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Design guidelines for turbulence in traffic on Dutch motorways

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ABSTRACT

Over the years the characteristics of traffic on Dutch motorways has changed, but its design guidelines did not develop as rapidly and large parts remain unchanged since the first guidelines from the 1970s. During the latest revision of the Dutch motorway design guidelines it became clear that a solid and comprehensive theoretical, or evidence based, background was lacking for the validity of the prescribed ramp spacing and required length for weaving segments. This article presents the underpinning of revising the Dutch design manual for motorways for turbulence in traffic. For this study loop detector data at eight on-ramps and five off-ramps were collected as well as empirical trajectory data at fourteen different on-ramps (three), off-ramps (three) and weaving segments (eight) in The Netherlands. The results show that the areas around ramps that are influenced by turbulence are smaller than described in the design manuals and that, in their present form, the microscopic simulation software packages VISSIM and MOTUS fail to simulate the number and location of lane-changes around ramps realistically.

1. Introduction

The Netherlands is a relatively small but dense populated country. It has a rather expanded motorway network today with relatively high traffic volumes. The first motorized vehicles entered the country around 1900. Motorization in traffic increased rapidly. The degree of motorization in traffic has had its impact on the road network. Initially, the construction element of paved roads was of relevance in road design and from the 1920´s also geometric design was taken into account by road designers. These developments led to changes in the structure of the total road network. Rijkswaterstaat, the Dutch National Roads Authority, introduced the motorway-concept officially in its "Rijkswegenplan" (Plan for National Roads) 1938, but the construction of the first motorway (between The Hague and Utrecht) started in 1932 and was opened for traffic already on April 15, 1937.

Where there used to be only one type of road in the past, nowadays there is a functional categorization of roads ('road hierarchy') as described for example in a report known as "Traffic in Towns", published by the UK Ministry of Transport in 1963, also known as the Buchanan-report after Sir Colin Buchanan who chaired the authors' team (Buchanan, 2015). Two major functions are distinguished for traffic: mobility and accessibility. These are very different functions, and both functions require a specific infrastructure, a specific design and specific use requirements to make safe(r) road traffic possible (Wegman et al., 2008). Motorways have only a flow function.

Within the concept of a functional categorization of roads, derived from the Buchanan report and later modified by Koornstra et al. (1992), a motorway fulfils the function of facilitating traffic flow. The Highway Capacity Manual (HCM, 2016) defines a motorway as: "*A divided highway with full control of access and two or more lanes for the exclusive use of traffic in each direction. Motorways provide uninterrupted flow. There are no signalized or stop-controlled at-grade intersections, and direct access to and from adjacent property is not permitted. Access to and from the motorway is limited to ramp locations. Opposing directions of flow are continuously separated by a raised barrier, an at-grade median, or a continuous raised median. Operating conditions on a motorway primarily result from interactions among vehicles and drivers in the traffic stream and among vehicles, drivers, and the geometric characteristics of the motorway*". Dutch motorways meet perfectly well all characteristics as described in the HCM-definition.

By separating vehicles, that move at a high speed and in opposing directions, by controlling access and by using grade separated intersections only, a motorway is relatively safe (Wegman et al., 2008). Because of the high travel speeds on motorways, it is important that the design of the road is predictable for its users. This means that the design needs to support the user's expectations of the road. The design of all road elements need to be in line with these expectations and should therefore be uniform throughout the motorway network (Wegman

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et al., 2008). To secure uniformity in motorway design, Rijkswaterstaat started to develop motorway design guidelines in the 1970s (Rijkswaterstaat, 1975). These guidelines were partly based on Rijkswaterstaat's own research and experience, but were also inspired by and partly based on US guidelines and manuals, such as: the "Policy Geometric Design Highways" by the American Association of State Highway Officials (AASTHO) and the "Highway Capacity Manual" (HCM) by the Transportation Research Board (TRB). Other sources of inspiration were the "Richtlinien für die Anlage von Autobahnen" (RAA, 2008) in Germany, and the" Design Manual for Roads and Bridges" (DMRB, 1994) in Great Britain.

Originally, the Dutch guidelines were only used by Rijkswaterstaat staff to share information regarding design policy, decisions made in the past and standard design solutions. Rijkswaterstaat's policy regarding motorway design has changed over the years, by outsourcing design work to the private sector. However, design solutions should not be dependent on the individual designer but guided by design guidelines (Wegman et al., 2008). Also the characteristics of vehicles and the penetration of technology in vehicles (e.g. ADAS, Advanced Driver-Assistance Systems) has changed. These changes led to several revisions of the design guidelines: in 1992 (Rijkswaterstaat, 1992), in 1999 (which was never published), in 2007 (Rijkswaterstaat, 2007), and recently in 2015 and 2017 (Rijkswaterstaat, 2017). But the guidelines did not develop as rapidly as technology, and large parts of the design guidelines remain unchanged since the first guidelines from the 1970s.

During the latest revision it became clear that, despite a long tradition of research within Rijkswaterstaat, a solid and comprehensive theoretical, or evidence based background was missing for different parts of the guideline. In a joint research project carried out in 2013 by SWOV (National Institute for Road Safety Research), Rijkswaterstaat (the National Roads Authority), the Information and Technology Platform for Infrastructure, Traffic, Transport and Public space (CROW), and Delft University of Technology, the validity of existing guidelines for the design of urban and rural distributor roads and the design of through roads were assessed (Schermers et al., 2013). In this study it was stated that, among a long list of other issues, the underpinning is lacking for turbulence in traffic and it was decided to carry out research, by means of a PhD study (Van Beinum, 2018b), on the following topics:

- the required ramp spacing on motorways, based on turbulence in traffic;
- the required length for weaving segments, based on turbulence in traffic.

This article presents the results of the van Beinum-study (2018b) and is of relevance for underpinning of revising the Dutch design manual for motorways for ramp spacing and weaving segment length, based on turbulence in traffic. We have focussed this study on driving behaviour and vehicle interaction, in nearly saturated free flow (no congestion) traffic conditions. The article is structured as follows: the first Section describes the theoretical background of the concept of turbulence and the available tools and methodologies to assess the characteristics of turbulence. The second Section presents the methodologies that were applied in this research and the third Section gives the main results. This article concludes with a discussion and a conclusion Section.

2. Theoretical background of turbulence

2.1. Concept of turbulence

The concept of turbulence, as it is used in motorway design guidelines, not only in the Netherlands but also elsewhere, implies a disturbance in the traffic stream, that is caused by vehicles that make mandatory lane-changes, causing additional lane-changes, speed changes, and headway changes by other surrounding road users. Mandatory lane-changes occur at locations where the number of lanes on the motorway changes. These locations are referred to as "discontinuities". Changing lanes, however, is a legitimate manoeuvre on a motorway. Turbulence is therefore regarded to be a common and unavoidable phenomenon in a traffic stream (HCM, 2016), and will have a higher magnitude around motorway discontinuities (Kondyli and Elefteriadou, 2011). Commonly known examples of discontinuities are on-ramps, off-ramps and weaving segments.

2.1.1. Definition of turbulence

In literature turbulence is mentioned, yet no explicit definition for turbulence is given. Only the effects and characteristics of turbulence are mentioned. These are some examples:

"Weaving segments require intense lane-changing manoeuvres as drivers must access lanes appropriate to their desired exit leg. Therefore, traffic in a weaving segment is subject to lane-changing turbulence in excess of that normally present on basic freeway segments. This additional turbulence presents operational problems and design requirements" (HCM, 2010);

"Ramp-freeway junctions create turbulence in the merging or diverging traffic stream. In general, the turbulence is the result of high lane-changing rates. The action of individual merging vehicles entering the traffic stream creates turbulence in the vicinity of the ramp. Approaching freeway vehicles move toward the left to avoid the turbulence. Thus, the ramp influence area experiences a higher rate of lane-changing than is normally present on ramp-free portions of freeway" (HCM, 2010);

Turbulence can be captured by four variables:" (1) variation in speeds in the left and interior lanes, (2) variation in speed in the right lane, (3) variation in flow in the left and interior lanes, and (4) variation in flow in the right lane" (Golob et al., 2004).

Since there is no explicit definition for turbulence available, two new definitions are proposed by Van Beinum et al. (2016):

• Turbulence:

- ○ individual changes in speed, headways, and lanes (i.e. lane-changes) in a certain road segment, regardless the cause of the change;

• Level of Turbulence:

- ○ the frequency and intensity of individual changes in speed, headways and lane-changes in a certain road segment, over a certain period of time.

2.1.2. The implications of turbulence

Kondyli and Elefteriadou (2012) found that turbulence due to merging manoeuvres initiates 110 m upstream of the on-ramp gore. According to the (HCM, 2016), the area in the vicinity of a ramp that is influenced by merging traffic stretches from about 460 m (1.500 ft.) upstream to 460 m downstream of the gore. To the best of our knowledge no other sources are available that describes the start or the end of a raised level of turbulence. Parts of the motorway that suffer high levels of turbulence more often function as bottlenecks and show higher crash rates, compared to road segments with low turbulence (Golob et al., 2004; Lee et al., 2003, 2002; HCM, 2016).

2.1.3. Impact of road design and driver behaviour on turbulence

The level of turbulence is expected to increase when the available length for performing mandatory lane-changes decreases. Therefore, turbulence has to be taken into account for ramp spacing and the length of weaving segments (HCM, 2016; AASHTO, 2011; DMRB, 1994; RAA, 2008; Rijkswaterstaat, 2017). To determine the correct lengths, it is

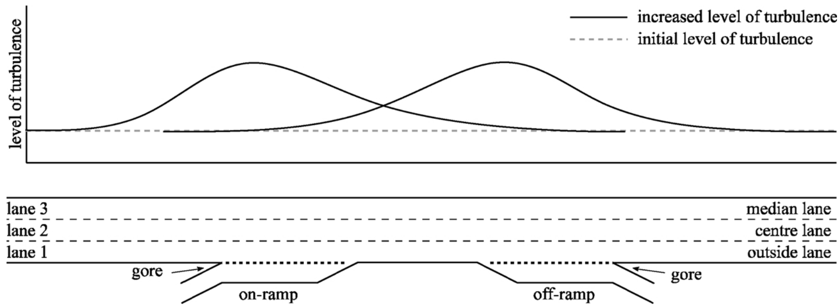

Fig. 1. Concept of the level of turbulence around succeeding ramps.

important to have knowledge about the location where the level of turbulence starts to increase upstream of a discontinuity, and where the turbulence dissolves downstream of a discontinuity. Furthermore, when two discontinuities are located close to each other, their turbulence impact areas might overlap. This concept is shown for an on-ramp that is succeeded by an off-ramp in Fig. 1. In this case, knowledge about the implications for traffic operations and traffic safety of the overlap and the severity of this overlap is required.

2.1.4. Ramps spacing in different guidelines

Different approaches for ramp spacing are used in the different guidelines and manuals. For example: the AASHTO Green Book (AASHTO, 2011) uses a set of minimum values for ramp spacing and the Dutch guidelines (Rijkswaterstaat, 2017) use a criteria called Turbulence length, which is dependent on the motorway's design speed. The prescribed lengths for ramp spacing differ per type of ramp and also per guideline. For example, Table 1 shows the different prescribed distances between an on-ramp followed by an off-ramp (measured from gore to gore). Furthermore, the guidelines do not indicate the implications of deviating from the guidelines in terms of traffic operations and traffic safety.

These guidelines are important tools for road designers, influence decision making in road design to a large extent, and can eventually have an enormous influence on the physical layout of a road. Currently, there are two major problems for applying current motorway design guidelines with respect to turbulence:

- a solid theoretical and empirical underpinning regarding the required length for a raised level of turbulence is lacking;
- to the best of our knowledge, a thorough understanding is missing of (quantitative) implications in terms of impacts on traffic operations and traffic safety, when deviating from the design guidelines.

Table 1
Distance between On-Ramp and Off-Ramp prescribed in different Guidelines.

country	distance	design criteria
The Netherlands (Rijkswaterstaat, 2017)	750 m	design speed
Germany (RAA, 2008)	1100 m*	minimum value for isolated intersection planning
USA (AASHTO, 2001)	600 m**	road category: freeway
	480 m***	road category: freeway
UK (DMRB, 1994), Vol.6, Sec. 2, Cpt 4.7	450 m****	3.75 V, where V = design speed = 120 km/h

* 250 m acceleration lane + 600 m between acceleration and deceleration lane + 250 m deceleration lane.
** system to service interchange (weaving).
*** service to service interchange (weaving).
**** may be increased to the minimum requirements for effective signing and motorway signalling.

2.2. Methodologies to collect empirical data to measure turbulence

There are different methods available to collect empirical data that could be used to quantify turbulence in motorway traffic. A method is regarded suitable if it is able to: 1) indicate the location where a raised level of turbulence starts and dissolves in the vicinity of ramps, 2) generate or measure trajectories of (all) individual vehicles in the vicinity of ramps and 3) give insight in the interaction between different vehicles.

Loop detectors are useful to collect data to investigate macroscopic traffic state variables such as density, speed and headway distributions (Xu et al., 2012; Treiber et al., 2000). Loop detector data is available in large quantities and is relatively easy to collect. Data from Dutch motorways, for example, can be accessed real time online. The disadvantage of using loop detector data is that it does not provide detailed information of individual manoeuvres, such as lane-change, acceleration and deceleration. For collecting this kind of detailed information, different methods are available. For this study we have considered: video recordings, driving simulators and instrumented vehicles / naturalistic driving. Video recordings can be used to generate trajectory data by which turbulence related driver manoeuvres such as merging, overtaking and acceleration can be studied in a detailed way (Daamen et al., 2010; Hoogendoorn et al., 2011; Marczak et al., 2013). Cameras can be mounted on a high observation point such as a helicopter (Hoogendoorn et al., 2003), a drone (Voorrips, 2013) or a building/structure (NGSIM, 2015). Trajectory data, however does not give an insight in choices made by drivers, is relatively expensive to collect and the data processing is time consuming. Behavioural aspects that explain the driver's choices can be researched by using data from a driving simulator (Van Winsum and Heino, 1996; De Waard et al., 2009). A driving simulator has several advantages: the ability to test a wide variety of different existing and non-existing road design layouts, control of the intervening variables and it is a safe environment. One of the disadvantages of driving simulators is that its measurements are taken from a simulated environment and does not necessarily reflect drivers' behaviour exactly as in reality (Farah et al., 2009). Driver behaviour data from a real life traffic environment can be acquired by the use of an instrumented vehicle. This can be done by using a vehicle in an experimental setting (Brackstone et al., 2002; Wu et al., 2003; Kesting and Treiber, 2008; McDonald et al., 1997), or by using vehicles that are operated daily (naturalistic driving) (Olson et al., 2009; Antin, 2011; Blanco et al., 2011; Chong et al., 2013; NDS, 2015). The disadvantage is that a relatively big organizational effort is required to equip and operate the vehicles. Other disadvantages include the effort to process the large amount of data and the need to mask/protect personally identifiable information. Based on the pros and contras of the different methodologies to collect data, it has been decided to work with video data collected by a camera mounted on a hovering helicopter.

2.3. Methodologies to collect simulated data to measure turbulence

The most direct way to study traffic operations is by studying empirical traffic data, such as trajectory data (Coifman et al., 2005; Laval and Leclercq, 2010; Laval, 2011; Zheng et al., 2011b, 2011a; Polus et al., 1985) or loop detector data (Treiber et al., 2000; Coifman and Kim, 2011; Coifman et al., 2005). The HCM suggests that traffic simulation can be used to assess the traffic operations performance of roads (HCM, 2010). When using microscopic simulation software, it is possible to take into account different road characteristics, different traffic characteristics and microscopic behaviour in order to evaluate traffic operations and traffic safety on a certain motorway segment. Known examples of commercial microscopic simulation software packages, which are widely used in research are: AIMSUN (Young et al., 2014), CORSIM (Sun and Kondyli, 2010), PARAMICS (Dijkstra, 2011) and VISSIM (Chih-Sheng and Nichols, 2015). Recently, also new and improved driving behaviour models are proposed (Ahmed, 1999; Toledo et al., 2007a; Schakel et al., 2012) and implemented in experimental setups like MITSIM and MOTUS.

For this study both a commercial microscopic simulation package (VISSIM (PTV, 2017)) and a recently developed model (MOTUS (Schakel et al., 2012)) were selected and applied. The details of the method and criteria that were used to select the most suitable microscopic simulation models, are described in (Van Beinum et al., 2019). A key- question to be answered is of course whether simulated driving behaviour from these packages is realistic enough for assessing the impact of design of on-ramps, off-ramps and weaving segments on the level of turbulence. This question is an important component of this study.

3. Data collection

For this study four datasets were generated: two sets with collected empirical data and two sets with simulated data. The empirical data consists of a set with macroscopic data (collected from loop detectors) and a set of trajectory data (collected from video recordings taken from a hovering helicopter). The empirical macroscopic data were used to indicate the dimensions of the area with a raised level of turbulence around off-ramps and on-ramps. Based on these results the requirements for the collection of the empirical trajectory data were established. The empirical trajectory data were used to calibrate both VISSIM and MOTUS (Van Beinum et al., 2019). The calibrated VISSIM model and the calibrated MOTUS model were used to generate the simulated data.

3.1. Macroscopic data

The macroscopic data were used to determine at what distance a raised level of turbulence starts upstream of a ramp and at what distance downstream of a ramp it dissolves. The data were collected from loop detectors at different on-ramps and off-ramps at several three-lane motorways in The Netherlands. To identify the location near the ramp where the level of turbulence starts to change, also data from three different continuous motorway segments were collected.

Detectors in The Netherlands provide 1-minute aggregated flow and mean speed data for each lane, which are used to calculate an approximate density. The measurements were taken at days with comparable conditions, such as: period of year, weather, daylight, amount of commuting and recreational traffic, and traffic density. A total of 34 days were selected. The details of this procedure are described in (Van Beinum et al., 2017).

The macroscopic data were collected at eight different on-ramps with a total of fourteen different detectors and at five different off-ramps with a total of eighteen different detectors. From these sites two data sets were generated with in total n = 38,638 on-ramp entries and n = 59,109 off-ramp entries. The measured mean speeds range between 97.9 km/h and 106.1 km/h at the on-ramps and between 96.2 km/h and 107.4 km/h at the off-ramps. At the on-ramps the lower speeds were measured only at the detectors located up to 150 m downstream of the ramp. At the off-ramps the lower speeds were measured at a range of 571 - 218 m upstream of the off-ramp. The measured traffic volumes were comparable at each detector and range between 3584 veh/h and 3885 veh/h at the on-ramps and between 3493 veh/h and 3917 veh/h at off-ramps.

3.2. Microscopic data

Empirical trajectory data were collected at fourteen sites in the Netherlands. The trajectories were collected using a camera mounted underneath a hovering helicopter, comparable to the method described in (Hoogendoorn et al., 2003). Using a 5120 × 3840 pixel camera and a 15 mm Zeiss lens enabled us to capture a road stretch of approximately 1200 m - 1500 m from an altitude of approximately 500 m. The length of the measured road stretch coincides with the findings from the empirical macroscopic data (Van Beinum et al., 2017), where we found that an increased level of turbulence at on-ramps starts at approximately 200 m upstream of the ramp gore and ends approximately 90 0 m downstream of the ramp gore. At off-ramps these values are respectively 1000 m upstream of the ramp gore and approximately 600 m downstream of the ramp gore. An overview of the different sites with

Table 2
Site characteristics;

road	site name	type	through lanes	length* [m]	speed limit [km/h]	number of vehicles		
						total	V/C**	trucks
A13	Delft	off-ramp	3	250	100	2.569	0.78	123
A59	Terheijden	off-ramp	2	250	130	1.599	0.57	200
A16	Zonzeel	off-ramp	3	210	130	1.943	0.69	444
A13	Delft	on-ramp	3	300	100	2.654	0.81	168
A59	Terheijden	on-ramp	2	320	130	1.422	0.51	109
A16	Zonzeel-north	on-ramp	3	340	130	1.679	0.58	508
A4	Bergen op Zoom-east	weaving	2	500	120	1.582	0.35	163
A4	Bergen op Zoom-west	weaving	2	400	120	1.434	0.55	118
A59	Klaverpolder-north	weaving	2	600	130	1.239	0.55	154
A59	Klaverpolder-south	weaving	2	500	130	1.760	0.74	274
A16	Princeville-east	weaving	3	1.000	130	2.396	0.58	629
A16	Princeville-west	weaving	3	1.100	130	2.082	0.52	410
A15	Ridderkerk-north	weaving	3	700	130	2.158	0.61	446
A15	Ridderkerk-south	weaving	3	1000	130	2.868	0.78	555

* Length of acceleration lane (on-ramp), deceleration lane (off-ramp) or weaving segment.
** Volume/Capacity (V/C) ratio.

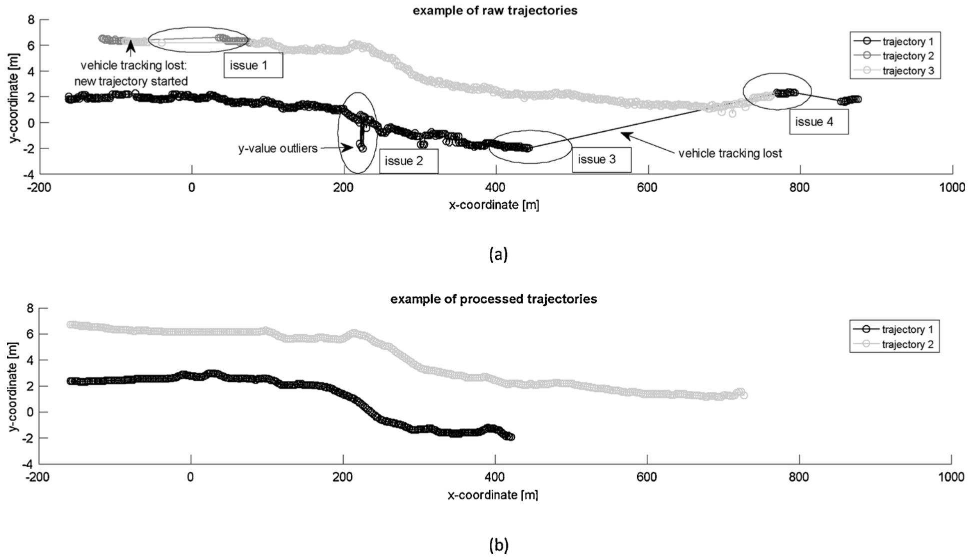

Fig. 2. Example of raw (a) and processed trajectories (b).

their characteristics is given in Table 2.

3.2.1. Smoothing

The trajectory data originates from video footage (12 fps), which were processed with automated vehicle recognition software to x, y, t -coordinates, which represent the centre of the vehicle at a specific time. The raw data were processed to reduce the noise due to measurement errors and inaccuracies. Fig. 2(a) shows an example of 4 different issues in the data that were encountered.

The automatic vehicle recognition and vehicle following software sometimes loses track of the vehicle due to objects overhead (e.g. a viaduct). When the vehicle is recognized again, it was sometimes recognized as a new vehicle (issue 1), as a different, wrong, vehicle (issue 3) or as the same, correct, vehicle further downstream (issue 4). Also unrealistic x- and -y values were measured (issue 2). These unrealistic values are caused by shadows besides the vehicle, that were sometimes recognized as part of the vehicle, or by vehicles driving closely next to each other that were recognized as one vehicle. These issues in the data were repaired. Finally all missing data points in the trajectories were interpolated and the trajectories were smoothed using a polynomial regression filter (Toledo et al., 2007b). Fig. 2(b) shows an example of two trajectories after processing.

3.3. Simulated data

From the empirical trajectory dataset the on-ramp, off-ramp, short weaving segment and long weaving segment with the highest traffic flow were selected for calibration of VISSIM and MOTUS. These sites are: on-ramp Delft, off-ramp Delft, weaving segment Klaverpolder-south and weaving segment Ridderkerk-south. The selected locations have a volume/capacity ratio (V/C) between 0.74 and 0.81, which is regarded to be reasonably high. It is expected that in this V/C range, entering and exiting traffic will have a significant effect on turbulence.

The different sites were modelled in VISSIM and MOTUS. The physical road characteristics, in terms of number of lanes and the length of the acceleration/deceleration lane, were modelled comparable to the measured sites. Also, the traffic conditions within the simulation were comparable to those during the field measurements. The following traffic conditions were used as an input for the simulation: 1) number of through going vehicles and vehicles that enter and/or exit the motorway, 2) the number of trucks and 3) the distribution of desired speeds. Furthermore, the simulation time was set equal to the duration of the field measurements.

4. Results

The increased level of turbulence in the vicinity of ramps is caused by drivers that perform manoeuvres to enter or exit the motorway. The

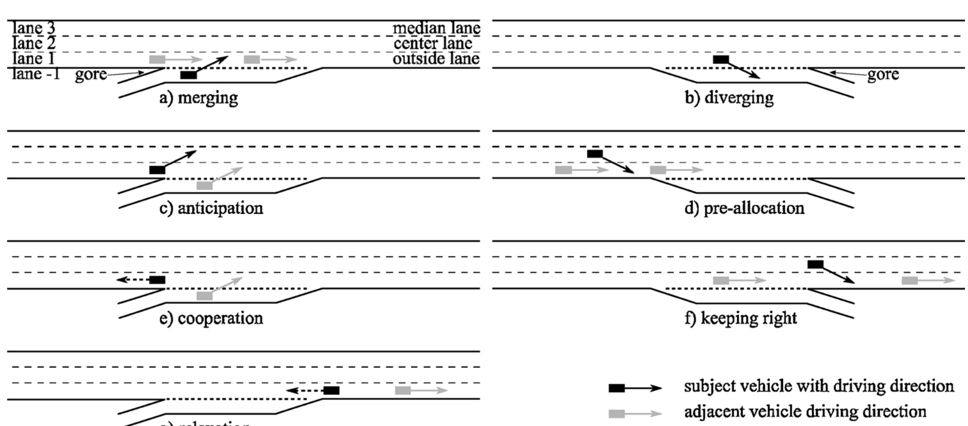

Fig. 3. Manoeuvres in the vicinity of ramps.

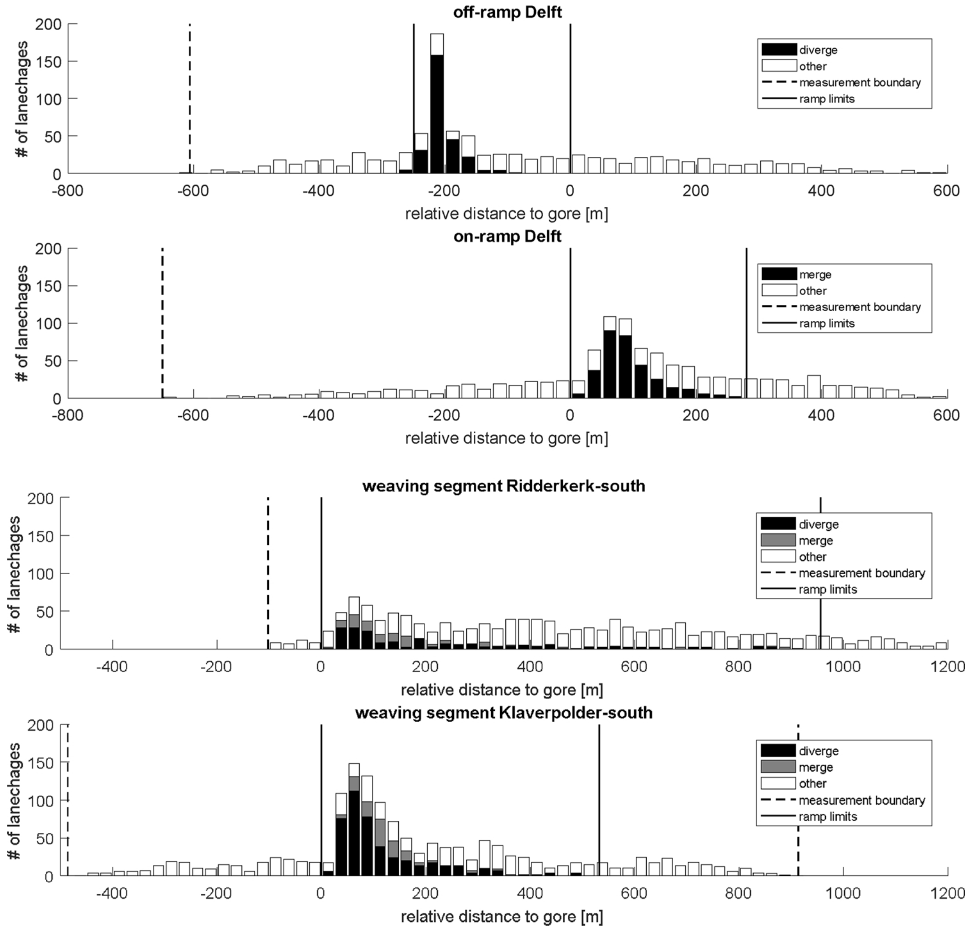

Fig. 4. Lane change locations near on-ramps and off-ramps.

following manoeuvres are performed in the vicinity of ramps: merging, diverging, pre-allocation, cooperation, anticipation keeping right and relaxation (Van Beinum et al., 2018). The different manoeuvres are graphically displayed in Fig. 3. A more detailed overview and description of these manoeuvres is given in (Van Beinum et al., 2016, 2018).

4.0.1. Location and number of lane-changes

The lane-change locations and number of lane-changes are presented in Fig. 4. The results show that the majority of the lane-changes occur at the acceleration lane or deceleration lane. Further upstream and downstream of a ramp only a limited increase in the level of turbulence was measured. The results also indicate that the ramp influence area for on-ramps is larger than for off-ramps and pre-allocation and anticipation were found to be of little influence for turbulence. For on-ramps mainly secondary lane-changes create turbulence downstream of the ramp. These secondary lane-changes might also explain the increased intensity of keeping right lane-changes downstream of the on-ramp. Not all measured lane-changes can directly be linked to entering or exiting traffic. Lane-changes to the inside and outside of the motorway, which are not triggered by entering or exiting vehicles nearby, are present over the whole measured area.

Most lane-changes were found to be located within close proximity of a ramp gore: a substantial amount of all lane-changes takes place at the acceleration lane (33–55%) and the deceleration lane (47–61%). Only a limited amount of lane-changes are performed further downstream or upstream of a ramp. For on-ramps it was found that 4–9% of all lane-changes involved motorway drivers that anticipated on entering traffic, by changing lanes towards the inside of the motorway, at about 25–100 m upstream of the on-ramp, in order to avoid or give room to entering vehicles. Drivers performed additional lane-changes towards the inside of the motorway (secondary merge) and towards the outside of the motorway (keeping right) until approximately 475–575 m downstream of the on-ramp. For off-ramps it was found that at the earliest start of the measured area (600, 750 and 500 m upstream of the off-ramp), most exiting drivers (96, 86 and 91%) were already driving on the outside lane. Drivers started to pre-allocate upstream of the off-ramp in three different stages: 1) at more than 750 m upstream of the ramp; 2) at approximately 600 m upstream of the ramp, where an exit sign is located; 3) at approximately 200–400 m upstream of the ramp. Downstream of the off-ramp the number of lane-changes was limited and mostly involved lane-changes towards the most right lane (keeping right rule). These lane-changes were performed until approximately 200–375 m downstream of the off-ramp gore.

4.0.2. Use of the acceleration and deceleration lane

Most of merging and diverging lane-changes were performed in the very first part of an acceleration lane, deceleration lane or weaving segment. Fig. 5 and Table 3 show that 65%–95% of the lane-changes are performed in the first 25% of the lane, even in heavy traffic. The corresponding percentages are displayed in Table 3. The lengths which are prescribed in the different design guidelines (see Table 1), to offer drivers space to make lane-changes, are hardly used by drivers.

The figure shows distributions with comparable shapes for a scenario with a low traffic flow. However, a two sample Kolmogorov Smirnov (KS) test showed that the difference between the distributions is significant. In the scenario with a high traffic flow the distribution shapes start to deviate at F(X) = 0.5. For both a high and a low traffic

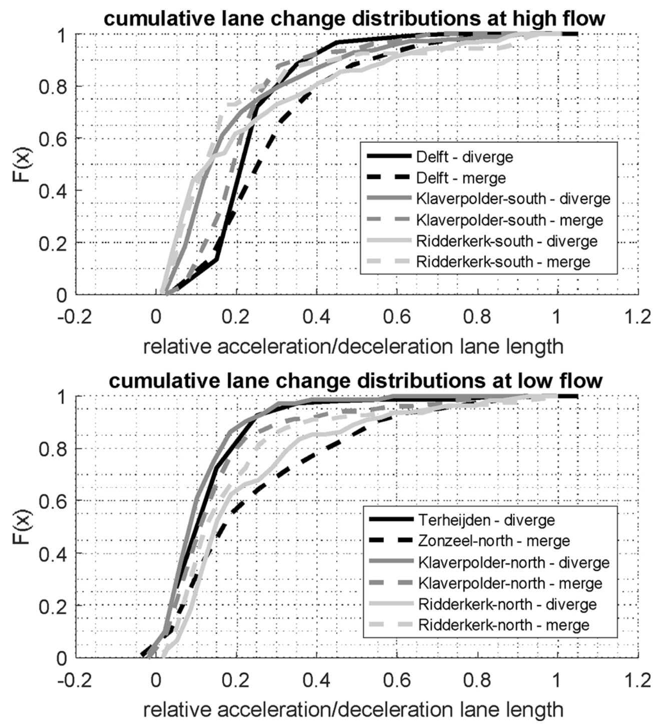

Fig. 5. Use of acceleration and deceleration lane under different conditions.

Table 3
Utilization of the available length for weaving.

	percentage of lane-changes performed in first 25% of the lane	
	high traffic flow (0.74 ≤ F/C ≤ 0.81)	low traffic flow (0.55 ≤ F/C ≤ 0.61)
off-ramp - diverge	80%	95%
on-ramp - merge	65%	68%
short weaving - diverge	80%	95%
short weaving - merge	85%	90%
long weaving - diverge	73%	74%
long weaving - merge	80%	86%

flow on the motorway the use of a long weaving segment by merging vehicles is comparable (KS-test: n1 = 107, n2 = 122, p = 0.624).

4.0.3. Where does turbulence start and end?

The location where turbulence starts and dissolves was also derived from the macroscopic data. The lane flow distribution has been calculated for both the on-ramp and the off-ramp. The fraction of flow was calculated per lane for each detector and was compared to a basic continuous motorway. Fig. 6 shows the results. The calculated fractions of flow are depicted by an 'o'. The thick line represents a fit (moving average over 5 points) and the dashed line represents the average value measured on the basic motorway.

The results show that the lane flow distribution changes near on-ramps and off-ramps. At on-ramps the changes start at about 300 - 200 m upstream where there is a slight shift of traffic from the right lane towards the left lane. Downstream of the on-ramp gore the fraction of flow on the right lane increases. This effect gradually reduces further downstream and is back to normal at about 900 m downstream.

At off-ramps the changes start about 1000 m upstream with a slight shift of traffic from the left to the right lane. At 250 m upstream of the gore the change in fraction of flow is at its highest and seems to be gradually reducing further downstream. However, at 600 m downstream the lane flow distribution is still not comparable to that of the basic continuous motorway.

4.0.4. How well are the characteristics of turbulence simulated

The results for the distribution of lane-change locations are shown in Fig. 7. In this figure the total number or lane-changes is displayed for the empirical data, and the simulated data. It shows that VISSIM generally overestimates the number of lane-changes. MOTUS on the other hand underestimates the number of lane-changes. When looking at the lane-change location it shows that VISSIM locates the lane-changes at the on-ramp (merging) too far upstream, while MOTUS locates these lane-changes too far downstream. For the off-ramp (diverging) VISSIM locates the lane-changes quite accurate, while MOTUS locates it too far downstream.

The simulated mandatory lane-changes were found to be accurate in number. However, the exact location of the simulated lane-changes were found to be too deterministic compared to the empirical data. Some of the entering drivers make an additional lane-change towards the inside of the motorway almost immediately after they have merged into the outside lane. Others stay in the outside lane. Simulations show a more step-wise process, where a vehicle first enter the motorway and then starts to consider an additional lane-change, when it's desired speed cannot be reached due to a slow driving leader. In this way simulated lane-changes for secondary merges are located further downstream than in reality.

The empirical data shows that some of the exiting drivers prefer to pre-allocate long in advance, while others prefer to make a last-moment lane-change. In current simulation models the location where vehicles pre-allocate has less variance.

5. Discussion

According to the motorway design guidelines in different countries, succeeding ramps should be sufficiently spaced to avoid a high level of turbulence in traffic, which is expected to have a negative impact on motorway capacity and traffic safety. The length of area around the ramps, where an increased level of turbulence related to entering or exiting traffic was found, is comparable to the lengths that are mentioned in manuals and guidelines. Therefore, no overlap of influence areas of succeeding ramps is expected to occur when the guidelines are followed.

The manuals and guidelines state that an increase in the length of a weaving segment will reduce the level of turbulence. A weaving segment should have such a length for drivers to perform their lane-changes safely. According to (Rijkswaterstaat, 2017) weaving segment lengths up to 1300 m is recommended for some configurations. By far most lane-changes (low traffic flow: 73–95%, high traffic flow: 74–85%) occurred in the first quarter of the weaving segment, leaving the remaining three quarters mostly unutilized. Looking at Fig. 5 and Fig. 7, a weaving segment longer than 700 m seems unnecessary. Based on these conclusions a revision to the Dutch motorway design guideline was suggested to Rijkswaterstaat and are currently under consideration.

The impact on motorway capacity and traffic safety when deviating from the guidelines and applying a shorter distance between ramps, remains unclear. Fig. 4 shows that the level of turbulence is much higher at the acceleration and deceleration lane, compared to further upstream and downstream. The most important implications for traffic safety and capacity are therefore expected close to the beginning of an off-ramp and an on-ramp. Since the increased level of turbulence at the borders of the ramp influence areas is relatively small, a limited level of overlap between ramp influence areas is not expected to be detrimental for the level of traffic safety and capacity of a ramp.

This differs from the concept that is currently used in motorway design guidelines and manuals, such as (Rijkswaterstaat, 2017; HCM,

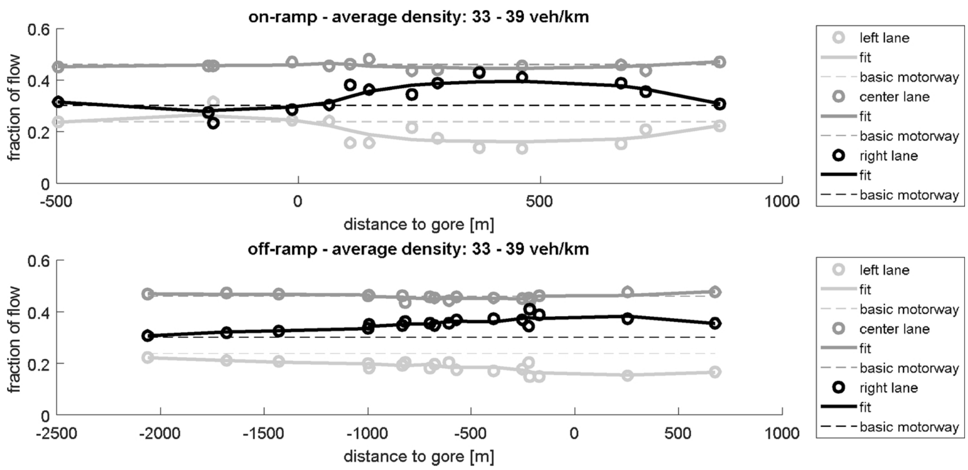

Fig. 6. Lane flow distribution at ramps.

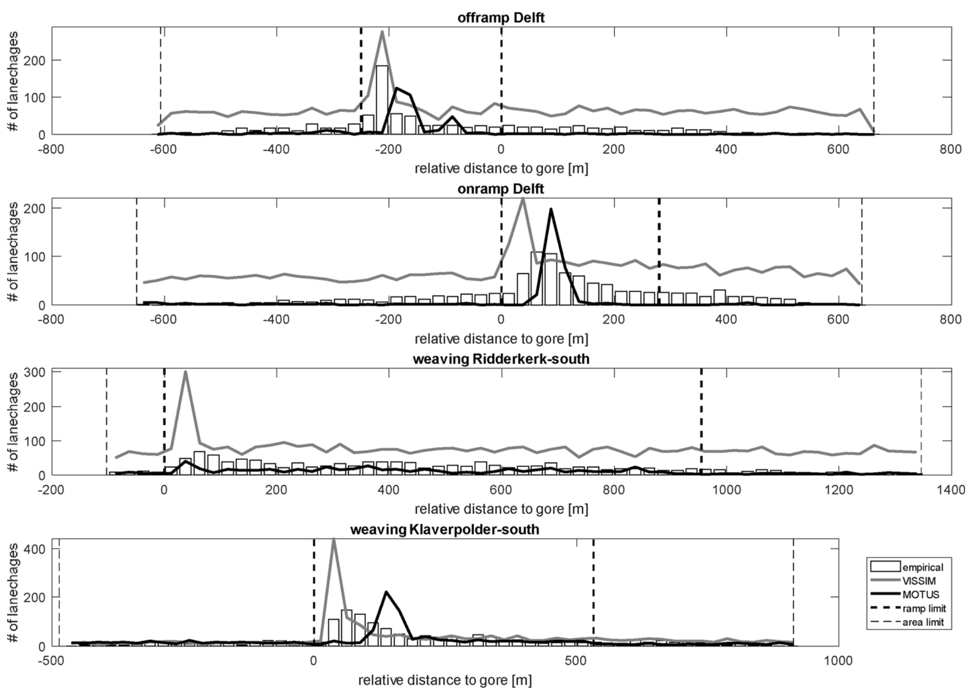

Fig. 7. Comparison of lane-change locations.

2016), which state that any overlap between ramp influence areas (areas around ramps with increased level of turbulence) should be avoided. Further research on the impact of overlapping areas with an increased level of turbulence on traffic safety and capacity is recommended.

The data also suggest that once a driver has the opportunity to change lanes to the deceleration lane he/she desires to changes lanes at the earliest opportunity. The same holds, although to a slightly lesser extent, for entering traffic, which desires to enter the motorway almost directly after the on-ramp gore. The characteristics of the observed manoeuvres by drivers around ramps, suggest that different drivers hold different strategies to enter and exit the motorway. The data suggest that drivers who plan to exit the motorway, base the location of their lane-change on sign posts.

The available length (and time) for path planning seems more important in motorway design than the length of the ramp influence area (turbulence). It is therefore recommended to focus Motorway design guidelines more on timely informing drivers to leave a motorway, and psychologically prepare drivers for that, by placing sign posts or by in-car route navigation systems, rather than on turbulence. The same holds for the guidelines for weaving segment lengths.

The currently availably microscopic simulation software packages seem yet unable to reproduce the location and intensity of lane-changes accurately, which are the key elements in driving behaviour with respect to turbulence. The data suggests that drivers plan their path to enter or exit the motorway in advance. The investigated microscopic simulation models fail to reproduce these characteristics realistically. For example: the current mechanisms in driver behaviour models seem to be unfit to simulate pre-allocation realistically. Furthermore, the data suggests that different drivers hold different strategies for planning their path. These differences in strategy are only programmed in microscopic simulation models to a limited extent, for example by implementing an “aggressiveness” factor that increases maximum acceleration and deceleration rates and decreases critical gap values.

In order to simulate driving behaviour around ramps accurately, microscopic simulation models need to reproduce these rather complex driver decision processes. The way driver behaviour is modelled is, for good reasons, often quite simplistic, and is mostly built upon a few

Table 4
Ramp influence areas.

on-ramp		off-ramp		source
upstream [m]	downstream [m]	upstream [m]	downstream [m]	
25-100	475-575	400-600*	200-375	(Van Beinum et al., 2018)
200	900	1000	–	(Van Beinum et al., 2017)
110	260	–	–	(Kondyli and Elefteriadou, 2012)
460	460	460	460	(HCM, 2010)
150	750	750	150	(Rijkswaterstaat, 2017)

* location of sign post.

basic assumptions and mechanisms. These simple mechanisms result in lane-change locations which are less spread out, as compared to the empirical data. In their present form, both VISSIM and MOTUS seem unsuitable for assessing the implications of turbulence realistically. The following recommendations for further research to improve driving behavioural models are given:

- categorize driving behaviour, not only by longitudinal and lateral behaviour, but categorize them by type of manoeuvre and model the behaviour during these manoeuvres accordingly. The most prominent manoeuvres to improve are: pre-allocation, secondary merges and keeping right;
- different drivers are expected to have different strategies when entering or exiting a motorway at ramps and at weaving segments. Additional research is recommended to identify these strategies;
- the number of discretionary lane-changes, as reproduced by microscopic simulation models, is not accurate. Additional research is recommended on discretionary lane-change incentives, the desire to change lanes, and the factors that influence lane-change decisions, for discretionary lane-changes.

Vehicle interactions were proven to be simulated relatively accurate for car following behaviour and gap acceptance. For the details on these results is referred to (Van Beinum, 2018b). Microscopic simulation models seem therefore fit to study the characteristics of vehicle interactions at specific locations in the design. For example by assessing surrogate safety measures.

For standard elements of a road design, such as a basic weaving segment, a standard on-ramp or a standard off-ramp, the inaccuracies of the investigated microscopic simulation models is expected to be limited, since a lot of research and experience is available for these situations. For unconventional, or ‘fit for purpose designs’ this problem is expected to be more important. It is recommended not to use microscopic simulation software to quantify traffic safety of complex, unconventional designs.

6. Conclusions

Schermers et al (2013) questioned the underpinning (with knowledge from research) of existing guidelines for motorways on the concept of turbulence in the vicinity of on- and off-ramps and in weaving segments. Inspired by their findings, the aim of this study was to gain more understanding on the characteristics of turbulence around on-ramps, off-ramps and in weaving segments, based on empirical data.

For this study, a unique set of trajectory data was collected (Van Beinum, 2018a). This dataset contains precise vehicle location information (*x,y,time*) of each individual vehicle at fourteen different locations in The Netherlands: three on-ramps, three off-ramps and eight weaving sections. The size, quality and characteristics of this data set are unprecedented. A thorough analysis of the data was performed and gave new, unique, insights in the empirical characteristics of turbulence in weaving segments and the vicinity of ramps.

From the collected empirical trajectory data, different manoeuvres were identified that are performed by drivers that either enter or exit the motorway, and by drivers that anticipate on or cooperate with entering or exiting vehicles. The observed manoeuvres were analysed in order to gain knowledge on the characteristics of turbulence and the appropriateness of motorway design guidelines. Furthermore, the characteristics of these manoeuvres were compared to the manoeuvres as replicated by two microscopic simulation software packages (VISSIM and MOTUS) to assess whether these simulation models are adequate for functioning as a design tool.

Lane-changes that are related to vehicles that enter or exit the motorway, were found to be the most important source of turbulence. The empirical observations indicate that most lane-changes are located in immediate proximity of a ramp, at the beginning of an acceleration or deceleration lane. The number of lane-changes further upstream or further downstream is much smaller than at the very beginning of an acceleration/deceleration lane. The distance over which the level of turbulence increases further upstream and further downstream of a ramp, is different for on-ramps and off-ramps. At on-ramps an increased level of turbulence is mainly present downstream of the on-ramp, and at off-ramps an increased level of turbulence is mainly present upstream of the off-ramp. Based on the measured increase in the level of turbulence, ramp influence areas were defined and summarized in Table 4.

The increased level of turbulence was found to be relatively small at the borders of the areas influenced by turbulence. In fact, only in the immediate proximity of a ramp - near the acceleration/deceleration lane - a significantly higher level of turbulence was observed.

Vehicles that exit the motorway were found to change lanes to the deceleration lane at the earliest opportunity. The same behaviour was, to a slight lesser extent, observed for vehicles that enter the motorway; most lane-changes from the acceleration lane are performed almost directly after the on-ramp gore. This is comparable to earlier findings (Polus et al., 1985; Daamen et al., 2010) and comparable for both ramps and weaving segments and it is the case for weaving segments with different lengths. The same holds for the guidelines for weaving segment lengths. Since 65%–95% of the lane-changes are performed in the first 25% of the weaving segment

The characteristics of the simulated manoeuvres deviate from the observed characteristics. With respect to turbulence; both the location and number of lane-changes are simulated inaccurately and inconsistently. The mandatory lane-changes were found to be accurate in number, but inaccurate in location, with considerable differences between VISSIM and MOTUS. For discretionary lane-changes, the simulated number of lane-changes were found to be inaccurate. VISSIM overestimates the number of lane-changes, while MOTUS underestimates the number of lane-changes.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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