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# Comparison of univariate and two-stage approaches for estimating crash frequency by severity—Case study for horizontal curves on two-lane rural roads

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ABSTRACT

The Highway Safety Manual (HSM) procedures apply specific safety performance functions (SPFs) and crash modification factors (CMFs) appropriate for estimating the safety effects of design and operational changes to a roadway. Although the applicability of the SPFs and CMFs may significantly vary by crash severity, they are mainly based on total crash counts, with different approaches for estimation of crashes by crash severity. The variety of approaches in the HSM and in the literature in general suggests that there may be no one best approach for all situations, and that there is a need to develop and compare alternative approaches for each potential application. This paper addresses this need by demonstrating the development and comparison of alternative approaches using horizontal curves on two-lane roads as a case study. This site type was chosen because of the high propensity for severe crashes and the potential for exploring a wide range of variables that affect this propensity. To facilitate this investigation, a two-stage modeling approach is developed whereby the proportion of crashes for each severity level is estimated as a function of roadway-specific factors and traffic attributes and then applied to an estimate of total crashes from an SPF. Using Highway Safety Information System (HSIS) curve data for Washington state, a heterogeneous negative binomial (HTNB) regression model is estimated for total crash counts and then applied with severity distribution functions (SDFs) developed by a generalized ordered probit model (GOP). To evaluate the performance of this two-stage approach, a comparison is made with predictions directly obtained from estimated univariate SPFs for crash frequency by severity and also from a fixed proportion method that has been suggested in the HSM. The results revealed that, while the two-stage SDF approach and univariate approach adopt different procedures for model estimation, their prediction accuracies are similar, and both are superior to the fixed proportion method. In short, this study highlights the potential of the two-stage SDF approach in accounting for crash frequency variations by severity levels, at least for curved two-lane road sections, and especially for the all too frequent cases where samples are too small to estimate viable univariate crash severity models.

## 1. Introduction

In the Highway Safety Manual (HSM), safety performance functions (SPFs) and crash modification factors (CMFs) are used to predict number of crashes in evaluating the safety effects of design and operational changes (AASHTO, 2010). Although the applicability of SPFs and CMFs may significantly vary by crash severity, the first release of the HSM focused on total crash counts, with only limited consideration of crash severity distributions (Geedipally et al., 2013). In that first release, when predicting crashes by severity using a safety performance function (SPF), a simple two-stage approach is used whereby the total number of crashes for a specific entity is first estimated; then crash frequency by severity is estimated by applying a fixed proportion for each severity to the estimate for total crashes. However, since severity distribution, and consequently the proportion for each severity level, may differ by segment characteristics and other factors, using a fixed proportion is questionable and may lead to biased predictions. A subsequent HSM update provided an improved two-stage methodology for

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estimating crashes by severity for freeway segments and interchanges; in this, the proportions applied to an SPF prediction vary according to a severity distribution function (SDF) that is estimated from a discrete choice probability model. In the literature there are many variations of the probability modeling aspect. For example, Geedipally et al. (2013) used a multinomial logit formulation for estimating crashes by severity for freeway segments and interchanges.

More recently, univariate count models have been proposed and developed for estimating crashes for various severity levels, where possible, for a planned update of the three HSM "predictive" chapters in the first HSM release (Ivan et al., 2018). This is the approach used in research related to this paper by Saleem and Persaud (2017), who disaggregated crashes on two-lane rural highway curves into two categories of no injury and injury (including fatal) crashes and developed separate negative binomial (NB) models.

While univariate models expand our understanding of crash severity distributions, they do so at the expense of neglecting the correlations that may exist among different severity levels. In other words, crash severities may share unobserved attributes at different severity levels, which a univariate modelling approach fails to take into account. To address this issue, several efforts have been made to jointly model crash counts by severity based on a multivariate modelling approach. Notable multivariate model specifications in the literature include multivariate Poisson (MVP) (Ma and Kockelman, 2006; Karlis, 2003), multivariate tobit regression (Anastasopoulos et al., 2012; Zeng et al., 2017), multivariate negative binomial regression (MVNB) (Rodgers and Leland, 2005), multivariate Poisson gamma mixture (MVPG) (Mothafer et al., 2016), and multivariate Poisson-lognormal (MVPLN) (Ye et al., 2009; Wang et al., 2017; El-Basyouny et al., 2014; Park and Lord, 2009).

A key difficulty when developing both univariate and multivariate crash count models is that they can be challenging to estimate in cases where crash data are characterized by a small sample size and low sample mean (Lord and Mannering, 2010; Wang et al., 2011), as this is typically the case for many crash severity specifications. The issue of small sample size and low sample mean may cause less stable results and biased parameter estimates (El-Basyouny and Sayed, 2009). Indeed, Ivan et al. (2018) failed to develop univariate models for several crash severities for various site types, which will leave HSM users to continue to resort to the simple two-stage approach for these cases.

To address the limitations of both univariate and multivariate models, other researchers, e.g., Wang et al. (2011) and Geedipally et al. (2013), suggested an alternative two-stage frequency-severity modeling approach that applied severity distribution functions (SDFs) to SPFs. A SDF, as it relates to Highway Safety Manual (HSM) applications, is based on a discrete choice probability model that estimates the proportion of crashes at each severity level using non-crash-specific data (i.e., variables describing a site's design and operational characteristics), resulting in the estimated proportions becoming specific to an individual site (segment or intersection) in a specific jurisdiction (Bonneson et al., 2012). The proportions obtained by the SDF can be applied to a higher level SPF to predict the number of crashes for each severity category (Geedipally et al., 2013). This modelling approach may lead to some loss in prediction accuracy since the SDF, of necessity, neglects crash-specific variables such as vehicle type and involved person age that are typically not available for HSM-type site-level predictions. Moreover, recent studies (Anastasopoulos and Mannering, 2011) noted that random parameter models using only non-crash-specific data still can provide accurate predictions, implying that the limitations of the SDF approach can be potentially minimized when such models are employed.

Given the lack of uniform definitive guidance on how crash frequency by severity should be estimated and the variety of modeling issues involved in developing count and probability severity models, it seems of interest to build on the research by Wang et al. and Geedipally et al. to explore the development of alternative approaches and to also demonstrate how one can be selected for a given application. In so doing, a case study pertaining to horizontal curves on two lane rural roads is investigated to see if the alternative approaches can be applied to specific highway design elements where crash severity may be more of a concern. The average crash rate for horizontal curves is about 1.5–4 times the average crash rate for straight sections (Zegeer et al., 1992), a disproportionality that can be more pronounced if crash severity is also taken into consideration. For example, based on Fatality Analysis Reporting System (FARS) and National Automotive Sampling System (NASS) data sets, Troxel et al. (1994) reported that the probability of fatal crashes per kilometer on curved roads is nearly three times higher than that on straight sections. Although studies have developed some probability models of crash severity at curves (Schneider et al., 2009), they could not offer a straightforward framework to predict the expected number of crashes for each severity level on any given curve. This is because employing crash-specific variables in the probability models makes the prediction dependent to non-segment-specific factors (e.g., weather or lighting conditions, driver features, etc.), which cannot be easily estimated in applying such models for crash prediction.

## 2. Methodology

This section presents an overview of three techniques for estimating crash counts by severity that are compared in the paper – the fixed proportion and SDF two-stage approaches and the univariate modeling approach. The goodness-of-fit (GOF) measures applied to compare the techniques are the Mean Prediction Bias (MPB), Mean Absolute Deviation (MAD), and Mean Squared Prediction Error (MSPE). Cumulative Residual (CURE) plots also provide further insights in evaluating and comparing approaches.

### *2.1. Univariate count-data models*

Univariate count-data models are applied in this study to separately estimate crash counts for different severity levels. In addition, a model is also developed for total number of crashes for use in the two-stage SDF approach.

Count-data models, such as Poisson and negative binomial (NB) regression models, are traditionally used to model the count of traffic crashes, which are viewed as a discrete, random, and non-negative integers. The NB model is preferred in that it accommodates the overdispersion that typically characterizes crash data by including an error term in the Poisson model and allowing the variance to differ from the mean. The heterogeneous negative binomial (HTNB) regression model is now commonly adopted to allow the overdispersion parameter ($\alpha_i$) to vary across road sites as a function of roadway traits, e.g., length of a road segment. If no variable is found to contribute significantly to the overdispersion parameter, then the term $\alpha_i$ will take a constant value, reverting to a standard NB model (Abdelwahab and Abdel-Aty, 2004). Indeed, as Mitra and Washington (2007) found, extra-variation is a function of covariates when the mean structure (expected crash count) is poorly specified or suffers from omitted variables. That the overdispersion parameter may be variable in this study is as likely to arise from omitted variables not available in the dataset used as it is from the tried and tested mean function adopted.

In this study, total crashes are disaggregated into different severity levels; thus, it is expected that there can be a large number of road segment curves for which no crashes occurred during the study period, especially for disabling or fatal injuries. This implies that the presence of excess zero counts is plausible, which may potentially affect overdispersion in crash data. In such conditions, standard count models, such as Poisson and NB, fail to adequately handle the overdispersion resulting from a proliferation of zero counts. To better fit such data, zero-altered models, including hurdle Poisson (HPO) and hurdle NB (HNB) models, have been proposed.

For the current study, four count models including NB, HTNB, HPO, and HNB models have been developed to determine factors associated

**Table 1**
Description of count-data models applied in this study.

| Model | $Pr(y_{it} \mid \mu_{it})$ | Model specification |
|---|---|---|
| NB | $\frac{\Gamma(y_i+\theta)}{\Gamma(\theta)y_i!}\left(\frac{\mu_i}{\mu_{it}+\theta}\right)^{y_i}\left(\frac{\theta}{\mu_i+\theta}\right)^{\theta}$ | To handle overdispersion due to unobserved heterogeneity with fixed dispersion parameter |
| HTNB | $\frac{\Gamma(y_i+\theta_i)}{\Gamma(\theta_i)y_i!}\left(\frac{\mu_i}{\mu_i+\theta_i}\right)^{y_{it}}\left(\frac{\theta_i}{\mu_i+\theta_i}\right)^{\theta_i}$ <br> $\alpha_i = \frac{1}{\theta_i} = exp(\gamma_0 + \gamma_1 Z_{i1} + \gamma_2 Z_{i2} + ...+\gamma_m Z_{im})$ | To handle overdispersion due to unobserved heterogeneity with varying dispersion parameter |
| HPO | $(Y = y_i) = \begin{cases} P_i\, y_{i=0} \\ (1-P_i)\frac{e^{-\mu_i}\mu_i^{y_i}}{(1-e^{-\mu_i})y_i!}\, y_i > 0 \end{cases}$ | To handle overdispersion due to excess zeros |
| HNB | $P(Y = y_i) = \begin{cases} P_i\ y_{i=0} \\ (1-P_i)\left(1-\frac{1}{(1+\alpha\mu_i)^{1/\alpha}}\right)\left(\frac{\Gamma(y_i+\frac{1}{\alpha})(\alpha\mu_i)^{y_i}}{\Gamma(y_i+1)\Gamma(\frac{1}{\alpha})(1+\alpha\mu_i)^{(y_i+1/\alpha)}}\right) y_i > 0 \end{cases}$ | To handle overdispersion due to both unobserved heterogeneity and excess zeros |

with traffic crashes in different injury severity categories. A logit model was used for the hurdle parts of HPO and HNB models. Table 1 presents the specification of these models. Detailed information on the zero-altered models is available in other studies (Easa and You, 2009; Hosseinpour et al., 2016; Lord et al., 2007, 2005).

### 2.1.1. Model specification tests

A number of statistical tests were used to select the best-fit model for each severity category as well as for total crashes. These include a deviance statistic used to check goodness-of-fit (GOP), a likelihood ratio test (LRT) to compare nested models (e.g., NB vs. HTNB or HPO vs. HNB), a Vuong test (Vuong, 1989, Shankar et al. 1997) to compare non-nested models (e.g., HTNB vs. HNB or PO vs. HPO), and information-based criteria, including Akaike's information criterion (AIC) and Bayesian information criterion (BIC), used to compare both nested and non-nested models (Hosseinpour et al., 2013). These selection criteria are summarized and presented in Table 2.

## 2.2. Two-stage SDF method

In the two-stage SDF method, as proposed by Geedipally et al. (2010), the process for predicting the number of crashes by severity involves two stages. First, a crash-frequency model is adopted to predict the count of crashes occurring at curve sections. Second, a SDF model is employed to estimate the proportions of different severity levels for a curve section. Then, the predicted crash counts for a certain crash severity can be obtained by multiplying the number of crashes by the proportion of that severity category. Details on these two stages are discussed in the following subsections.

### 2.2.1. Estimating a SDF model

As mentioned earlier, a SDF is a probability model for estimating crashes by severity levels based on roadway-specific variables. Since severity levels are reported as categorical data, discrete outcome models constitute the most widely-used modelling approach for such data (Kockelman and Kweon, 2002; Xie et al., 2009; Anarkooli et al., 2017; Geedipally et al., 2013; Anarkooli and Hosseinlou, 2016). These types of models can be classified as either nominal (e.g., multinomial logit models, nested logit models, and mixed logit models) or ordinal (e.g., ordered probit/logit models). There is no consensus on which model performs the best, as the selection of the appropriate methodological approach depends heavily on the characteristics of the data (Savolainen et al., 2011). Since injury severity levels are commonly recorded in the ordinal scale, some studies suggested that nominal models, while accounting for the categorical nature of dependent variables, do so at the expense of neglecting the ordered nature of the injury levels (Mooradian et al., 2013; Anarkooli et al., 2017). More specifically, multinomial logit models, arguably the most widely used nominal response variable, completely ignore the sequential order of injury severity levels, which can lead to biased estimations.

Among ordered discrete models, ordered probit model is the most prominent approach used for traffic crash severity analysis (Ye and

**Table 2**
Model selection criteria used for nested and non-nested models.

| Models | Tests | Decision |
|---|---|---|
| **Overall GOF** | Deviance $= \chi^2 = -2*[LL_\beta - LL_O]$ <br> where $LL_\beta$ and $LL_0$ are respectively the log-likelihood of the current model and the null model. | A significant value for the deviance statistic indicates that the model is preferred to its null counterpart, implying a good fit for that model (Hilbe, 2011; Lovegrove and Sayed, 2007). |
| **Nested** | $LRT = -2*(LL_{HTNB\ or\ HNB} - LL_{NB\ or\ HP}) \cong \chi^2$ | A significant value for LRT indicates that the overdispersion in the crash data is present. In this case, the NB-based models would be preferred to the Poisson counterparts. Otherwise, the Poisson-based models are more plausible (Khan et al., 2011; Gao and Khoshgoftaar, 2007; Lewin et al., 2010). |
| **Non-nested** | Vuong test: $V = \frac{\overline{m}\sqrt{n}}{SD(m)}$ <br> where $m_i = ln\left(\frac{\sum_i P_1(y_i \mid x_i)}{\sum_i P_2(y_i \mid x_i)}\right)$; $\overline{m}$ is the mean of $m_i$ and $SD(m)$ is the standard deviation of $m_i$; $P_1(y_i \mid x_i)$ and $P_2(y_i \mid x_i)$ are the predicted probability of the standard models (PO and NB) and the two-state model (HPO and HNB models), respectively. | If $V > 1.96$, then the test favours HPO/HNB over PO/NB, and if $V$ is lower than $-1.96$, the parent PO or NB model is favored. A value of $-1.96 < V < 1.96$ indicates neither model is preferred over the other (Khan et al., 2011; Kweon, 2011). |
| **Nested & Non-nested** | $AIC = -2LL + 2P$ <br> $BIC = -2LL + P(ln(n))$ <br> where $LL$ is the logarithm of the maximum likelihood estimation for each model, $P$ is the number of model parameters, and $n$ is the number of observations ($n = 4059$). | A model with the lowest AIC and BIC values is preferred. To decide whether there is a statistically significant difference between two models, Hilbe's AIC and Raftery's BIC rule-of-thumb criteria were adopted in this study (Hilbe, 2011; Raftery, 1995). |

**Table 3**
Summary statistics of data used for the model development.

| Variable | Mean | SD | Min | Max | Frequency |
|---|---|---|---|---|---|
| Speed limit (mph) | 52.42 | 8.28 | 25 | 65 | 4059 |
| Ln(AADT) | 8.25 | 0.91 | 10.21 | 4.95 | |
| Truck percentage (%) | 15.18 | 8.23 | 0 | 61.52 | |
| Lane width (ft) | 11.75 | 1.64 | 9.00 | 31.00 | |
| Curve length (ft) (Thousands) | 0.996 | 0.914 | 0.037 | 9.747 | |
| Curve radius (ft) (Thousands) | 2.222 | 2.166 | 0.100 | 11.460 | |
| Grade percentage (%) | 2.07 | 1.96 | 0 | 10.25 | |
| Vertical curve length (ft) (Thousands) | 0.432 | 0.508 | 0 | 4.800 | |
| Shoulder width (ft) | 4.83 | 2.44 | 0 | 16.00 | |
| Shoulder type | | | | | |
| Asphalt[a] (1 if asphalt or bituminous; otherwise 0) | n/a | n/a | n/a | n/a | 3815 |
| Gravel (1 if gavel, otherwise 0) | n/a | n/a | n/a | n/a | 82 |
| None (1 if soil and others; otherwise 0) | n/a | n/a | n/a | n/a | 122 |
| Road functional class | | | | | |
| Rural collector[a] (1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 731 |
| Rural principal arterial (1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 2029 |
| Rural minor arterial(1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 1299 |
| Severity | | | | | |
| PDO (1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 5064 |
| Possible (1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 1519 |
| Non-disabling (1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 1350 |
| KSI (1 if true; otherwise 0) | n/a | n/a | n/a | n/a | 506 |

[a] Reference case for categorical variables.

Lord, 2014). The model is formulated by defining a latent and continuous variable $y_i^*$ as follows (Washington et al., 2009):

$$y_i^* = X_i\beta + \varepsilon_i \ \ \forall \ i \tag{1}$$

where $X_i$ is a vector of explanatory variables for crash i; $\beta$ is a vector of estimable parameters; and $\varepsilon_i$ is the random error term capturing the effect of unobserved factors, which is assumed to be normally distributed with a mean of zero and a variance of 1.

For any crash $i$ on the curves, the observed injury severity level ($y_i$) can be attributed to the latent counterpart ($y_i^*$) in the following way:

$$y_i = j \Rightarrow \mu_{j-1} \le y_i^* \le \mu_j \Leftrightarrow \begin{cases} 1 \ if \ -\infty \le y_i^* \le \mu_1 \\ 2 \ if \ \mu_1 < y_i^* \le \mu_2 \\ 3 \ if \ \mu_2 < y_i^* \le \mu_3 \\ 4 \ if \ \mu_3 < y_i^* \le \infty \end{cases} \tag{2}$$

where j represents severity level (e.g., in this study j = 1 corresponds to PDO category), $\mu_1$ and $\mu_2$ are threshold parameters to be estimated.

Given the value of X, the probability of an individual crash resulting in injury severity level j can be defined as follows:

$$P(y_i = j) = F(\mu_j - X_i\beta) - F(\mu_{j-1} - X_i\beta) \tag{3}$$

where $F(.)$ stands for standard normal cumulative distribution function.

However, there are some restrictions in a standard ordered probit (OP) model, which can make this approach a questionable one for crash severity analysis. First, ordered probability models can be particularly susceptible to underreporting of crash-injury data, resulting in biased or inconsistent parameter estimates. This underreporting is not expected to be a significant issue for this study, given the recognized high quality of HSIS data used. Second, the standard OP model adopts the parallel-slope assumption, which constrains the effect of regression parameters to be consistent across different severity levels, can restrict the influence of explanatory variables on severity outcomes (Yasmin and Eluru, 2013). More specifically, when a variable increases (or decreases) the probability of the highest severity level, it necessarily decreases (or increases) the probability of the lowest severity level, an outcome that is not always true.

The generalized ordered probit (GOP) model is an efficient way to address the parallel-slope assumption limitation and was found to be the best for the data used in this study. This approach, while controlling the ordinal nature of the severity data, relaxes the parallel-slope assumption by allowing the threshold values to vary across road segments. This property of the GOP model makes it similar to the partial proportional odds (PPO) model (Sasidharan and Menendez, 2014); however the two models differ in the assumptions for the error term.

The threshold values for the GOP model adopted for this research are a function of the explanatory variables as follows:

$$\tilde{\mu}_j = \mu_{ij} - X_i' k_j \tag{4}$$

where $\tilde{\mu}_j$ is a constant term; $\mu_{ij}$ is the threshold value for segment i and severity level j; $k_j$ represents the influence parameter of the covariates on the thresholds;

Substituting $\tilde{\mu}_j$ for $\mu_j$ in Eq. (4) leads to

$$P(Y_i = j) = \Phi(\tilde{\mu}_j - X_i\beta_j) - \Phi(\tilde{\mu}_{j-1} - X_i\beta_{j-1}) \tag{5}$$

### 2.2.2. Estimating the proportion for each severity level

Based on the estimated results of the GOP and total-crash models, the predicted crash counts for each severity level at a curved segment can be determined by multiplying the estimated total number of crashes from a count model by the predicted probabilities of the severity categories, which are obtained from the GOP model. This can be calculated using the following equation:

$$\hat{N}_{ij} = \hat{P}_{ij}.\ \hat{N}_i \tag{6}$$

where $\hat{N}_{ij}$ is the predicted crash counts of injury severity j at curved section i; $\hat{N}_i$ is the total number of crashes for section i, as estimated by the total-crash model; and $\hat{P}_{ij}$ is the probability of occurrence of the severity level j on section i.

### 2.3. Fixed proportion method

As noted in the introduction, this is a simple two-stage approach as used in the Highway Safety Manual. In this, the crash frequency by severity is estimated by applying a fixed proportion for each severity to the total crashes predicted from the same SPFs estimated for the two-stage SDF approach. To obtain the fixed proportions, the observed crash counts for each severity level in the estimation dataset is simply divided by the observed total crash counts.

## 3. DATA

Table 3 presents the summary statistics of data used for model development. The data pertain to two-lane rural roads in the state of Washington and are provided by the Highway Safety Information System (HSIS). The dataset consisted of a road inventory file and a curve file, which were merged on the basis of unique location identifiers. Based on Highway Safety Manual (AASHTO, 2010), the minimum curve radius of 100 ft. was used. Likewise, as suggested by Bauer and Harwood (2013), curves with radius larger than 11,460 ft. can be classified as tangents for practical purposes, and so were excluded. Ultimately, the analyses were conducted on 4059 curves, which had 8440 crashes during a 6-year period from 2009 to 2014.

Crashes recorded in HSIS dataset are categorized into five severity levels, constituting the KABCO injury scale, namely, fatality (K), disabling injury (A), non-disabling injury (B), possible injury (C), and PDO (O). However, due to the limited number of K and A crashes, these two injury levels were combined to produce a new category named KSI (killed or seriously injured). The KABCO injury scale was collapsed into four categories: KSI (KA), non-disabling injury (B), possible injury (C),

**Table 4**
Parameter estimates of the fitted models.

| | PDO | | Possible Injury | | Non-disabling Injury | | KSI | | TOTAL | |
|---|---|---|---|---|---|---|---|---|---|---|
| The best-fit model | HNB | | HTNB | | HPO | | HPO | | HTNB | |
| | Mean part | Zero part | Mean part | Dispersion part | Mean part | Zero part | Mean part | Zero part | Mean part | Dispersion part |
| Intercept | −8.388 | −2.189 | −6.544 | −9.526 | −5.913 | −2.372 | −1.603 | −3.897 | −2.472 | −16.884 |
| Ln(AADT) | | 0.368 | 0.670 | 0.929 | 0.604 | 0.097 | | 0.157 | 0.378 | 1.623 |
| Shoulder width | | | −0.028 | | | | | | −0.01 | |
| Shoulder type | | | | | | | | | | |
| Asphalt (as the BC) | | | | | | | | | | |
| Gravel | | | 0.507 | | | | | | | |
| Speed limit | | | | | | 0.010 | | 0.013 | | |
| Curve length | 0.564 | 0.299 | 0.214 | | 0.305 | 0.180 | | 0.195 | 0.254 | 0.276 |
| Curve radius | −0.079 | | −0.047 | | −0.088 | −0.056 | | −0.098 | −0.045 | |
| Vertical curve length | −0.145 | | | | −0.404 | | | | | −0.626 |
| Road functional class | | | | | | | | | | |
| Rural collector (as the BC) | | | | | | | | | | |
| Rural minor arterial | 0.385 | | 0.291 | | | 0.172 | −1.018 | | 0.116 | 1.048 |
| Rural principal arterial | 0.216 | | | | | | | | | 0.653 |
| Dispersion ($\alpha$) for HNB | 0.580 | – | | | | | | | | |
| *Summary statistics* | | | | | | | | | | |
| No. of observations | 4059 | | 4059 | | 4059 | | 4059 | | 4059 | |
| No. of parameters | 11 | | 9 | | 11 | | 7 | | 12 | |
| Log-likelihood at zero ($LL_O$) | −5747.9 | | −3232.5 | | −3005.7 | | −1637.8 | | −6988.5 | |
| Log-likelihood at converge ($LL_\beta$) | −5398.6 | | −3038.8 | | −2944.6 | | −1623.1 | | −6643.0 | |
| ***Deviance*** $= -2[LL_\beta - LL_O]$, (*d.f.*) (*p*-value) | 698.74, (8) (*p*-value < 0.001) | | 387.41, (6) (*p*-value < 0.001) | | 122.18, (9) (*p*-value < 0.001) | | 29.35, (5) (*p*-value < 0.001) | | 691.07, (5) (*p*-value < 0.001) | |
| LRT for nested models, (*d.f.*) | 461.0[a], (1) (*p*-value < 0.001) | | 11.598[b], (1) (*p*-value < 0.001) | | – | | – | | 277.8[c], 5 (*p*-value < 0.001) | |
| Vuong test for non-nested models | 11.12[d] (*p*-value < 0.001) | | – | | 2.766[e] (*p*-value < 0.0028) | | 1.424 [f] (*p*-value < 0.077) | | – | |
| AIC | 10819.13 | | 6095.65 | | 5911.26 | | 3260.29 | | 13309.96 | |
| BIC | 10888.53 | | 6152.43 | | 5980.66 | | 3304.45 | | 13385.66 | |

Note: “BC” stands for reference category. The p-value of all variables is less than 0.05.
[a] LRT for HNB versus HPO.
[b] LRT for HTNB versus NB.
[c] LRT for HTNB versus NB.
[d] Vuong test for HNB versus NB.
[e] Vuong test for HPO versus PO.
[f] Vuong test for HPO versus PO.

and PDO (O). It should also be noted that, in the case of multiple-injury crashes, the crash severity level pertains to the most severely injured occupant in the crash.

## 4. Results and discussion

### 4.1. Crash frequency models

Table 4 presents the results of the ultimately selected count models for four different severity levels as well as for the total number of crashes. The *p*-value of all variables included is less than 0.05, implying a significant effect of the corresponding variable on the response variable. As can be seen in Table 4, the crash counts at different severity levels are attributed to different subsets of explanatory variables. In general, the implied effects of all variables in terms of direction are consistent with logic and with previous research findings. For example, wider shoulder widths are associated with fewer crashes, which is likely explained by the fact that errant vehicles have more recovery area on curves with wider shoulders. This effect may be more pronounced on curved sections since the proportion of single-vehicle crashes on curved sections due to errant vehicles is markedly higher than on straight segments (Schneider et al., 2009). Similarly, in the case of opposite-direction crashes (i.e., head-on and opposite sideswipe), recent studies pointed out that wide shoulders could provide more room for a vehicle in the opposing lane to avoid colliding with an errant vehicle (Ivan et al., 2006; Hosseinpour et al., 2014).

The values of the deviance statistic for all models were estimated to be highly significant, indicating a reasonable statistical fit for each model. The values for the Likelihood Ratio Test (LRTs) for HNB versus HPO and HTNB versus NB show that the selected models are superior to their nested counterparts. In addition, to test the association of excess zeros to overdispersion, a Vuong test was also applied for pairs of HPO vs. PO and HNB vs. NB. The values of Vuong test showed that both HPO and HNB models are preferred over PO and NB models, respectively. Based on the model selection criteria used in this study, the HTNB model was found as the best-fit model for possible injury crashes and total crashes. The appropriateness of the HTNB model over other candidate models is attributed to the fact that the overdispersion in the possible injury and the total crash data arises from unobserved heterogeneity rather than excess zeros, and that the HTNB model is a more flexible approach to address the issue by relating the dispersion parameter to a set of roadway characteristics; thus, that extra variability in crash data could be adequately addressed in the HTNB model compared to the NB model with a constant dispersion parameter. For non-disabling and KSI crashes, the HPO was found to be the best model, while the HNB model was selected as the best-fit model for the PDO injury crashes. The superiority of the HNB model indicates that overdispersion in the crash data is due to both unobserved heterogeneity and excess zeros in the crash data. To sum up, the results indicate that different modelling specifications may be required for crashes at different severity levels.

In the HPO and HNB hurdle models, different explanatory variables are associated in both or either the count part or zero part. For example, for the PDO model, curve length is statistically significant in both the

**Table 5**
GOP modelling results.

| Variable | Possible Injury | | Non-disabling Injury | | KSI | |
|---|---|---|---|---|---|---|
| | Coeff. | *p*-value | Coeff. | *p*-value | Coeff. | p-value |
| Constant | −0.125 | 0.554 | 0.245 | 0.297 | −0.572 | 0.106 |
| Ln(AADT) | −0.019 | 0.254 | −0.14 | < 0.001 | −0.099 | < 0.001 |
| Lane width (ft) | −0.007 | 0.456 | −0.018 | 0.108 | −0.037 | 0.062 |
| Speed limit (mph) | 0.003 | 0.097 | 0.007 | 0.001 | 0.006 | 0.054 |
| Curve radius (ft) | −0.015 | 0.028 | −0.015 | 0.044 | −0.02 | 0.068 |
| Grade percentage (%) | 0.012 | 0.092 | 0.006 | 0.416 | 0.005 | 0.644 |
| Facility type (Rural collector as the BC) | | | | | | |
| Rural principal arterial | −0.091 | 0.002 | −0.026 | 0.41 | 0.021 | 0.645 |
| ***Summary statistics*** | | | | | | |
| No. of observations | | | | | | 8440 |
| No. of parameters | | | | | | 21 |
| Log likelihood at zero | | | | | | −9123.90 |
| Log likelihood at convergence | | | | | | -9038.32 |
| *Deviance* = $-2*[LL_\beta - LL_O]$, (*d.f.*) | | | | | | 171.16, (17) (*p*-value < 0.001) |
| LRT (GOP vs. OP), *(d.f.)* (parallel slope assumption test) | | | | | | 111.3, (12) (*p*-value < 0.001) |

*Note*: BC stands for "based category". The base case for injury severity is "PDO".

count and zero parts, indicating that the length of curve is positively associated with both the likelihood and the frequency of PDO crashes. On the other hand, curve radius is only significant in the count part, not in the zero part.

A similar conclusion can be reached for the HTNB model where several explanatory variables are statistically significant in the dispersion part. For example, in the possible injury model, in addition to the constant term, the natural logarithm of AADT also contributes to the dispersion parameter of the model. Further discussion of the significant variables associated with the severity models follows.

### *4.2. Crash severity models*

Table 5 shows the parameter estimates and the summary statistics (e.g., log-likelihood at convergence, deviance statistic, LRT) for injury severity probability models. The deviance statistic for the GOP model was estimated to be 171.16, indicating an overall good fit for the model. This rejects the null hypothesis that the GOP model is equal to its corresponding constant-only model. The LRT for the GOP model versus the OP model was estimated as 111.3, which is greater than the critical value of 21.03 with 12 degrees of freedom, indicating that the former is significantly superior to the latter. This rejects the hypothesis that the effects of included variables are consistent across severity levels (i.e., the parallel slope assumption).

In general, the implied effects of all variables in terms of direction are logical and in line with previous research findings. For example, speed limit was found to be positively associated with increased levels of injury severity, which could be interpreted for different crash types separately. In case of single vehicle crashes, which are mainly due to centripetal force at horizontal curves, higher running speed increases the risk of losing control of the vehicle, leading to severe collision types such as run-off-road and vehicle rollovers. For opposite direction crashes, since the visibility distance is limited at curved sections, drivers adopting higher speeds have less reaction time to avoid colliding with an errant oncoming vehicle (Hosseinpour et al., 2014), resulting in increased impact speed and severe injuries. This result is also in line with prior studies and reports (Kloeden et al., 2001; Rakotonirainy et al., 2015) that suggest that severe injury crashes at curves are usually associated with a high travel speed. For instance, Rakotonirainy et al. (2015) reported that 73% of fatal crashes occurring on road curves involve travelling at speeds in excess of the posted speed limit.

As another case, it was found that rural principal arterials are associated with decreasing crash severity compared to rural minor arterials and collectors. This finding may be rationalized by higher design standards and better road designs on these roads and is in line with the studies of Chang and Mannering (1999) and Wang et al. (2011), which reported that when a roadway is mainly designed for moving traffic, such as an interstate highway, crashes are more likely to be property damage only.

The results of the three performance measures for the two-stage SDF, univariate, and fixed-proportion approaches are shown in Table 6. Based on the values of MPB, MAD, and MSPE measures, the two-stage SDF and univariate approaches are, by and large, similar in predictive performance and they both perform generally better than the fixed proportion method. The Cumulative Residual (CURE) plots reinforce these observations. The plots for KSI crashes, for example, which are shown in Figs. 1–3, indicate that the residuals plotted by predicted crashes from the two-stage SDF approach and univariate models lie almost entirely within the two standard deviation lines, while the opposite is the case for the fixed proportions method.

These results suggest that the two-stage SDF approach as applied here can be seriously considered for estimating crash counts by injury severity. It is conceptually appealing in that it combines the results of the SDF and total-crash count model while accounting for a number of

**Table 6**
Performance measures for two-stage, univariate, and fixed proportion approaches.

| Measures | PDO | | | POSSIBLE INJURY | | | NON-DISABLING INJURY | | | KSI | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2-Stg[a] | Uni | Fixed | 2-Stg | Uni | Fixed | 2-Stg | Uni | Fixed | 2-Stg | Uni | Fixedd |
| **MPB** | −0.032 | 0.029 | −0.029 | −0.010 | −0.004 | −0.007 | −0.005 | −0.001 | −0.004 | 0.000 | 0.000 | −0.009 |
| **MAD** | 0.858 | 0.887 | 0.982 | 0.484 | 0.481 | 0.529 | 0.469 | 0.471 | 0.473 | 0.226 | 0.228 | 0.224 |
| **MSPE** | 1.895 | 2.883 | 2.458 | 0.452 | 0.444 | 0.528 | 0.366 | 0.365 | 0.387 | 0.135 | 0.135 | 0.139 |

[a] 2-Stg, Uni, and Fixed stand for two-stage SDF, univariate, and fixed proportion methods, respectively.

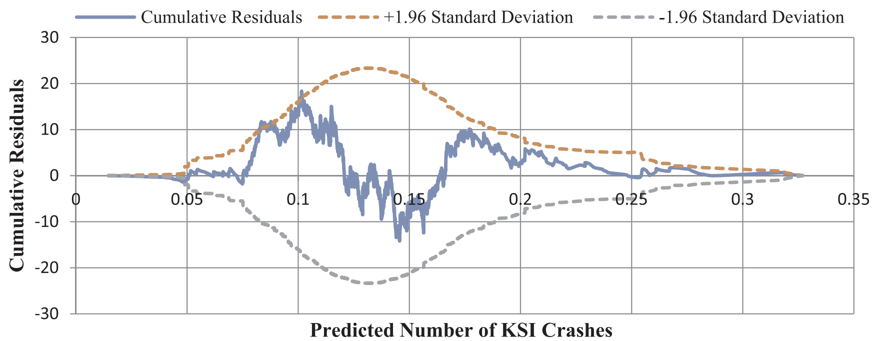


**Fig. 1.** CURE Plot for KSI Crashes Using the Two-stage SDF Method.

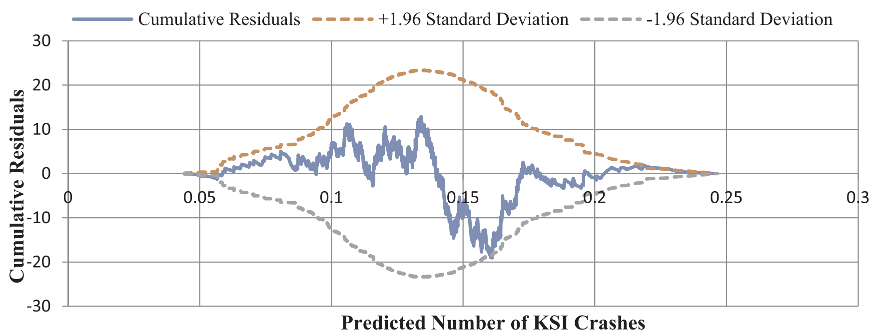


**Fig. 2.** CURE Plot for KSI Crashes Using the Univariate Method.

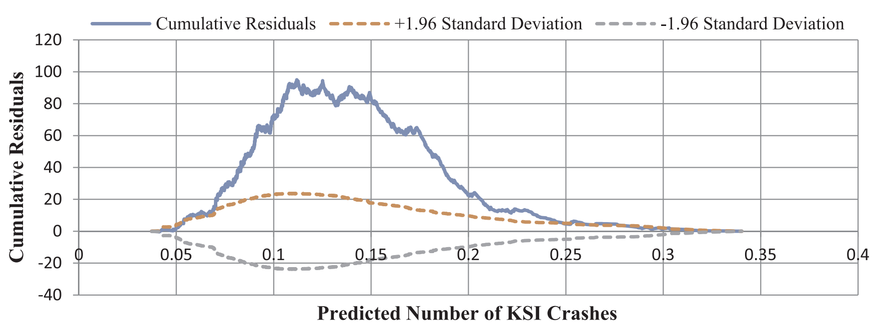


**Fig. 3.** CURE Plot for KSI Crashes Using the Fixed Proportion Method.

issues, such as the ordinal nature of crash severity, unobserved heterogeneity, and parallel slope assumption. In addition, as noted by Bonneson et al. (2012), unlike univariate models, the two-stage SDF approach considers all severity levels together and thus can be used to predict the shift in crashes among levels due to a change in roadway characteristics, while univariate models fail to account the correlations that may exist among different severity levels. Moreover, the development of different univariate frequency models for each possible injury severity level could be challenging with relatively small model estimation samples, as was evidenced with the recent experience in developing crash severity models for the HSM update (Ivan et al., 2018). However, in the two-stage SDF approach, it is only necessary to develop a probability model (i.e., to estimate the proportions for all severity levels) and a frequency model for predicting total number of crashes, both of which could be accomplished with relatively smaller samples.

## 5. Conclusions

The main objective of this study was to demonstrate the development and comparison of alternative approaches for estimating crashes by severity using horizontal curves on two-lane roads as a case study. Specifically, a comparison was made between: a) the application of the two-stage method that applies a severity distribution function (SDF) based on a probability model to a safety performance function (SPF) prediction; b) a univariate method that estimates and applies SPFs for each crash severity level; and c) the method suggested in the initial release of the Highway Safety Manual that applies a fixed proportion for each severity to an SPF prediction for total crashes. Crash data obtained from two-lane rural roads in Washington State for the years of 2009 to 2014 were used to estimate the models for these approaches.

This investigation highlights the potential of the two-stage SDF approach in considering injury severity when estimating SPFs and CMFs. In terms of model fit, coefficients, and significance level of the covariates, the results revealed that the two-stage SDF approach can provide reasonable results for the dataset used. Moreover, when compared with the other two methods, it was found that it performs significantly better than the fixed proportion method and at least as well as the univariate method. Considering that the two-stage SDF approach addresses the limitations of univariate models, such as neglecting the correlations among different severity levels, it may be preferred conceptually over the latter method. Regardless, it can be considered for prediction where samples are too small for estimating viable univariate models, as has frequently been the case in developing crash severity models for the impending Highway Safety Manual update.

The effects of the explanatory variables on crash severity in the probability models generally meet prior expectations. Higher speed limit and vertical grade were found to be two important factors increasing the severity of crashes occurring on curved sections, while increasing curve radius, lane width, traffic volume, and rural principal arterial classification were found to be negatively associated with the increased risks of severe crashes. The results from the two-stage SDF model do support the prevailing belief that flattening curves is a potential safety countermeasure to decrease both frequency and severity of crashes.

The research was as much about methodological approaches as it was about application. In the latter regard, the findings of this study are limited to the data collected on two-lane rural highway curves in the state of Washington, and so are not generalizable. In terms of future research, an evaluation of the approach can be undertaken on data collected on other entities (e.g., multilane highways or intersections) and from other regions where roadway geometry, the environment, and traffic characteristics may differ from the data used in this research. Moreover, depending on different research purposes, a more in-depth understanding of injury severities at curves can be obtained by focusing on specific types of crashes using the two-stage SDF modelling approach. Incorporating both crash severity and crash type does improve SPF and CMF estimation, which in turn leads to the development of effective countermeasures to reduce crash numbers and causalities.

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