematics in two ways: completely alone (the second experiment, E2) or with the help of an assistant (the third experiment, E3).

After each experiment, users (blind and visually impaired students) completed the questionnaire presented in Table 1. The conducted research was approved by the Ethics Committee of the Medical University of Silesia, Poland.

4. Results

The results from the questionnaires completed after the conducted experiments are presented in a normalized form (0–100%) in Table 3.

Table 4 presents both the standard deviation (SD) and effect size (Cohen's d test) for individual categories (large – dark gray; medium – light gray).

Then, statistical analysis was performed to compare the medians in each category (Wilcoxon test).

Figs. 2 and 3 graphically show a summary of the results obtained in individual categories.

5. Discussion

Analysis of the obtained results (Tables 3–5, Figs. 2 and 3) indicated that the use of the designed system significantly improved six out of the seven categories (behavioral, emotional, cognitive, social, modeling, motivational) comparing self-use of the system by a blind person (E2) to independent task solving only with the screen reader (E1), which confirms the validity of 6 out of 7 research hypotheses (described in

Table 3
Standard deviations (SD) and effect size (Cohen's d test) in individual categories.

No.	Category	E1	E2	E3	E1-E2	E1-E3	E2-E3
		SD	SD	SD	effect size	effect size	effect size
I	Behavioral aspects	0.07	0.08	0.06	1.14	0.65	0.62
II	Emotional aspects	0.12	0.11	0.15	0.52	0.14	0.31
III	Cognitive aspects	0.11	0.08	0.11	0.89	0.70	0.10
IV	Social aspects	0.13	0.11	0.14	0.56	0.29	0.23
V	Distractors	0.15	0.10	0.12	0.51	0.50	-0.02
VI	Motivation	0.13	0.09	0.13	1.29	0.55	0.64
VII	Modeling	0.08	0.10	0.12	0.73	0.37	0.25

Table 4
Statistical tests on equality of medians in experiments in individual categories.

No.			E1	E2	E3
I	Behavioral aspects	**E1**	x	False p = 0.0117	True p = 0.1083
		E2		x	True p = 0.1200
		E3			x
II	Emotional aspects	**E1**	x	False p = 0.0354	True p = 0.4661
		E2		x	True p = 0.0781
		E3			x
III	Cognitive aspects	**E1**	x	False p = 0.0021	False p = 0.0061
		E2		x	True p = 0.4481
		E3			x
IV	Social aspects	**E1**	x	False p = 0.0183	True p = 0.3078
		E2		x	True p = 0.2390
		E3			x
V	Distractors	**E1**	x	True p = 0.2179	True p = 0.1284
		E2		x	True p = 0.5924
		E3			x
VI	Motivation	**E1**	x	False p = 0.0032	True p = 0.1395
		E2		x	True p = 0.0978
		E3			x
VII	Modeling	**E1**	x	False p = 0.0285	True p = 0.1173
		E2		x	True p = 0.4445
		E3			x

section 2.3). The distractors category was the only one in which no significant improvements were observed. At the same time, without a significant difference, in the same 6 categories (excluding distractors), the participants assessed the independent use of the system more positively (E2) than the assisted use of the system (E3). In the distractors category, the results of the experiments were ranked best to worst as follows: E3, E2, and E1 (Table 3).

The designed system allows the more effective independent learning of mathematical topics according to the proposed method of task decomposition [13,14] combined with the method of evaluating results based on the knowledge vector [33].

Analyzing the obtained results more carefully revealed several potential issues. First, attention should be paid to the limited number of respondents. However, due to the lack of studies on the discussed topic, the results obtained can be considered a valuable contribution to the development of computer-assisted systems for the disabled and the more widely understood discipline of ambient assisted living.

It should also be noted that an improvement of 10% points was obtained when the designed system was used independently (E2) compared to when only the screen reader was used (E1, Table 2). Therefore, it should be noted that no maximum score was obtained in individual categories. The greatest improvement was obtained in cognitive and motivational categories, which pleased the authors. These results show that at the assumed goal of the system being developed, which was to increase the motivation of blind students to learn mathematics, was achieved. Longer use of the system with its continuous evaluation and adaptation may lead to even better results in these categories.

The lowest absolute results and the lowest relative improvement were obtained in the emotional category, which indicates that there is still much work to be done to prepare blind people for the challenges of acquiring structural information. Future work will investigate methodological activities that will allow better results in this category.

In turn, the results obtained in the behavioral, social, and modeling categories tended to indicate improvements in other scenarios of using the system, which enabled group work by blind people using the designed system.

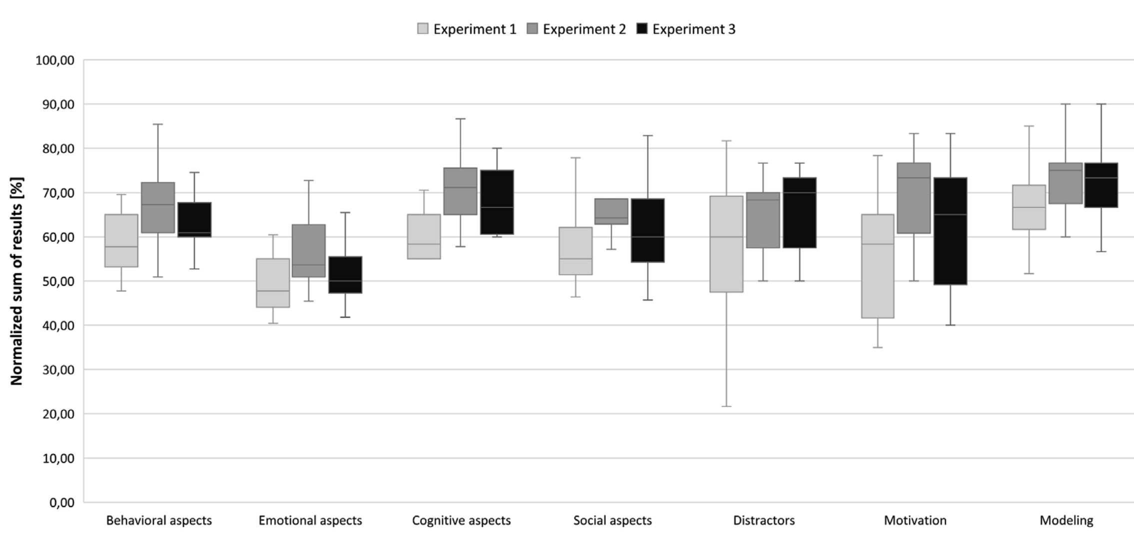

Fig. 2. Box plot of the normalized results obtained in E1, E2, and E3 in individual categories (based on Table 3).

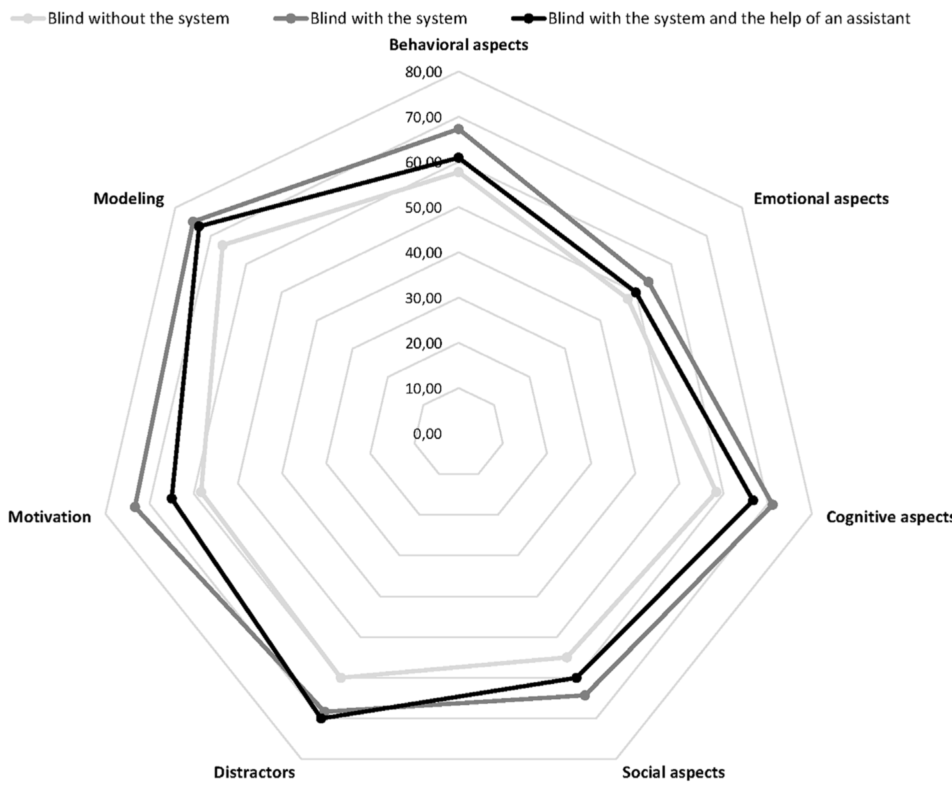

Fig. 3. Radar plot of the results obtained in E1, E2, and E3 in individual categories (based on Table 3).

Compared with the research mentioned in the Introduction, which reported the increased motivation of students using the tactile audio system [18] and the increased possibility of students solving problems with the use of modeling [19], the results obtained in the present study are consistent. Significant improvements in behavioral, motivational and modeling aspects were achieved using the developed alternative method.

From the math teacher's point of view, the developed method can be used freely, especially for covering difficult parts of the material, as the use of the developed platform raises the sense of self-efficacy when tasks are correctly solved. The teacher, based on his or her experience and knowledge of the individual characteristics of his or her students, can use the platform to further influence the level of perceived satisfaction of individual students, which can lead to improvements in the category with the poorest results, the category associated with emotions.

The possibility of using the developed system with a sound interface in other languages should also be discussed. Currently developed exercises have been developed in Polish on the basis of the rules developed by the authors after consulting with mathematicians and math teachers of blind people [14]. That article also provides an overview of other alternative tools that support the learning of mathematics by

Table 5
Comparison of the characteristics of the developed method with the classical method of teaching and learning mathematics by blind people.

Criterion	The classic method with the teacher	The proposed method
Behavioral aspects	Very limited cognitive autonomy - the need to follow the teacher	An independent plan of learning and repetition; the need to reach external sources of knowledge is kept to a minimum
Emotional aspects	The necessity of individual learning, otherwise (learning in a group) there is no possibility to adjust the pace of learning	The ability to adjust the pace of learning to the individual student's abilities.
Cognitive aspects	Difficulty in understanding the structural information of a mathematical formula (the necessity of repeating it repeatedly by a teacher).	The ability to repeat the task solving from any stage. Providing knowledge consistent with the context (no need to search knowledge in extensive documents)
Social aspects	Difficulty in tracing the entire structure of the task.	The ability to acquire structural information through an alternative description, the ability to present your ideas as part of navigating a set of elaborated tasks
Distractors	Limited possibility of interaction in terms of acquisition and transfer of structural information.	Necessary ability to focus on the voice presentation of the system with the possibility of a flexible learning plan with adaptation to personal preferences.
Motivation	Necessary ability to focus on the teacher's voice presentation limited to the schedule of classes	Increased motivation by adjusting the learning speed to individual person's abilities. Solved specific stages of the task with success increase motivation to continue teaching and learning
Modeling	The need to acquire structural information that is difficult to access	Breaking down the task into elementary stages allows the student to better solve similar tasks.
Accessibility and Usability	Limitation of imitation to follow the teacher	An alternative, unambiguous description of structural information (containing an additional description of the structure)
Learning Progress	Limited availability in particular to structural information.	Bug reports on individual stages; Automatic evaluation of the blind student's independent work. Time of solving and number of tasks, degree of difficulty. The ability to automatically select a task at a given level of difficulty. The possibility of re-tracking the course of the solution from the beginning to a given place

blind people. Using the system in a different language requires the descriptions to be translated. Technical details regarding the topic discussed can also be found in the publication.

Wongkia et al. [34] offered automatic loud reading of mathematical formulae available in xml materials in Thai. The evaluation results indicate that i-Math is able to produce acceptable speech output with the intended meaning.

In contrast to previous work carried out in the EUAIN project (European Accessible Information Network) [13], the presented approach is based on LATEX. Some researchers have raised the issue of ambiguity in converting between LATEX and MathML notation [35]. Additionally, another report concluded that TeX does not provide an encoding of the semantics for the mathematical material and that this information must be derived from some (external) context [12]. Thus, to guarantee high-quality educational materials, the authors decided to avoid automatic conversion to alternative text descriptions.

As mentioned in section 2.4, work is underway to use knowledge vectors [33] to choose the order in which exercises are solved. In relation to the theory of games [36,37], the proposed strategy allows minimization of the length of the game, which in the considered application, leads to minimization of the effort required to master all the concepts contained in the knowledge vector.

One of the important factors that characterizes the process by which blind people learn mathematics, as mentioned above, in section 2.1, is the ability to maintain concentration and process sensory information [21,23]. The most similar category of assessment for these factors is the distractors category. This is the only category in which no improvement was observed, which we interpret as an open problem to be addressed by further research. Considering the nature of learning among the blind, the main distractors are sounds. In the future, is the authors plan to develop a system version that allows the teacher to moderate the selection of which auditory information reaches the individual and the adaptive method of filtration according to personalized criteria indicated individually by the blind person.

Recently, there have also been works verifying the possibility of using the linear LAMBDA notation to support the learning of mathematical formulae by blind people [38], which was originally used with Braille terminals for use with the free screen reader NonVisual Desktop Access [39]. One study [40] showed the possibility of combining NVDA and LAMBDA notation. In the future, the authors plan to integrate the developed rules of an alternative presentation with the LAMBDA notation, which would allow the automatic preparation of exercises and their eventual adaptation to different languages.

According to the authors' experience, the structural information contained in mathematical textbooks can be divided into three levels – information contained at the chapter level, the exercises and the mathematical formulae. The presented approach takes into account the last two levels through the developed method of decomposing the exercises and providing an alternative description of the mathematical formulae containing structural information. In addition to the presentation of structural information included only in mathematical formulae, it is worth considering the structure of mathematical textbooks in the development of alternative presentation methods. Generally, Garderen et al. provided very little explicit instructional information about representations in classical mathematical textbooks [4]. However, other authors are working on the extraction of implicit didactic information from mathematical books [41]. Currently, the advancement of the developed methodology, as the authors point out, allows the submission of only basic mathematical operations, such as division and multiplication. The authors plan to extend the methodology of the extraction of implicit didactic information contained in the structure of mathematical textbooks [41] to include this information into the alternative presentation method.

6. Conclusion

The results of these experiments suggest that teaching and learning of mathematics among blind people by the decomposition of tasks combined with the knowledge vector-based evaluation of results contribute to universal education through:

1. Increasing the availability of content containing structured information (mathematical formulae) to the blind.
2. Limiting the importance of communication barriers in math education (the possibility of self-study, reducing costs by reducing the number of individual hours with the teacher).
3. Developing universal rules for the presentation of structural information.

Conflict of interest

There is no conflict of interest.

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