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Perceived speed of motion in depth modulates misjudgements of approaching trajectories consistently with a slow prior

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ABSTRACT

Previous studies have shown that the angle of approach is consistently overestimated for approaching (but passing-by) objects. An explanation based on a slow-motion prior has been proposed in the past to account for this bias. The mechanism relies on the (less reliable) in-depth component of the motion being more attracted towards the slow motion prior than the (more reliable) lateral component. This hypothesis predicts that faster speeds in depth will translate into a greater bias if the perception of velocity in depth follows Weber's law. Our approach is different than the one used in previous studies where perceived speed and direction were measured in different experiments. To test our hypothesis, we conducted an experiment in which participants estimated approaching angles via a pointing device, while at the same time comparing the speed of the approaching object with a lateral velocity reference. This way, we couple perceived speed with perceived trajectory for each approaching angle in the same trial. Our results show that the directional bias is larger for faster objects, which is consistent with motion in depth following Weber's law. The differential biases can be accounted for by a Bayesian model that includes a slow motion prior.

1. Introduction

One of the main functions of the visual system is to recover the 3D structure of the environment. This is particularly important when we need to estimate the direction and speed of moving objects on a collision (or near-collision) course with us.

Knowing how different cues, both monocular and binocular, contribute to estimate direction and motion in depth (MID) has attracted interest in the past (Beverley & Regan, 1973; Cumming & Bruce, 1994), but still is an active field of research (Harris, Nefs, & Grafton, 2008; Rokers, Fulvio, Pillow, & Cooper, 2018). Past work on MID, however, has mainly focused on precision and accuracy of motion estimates (Harris & Dean, 2003; Rushton & Duke, 2009).

Regarding the direction of approach, several studies have shown that we tend to overestimate the bearing angle (from now on β; see Fig. 1) of the trajectory of an approaching target (Harris & Drga, 2005; Lages, 2006; Poljac, Neggers, & Van Den Berg, 2006; Welchman, Tuck, & Harris, 2004). This is, we overestimate the lateral distance by which a ball passes us. This can be counterintuitive, given that we are very sensitive to the motion direction of objects on a collision course (Regan, Erkelens, & Collewijn, 1986).

To explain this bias, Welchman, Lam, and Bülthoff (2008) put forward a Bayesian explanation that included the so-called *Slow Motion Prior*, which is a main component of a motion perception model by Stocker and Simoncelli (2006). Sensory estimates (likelihood) are combined with an expectation of nearly zero motion in the environment (prior) resulting in consistent underestimations of speed (posterior), with the extent of underestimation depending on the reliability of the likelihood (e.g. contrast of a grating; Stocker & Simoncelli (2006)). Therefore, if the reliability of the signal is low, the slow prior will be weighted more, resulting in a slower posterior and, consequently, the speed of the stimulus will be underestimated more strongly.

In the same study, Stocker and Simoncelli (2006) found that the width of likelihood estimates for speed discrimination tasks increases logarithmically as a function of speed approximately following Weber's law (for targets moving faster than 1 deg/s), as suggested by previous literature in the field (McKee, Silverman, & Nakayama, 1986; Welch, 1989). Furthermore, they used a Bayesian Observer model to infer the shape (SD) of the Slow Motion Prior, which falls from a peak at slow speeds becoming shallower for faster ones. As a result, the prior expectation introduces increasingly biases for the posterior as a function of the perceived speed.

Welchman et al. (2008) explained the underestimation of approaching angles in terms of this slow prior: The estimate of the lateral

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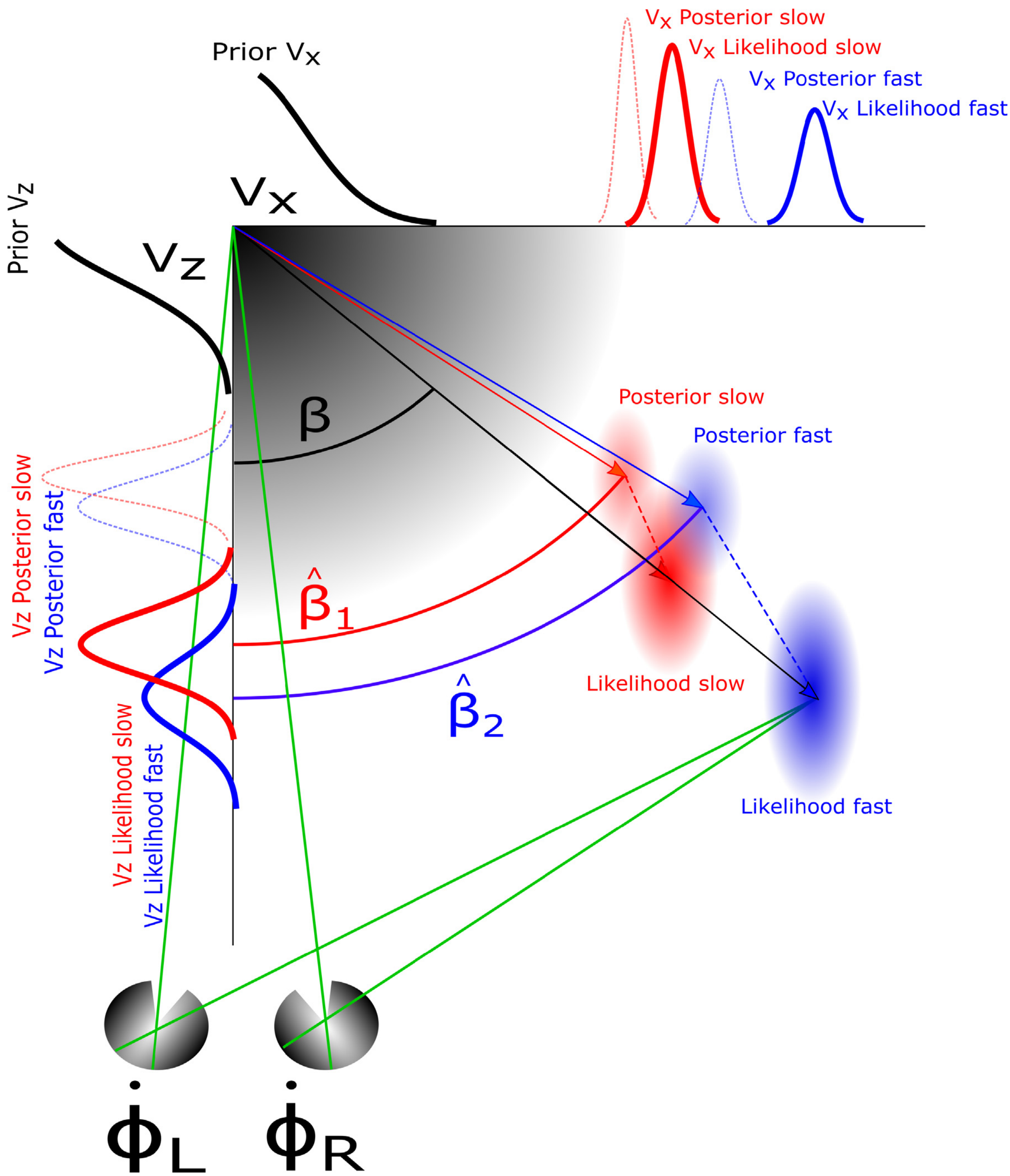

Fig. 1. The prior is represented by the grey radial gradient centered at $V_x = V_z = 0$. Two different movements with the same bearing angle β are depicted in this scene. The speed of each movement is indicated by the color of the arrow: the slow movement is indicated in red whereas the fast movement is indicated in blue. Assuming Weber's law, the faster movement is noisier than the slower one, which is denoted by the SD of the respective likelihood gaussian ones. In addition, V_z is less reliable than V_x (depicted by the spread of the likelihood distributions for each vector, represented as thick lines at the margin). The effect of the prior (grey radial gradient, centered at $V_x = V_z = 0$) affects each speed vector differently. This effect is represented by the shift of the posterior distribution (distributions represented with dotted lines for each vector at the margin). Given that the prior will affect the slow and fast movements differently, the perceived trajectory would depend on the physical speed of the movement while keeping the physical trajectory constant (β). The perceived trajectory is denoted by $\hat{\beta}_1$ for a slow movement (red) and $\hat{\beta}_2$ for a fast movement (blue). The dashed segments connecting the centroids of likelihood and posteriors denote the speed bias for each movement. $\dot{\phi}$ represents rate of azimuth change for each eye. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

component (V_x; Eq. (1)) is more reliable than the estimate of the depth component in MID (V_z; Eq. (2)see Fig. 1).

$$V_x \approx \frac{\dot{\phi} i d}{i + \dot{\delta} d} \tag{1}$$

$$V_z \approx \frac{\dot{\delta} d^2}{i + \dot{\delta} d} \tag{2}$$

V_x estimates are based on the rate of change of the azimuth ($\dot{\phi}$; Eq. (3); Fig. 1) and the rate of change of the disparity ($\dot{\delta}$; Eq. (4)), while V_z depends solely on the rate of change of the disparity. The variance of the azimuth signal is up to two times lower than for disparity. As a consequence, V_x, as it combines both signals, is much more reliable than V_z, which only depends on the less reliable signal, the rate of change of the disparity ($\dot{\delta}$) (see Gaussian curves in Fig. 1). In Eqs. (1) and (2): i

stands for interocular distance and d for viewing distance.

$$\dot{\phi} \approx \frac{\dot{\phi}_L + \dot{\phi}_R}{2} \tag{3}$$

$$\dot{\delta} \approx \dot{\phi}_L - \dot{\phi}_R \tag{4}$$

As Stocker and Simoncelli (2006) pointed out, the greater the reliability of the measure, the lower the effect of the prior and vice versa (Fig. 1). As a consequence, the fact that V_x is much more reliable than V_z should result in a differential influence of the prior for the posterior speed estimate. Under the assumption that Weber's Law holds for motion in depth, yields the prediction that faster velocities should lead to more biased trajectories.

Specifically, our hypothesis suggests that higher speeds in depth would lead to a larger overestimation of the bearing angle. Since we assume that the perception of the trajectory depends on the perceived speed (i.e. how the depth component is affected by the slow motion prior), we can test the hypothesis that the bias can be accounted for by how different speeds are encoded through the mediation of a slow prior. Thus, we investigated if different perceived speeds in depth lead to different degrees of bias on the perceived angle. To do so, we followed a different approach than Welchman et al. (2008). In the present study, we asked for the perceived speed in the same trials used for adjusting the direction. In this way, we can study if perceived speed is related to perceived direction for the same physical trajectories. In this study, only the case in which the initial lateral position of an object is the same as the position of the observer will be taken into account. For a more general approach, see Rokers et al. (2018). As a final step, we formulated a Bayesian model to test the predictions of a slow motion prior model.

2. Methods

2.1. Participants

Eleven observers participated in the experiment. All of them had normal or corrected-to-normal vision and were naive with respect to the purpose of the experiment. One subject was stereo-blind as tested with StereoFly test (Stereo Optical Co.) and had to be excluded from further analysis. The final sample consisted thus of ten participants (n = 10).

The research in this study is part of an ongoing research program that has been approved by the local ethics committee of the University of Barcelona. This study is in accordance with the Code of Ethics of the World Medical Association (Declaration of Helsinki). Before taking part in the experiment, every subject signed an informed consent form.

2.2. Material

2.2.1. Apparatus

Two Sony laser projectors (VPL-FHZ57) were used to provide overlaid images on a back-projection screen (244 cm height and 184 cm width) with a resolution of 1920x1080 pixels. The refresh rate of the image was 85 Hz for each eye. Circular polarizing filters were used to provide stereoscopic images. A pointing device (see Procedure; Fig. 2) was used to record the perceived direction with the position data being acquired by an Arduino board. The device was calibrated to provide linear measures of azimuth within a 180 degrees space. It was calibrated only once to make sure that the measurements were consistent across the participants. A custom input device with 2 buttons was used for the participants to indicate whether the first or second ball moved faster within each trial.

2.2.2. Stimulus

The stimulus consisted of two spheres with a checked texture presented consecutively. The first ball (reference target) moved along the x axis without any depth or vertical component. The second ball (test target) also included a z (depth component) approaching to the observer at different bearing angles (β = 2, 4, 8, 16, 32, 64 degrees) which would pass the observer either to their left (negative β) or right (positive β) (see color code for each β in Fig. 2). The simulated vertical position of the target was 1.48 m above the ground. The vertical speed component was 0, that is, there was no change in vertical position. The initial distances from the observer were 4 and 5 meters for the reference and test target respectively. We chose these values to match the average visual angle for both targets. The physical radius of the target was 3.3 cm (the size of a standard tennis ball). The presentation time was fixed to 1 s. The reference speed and β were pseudorandomized within participants and angles taking the values 2, 2.5 or 3 m/s (26.5, 32 and 37.5 deg/s). Once determined, the reference target was the same for a given angle and participant, while the speed of the test target (with depth component) varied according to a Bayesian staircase (see Procedure). Since the estimated duration of the experiment using all possible combinations of direction and velocity (3 speeds and 12 directions) was too high, each participant observed only 12 different pseudorandomized conditions.

2.3. Procedure

The experiment was performed in a dim room. Participants stood centrally at 2 m distance from the screen. The projected disparity was adapted to each participant's inter-ocular distance.

One session consisted of 360 trials (30 trials per approaching angle). Each trial consisted of two trajectories. We first showed the reference target that moved laterally (V_x component only: reference speed), followed by the test target (test speed: motion in depth with both V_x and V_z components). The speed of the test target varied according to a QUEST procedure (Watson & Pelli, 1983). We ran a total of twelve interleaved QUEST staircases (30 trials each), one for each combination of speed and approaching angle to compute the point of subjective equality (PSE) of speed between the test and reference target, that is, the speed of the motion in depth that is perceived as fast as the reference movement.

The participants completed 10–15 training trials previous to the main experimental procedure in order to familiarize themselves with the task. No feedback about response performance was provided during any part of the experiment.

After the two stimuli disappeared, an auditory signal prompted the participants to perform 2 different tasks in each trial:

1. *Trajectory estimation*: The observers were required to estimate the trajectory of the test target with the pointing device. To this end, the participants aligned the pointer with the perceived direction of the movement.
2. *Speed estimation*: In a two alternative forced choice (2AFC) task, the participants were instructed to press a button indicating which ball had moved faster (left for first/*reference* or right for second/*test*).

The participants gave their response for the speed estimation task while keeping the pointing device aligned with the estimated trajectory (from now on $\hat{\beta}$), such that both responses were registered simultaneously.

3. Model Specification

We developed a Bayesian model to estimate the variability of a prior in the $x - z$ plane that can describe the perceived trajectories in our experimental results. To define this model we assumed that motion and direction were estimated consistently with one another. For the sake of simplicity we introduced the model assuming that $\beta > 0$, but the same would apply for $\beta < 0$. Stocker and Simoncelli (2005) found that

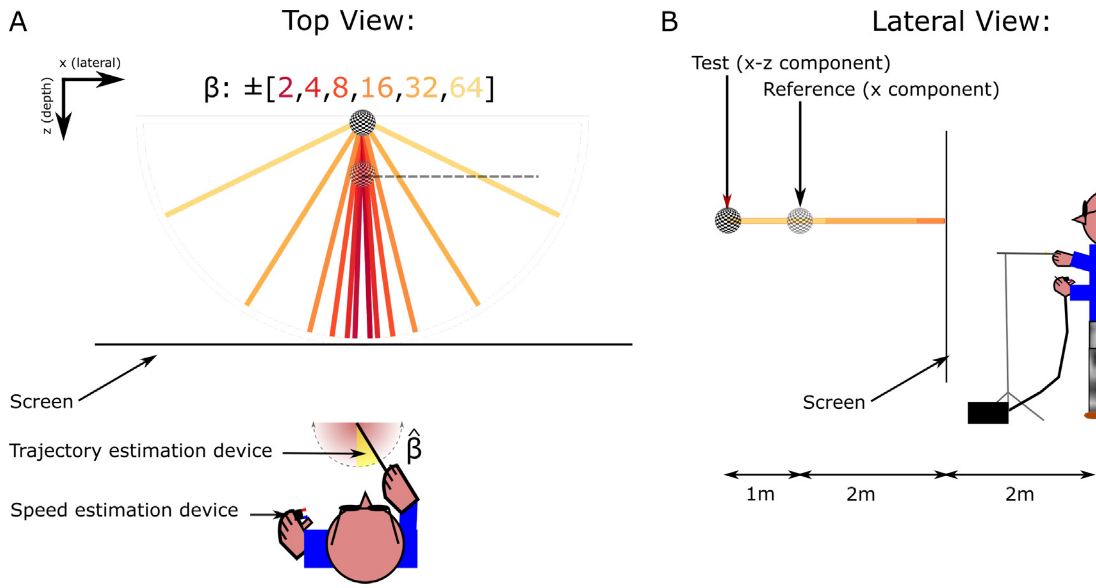

Fig. 2. The figure represents a top (A) and lateral (B) view of the setup. (A) The color of each trajectory indicates the bearing angle. In order to produce an estimate of the bearing angle, the participants aligned the pointer with the perceived direction of movement. To estimate the speed of the target moving in depth the participants were instructed to press a button indicating which ball had moved faster (blue for first/reference or red for second/test). As illustrated in the top view, the reference target only included a lateral component of movement (grey dashed line). However, test target included depth and lateral speed components (no vertical speed component was involved). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

motion estimates deviated from the weber's law for low angular speeds (< 1 deg/s). In order to account for this deviation, they decided to introduce a correction for speed estimates in their model. Even though Welchman et al. (2008) introduced this adjustment, since the angular velocities of our experiment are much greater, we have not corrected for this deviation.

We defined the likelihood distribution of our test stimulus as a 2D Gaussian (N) for each β, that is, a join of two 1D distributions, one for each axis or component (i.e. V_x and V_z as viewed by an observer).

$$p\left(\dot{\phi}\middle|V_x, V_z\right) = N_{(\mu_{xz},\sigma_{xz}^2)} \tag{5}$$

$$p(\dot{\phi}|V_x) = N_{(\mu_x,\sigma_x^2)} \tag{6a}$$

$$p\left(\dot{\phi}\middle|V_z\right) = N_{(\mu_z,\sigma_z^2)} \tag{6b}$$

Means for V_x and V_z were defined from the estimated PSE for each reference speed and β:

$$PSE_x = PSE\ sin(\beta) \tag{7a}$$

$$PSE_z = PSE\ cos(\beta) \tag{7b}$$

Standard deviations for V_x and V_z were defined as the discrimination thresholds for each component. Assuming that Weber's law holds for the range of speeds shown in this experiment, we calculated the discrimination thresholds for each component. Lateral discrimination thresholds were calculated assuming a Weber fraction of 10% as previously found in the literature (Portfors-Yeomans & Regan, 1996; Welchman et al., 2008). Depth discrimination thresholds, similarly, were obtained assuming that the Weber fraction at $\beta = 2$ is approximately equivalent to that of a stimulus in a collision course with the observer ($\beta = 0$). Therefore, we obtained the standard deviations from the respective Weber fractions (from now on *WF*) for x and z axis by β and participant (see inset at Fig. 5B):

$$\sigma_x = WF_x\ V_{Ref}\ sin(\beta) \Rightarrow \sigma_x \approx 0.1\ V_{Ref}\ sin(\beta) \tag{8a}$$

$$\sigma_z = WF_z\ V_{Ref}\ cos(\beta) \Rightarrow \sigma_z \approx WF_{\beta\pm2}\ V_{Ref}\ cos(\beta) \tag{8b}$$

Therefore, V_x and V_z follow a 2D Gaussian distribution with components:

$$p(\dot{\phi}|V_x) = N_{(PSE_x,\sigma_x^2)} \tag{9a}$$

$$p\left(\dot{\phi}\middle|V_z\right) = N_{(PSE_z,\sigma_z^2)} \tag{9b}$$

The distribution of the prior was defined as an isotropic 2D Gaussian in real world speed with mean 0 and a free parameter (variance). We chose a Gaussian with mean 0 since, in practical terms, a Gaussian approximates very well to a prior distribution following a power law as used by Stocker and Simoncelli (2006) and Welchman et al. (2008) ($p(|v|) = exp^{-7.04(\dot{x}^2+\dot{z}^2)-8.49}$). Additionally, this is also motivated by the assumption that speeds > 1(deg/s) follow Weber's law.

$$p_{v_x} = N_{(0,\sigma_v^2)} \tag{10a}$$

$$p_{v_z} = N_{(0,\sigma_v^2)} \tag{10b}$$

The variance of the prior (σ_v^2) is the only free parameter in this model. To estimate it, we obtained the posterior of each component ($\widehat{V}_x$ and $\widehat{V}_z$) by means of a MLE procedure (Ernst & Banks, 2002). This procedure combines different sources of information (prior and likelihood) in a weighted fashion to obtain an optimal posterior estimate (Fig. 1):

$$\widehat{V}_{xPred} = \frac{PSE_x}{1 + \left(\frac{\sigma_x}{\sigma_v}\right)^2} \tag{11a}$$

$$\widehat{V}_{zPred} = \frac{PSE_z}{1 + \left(\frac{\sigma_z}{\sigma_v}\right)^2} \tag{11b}$$

By simple trigonometry, the trajectories were calculated as a function of the predicted components of speed.

$$\widehat{\beta}_{Pred} = arctan\left(\frac{\widehat{V}_{xPred}}{\widehat{V}_{zPred}}\right) \tag{12}$$

As a result:

$$Bias_{Pred} = \hat{\beta}_{Pred} - \beta \quad (13)$$

We estimated the free parameter (prior variance) in a optimization routine using the *optim* function included in R (R Core Team, 2017). The objective function was formulated as a minimization of the sum of squared differences between the predicted ($\hat{\beta}_{Pred}$) and the measured trajectories ($\hat{\beta}$) for each trial.

$$Min: \sum (\hat{\beta}_{Pred} - \hat{\beta})^2 \quad (14)$$

Substituting in Eq. (12) with (11a) and (11b):

$$\hat{\beta}_{Pred} = arctan\left(\frac{PSE\ \ sin(\beta)/(1 + (WF_x\ V_{Ref}\ sin(\beta)/\sigma_v)^2)}{PSE\ \ cos(\beta)/(1 + (WF_z\ V_{Ref}\ cos(\beta)/\sigma_v)^2)}\right) \quad (15)$$

In Fig. 5B we use 15 to show the predicted perceived trajectories for each reference speed using parameters $WF_x = 0.1; WF_z = 0.28$; and the prior standard deviation obtained by the optimization routine $\sigma_v = 0.33$.

4. Data analysis

4.1. Speed estimation

We fit a cumulative Gaussian curve (mean and SD) to the proportion of faster than standard responses for each β and participant using the R (R Core Team, 2017) package *quickpsy* (Linares & López-Moliner, 2016) in order to obtain the PSE (i.e speed in the cumulative Gaussian curve that elicited 50% *faster from the standard*). We defined the discrimination thresholds as the half difference between 16% and 84% *faster from the standard* response probabilities in the psychometric function (i.e. 1 standard deviation).

Then, we calculated the Weber fractions for each β and reference speed as the discrimination threshold divided by the speed of the reference motion. We calculated this value for each participant and bearing angle separately.

Next, we obtained a ratio between the PSE and the reference speed. This value represents a measure of the degree of underestimation of the test relative to the reference speed. Ratios larger than 1 would denote an underestimation of test speed compared to reference speed.

Then, we used the PSE as a boundary to classify each trial as perceived as *slow* and *fast*. We made sure that the mean speed difference between both categories (i.e. *slow* and *fast*) was above discrimination threshold for speed by comparing the discrimination threshold with the speed difference between *slow* and *fast* groups across β in a two-way ANOVA.

4.2. Trajectory estimation

First, we filtered out those trials in which participants misjudged the absolute direction of the test movement (either left or right; less than 1%). Then we fitted a Linear Mixed Model to disentangle the effect of the speed group and β over $\hat{\beta}$ with the R-package *lme4* (Douglas, Mächler, Bolker, & Walker, 2014). We transformed the dependent variable $\hat{\beta}$ into its absolute value and β into the logarithm of its absolute variable to linearize the data. Speed group, β and their interaction were introduced as fixed effects. Slopes of β by participant and trial were introduced as random effects.

5. Results

5.1. Speed estimation

Fig. 3A shows the psychometric fit for the speed judgement of a representative participant. In this example, both psychometric fits were carried out for the same reference speed (2 m/s), but for two different bearing angles (β). Fig. 3B illustrates to what extent speed in depth was underestimated relative to lateral speed (ratios larger than one denote underestimation of the test target speed). Speed in depth was strongly underestimated for trajectories closer to the observer (smaller β). For these trajectories, the velocity had to be increased by a factor of 1.5–2 in order to be perceived as moving as fast as the reference. The underestimation attenuates and disappears for trajectories approaching a lateral movement. These results are in line with the matching speeds for lateral and depth motion in Welchman et al. (2008).

Fig. 3C shows Weber fractions for the estimation of speed in depth. As expected, Weber fractions depend on the presented trajectory ($F(11, 85) = 2.73$, $p = .005$) and increased as the V_z component (i.e. β) increased. Participants thus judged the speed of MID less accurately than for lateral motion (Table 1). Our results show a Weber fraction close to 0.25 for depth speed estimation, which is in agreement with those values reported by Rushton and Duke (2009) and Welchman et al. (2008). However, we found no significative effect of the reference speed ($F(1, 85) = 0.001$ $p = .975$, see Table 2) or interaction effect ($F(11, 85) = 0.91$, $p = .538$) suggesting that MID estimates follow Weber's law.

Finally, Fig. 3D illustrates, for the sake of comparison, the speed differences between slow and fast trials and the measured differential thresholds. The speed difference (fast-slow) was significantly larger than the differential threshold ($F(1, 198) = 24.63$, $p < .001$) and did not change significantly with β (i.e. interaction with β failed to reach significance, $F(11, 198) = 0.82$, $p = .620$). This provides sufficient grounds to assume that fast and slow speeds were perceived differently for further trajectory analysis.

5.2. Trajectory estimation

Fig. 4 displays the average perceived trajectory across subjects and β split by slow/fast trials in a polar representation. Fig. 5A shows the adjusted trajectory ($\hat{\beta}$) as a function of the physical trajectory of the stimulus (β).

An ANOVA on $\hat{\beta}$ yielded a significant (trivial though) effect of β ($F(1, 3560) = 7495.39$, $p < .001$) and, more importantly, speed group ($F(1, 3560) = 100.77$, $p < .001$) indicating that the perceived speed of the physical movement had an effect on the perceived trajectory and thus confirming our hypothesis showing that faster perceived speeds lead to more biased trajectory estimates. The interaction failed to reach significance ($F(1, 3560) = 2.44$, $p = .118$), indicating that the effect of the physical speed is independent of the presented trajectories in our study.

5.3. Bayesian model

As obtained by the optimization routine, the standard deviation of the slow prior is $\sigma_v = 0.329$. Fig. 5B shows the performance of the Bayesian model split by reference speed and mean participant reports of the participants.

The predictions of the model strongly correlate with the reported estimates $\hat{\beta}$ ($r(3481) = 0.91$, $p < .001$). To check the suitability of our model predicting $\hat{\beta}$, we computed and compared the log likelihood of a model with real β and reference speed as a dependent variables against a model including only the predicted trajectories in the Bayesian model $\hat{\beta}_{Pred}$. The results show that the Bayesian model has a larger likelihood ($LogLik\ (\hat{\beta}_{Pred})$: -15625.86) than the model based on β and reference speed ($LogLik\ (\beta \times V_{Ref})$: -16414.7). This shows that our model predicts better $\hat{\beta}$ than a model reliyng on the physical trajectory (β) and reference speed.

5.4. Initial distance under-estimation

A Bayesian model can successfully explain several characteristics of this bias in the perceived trajectory (i.e. superior bias close to $\beta = 0$), participants variability (Welchman et al., 2008 and dependence with speed). However, a more parsimonious explanation for these results

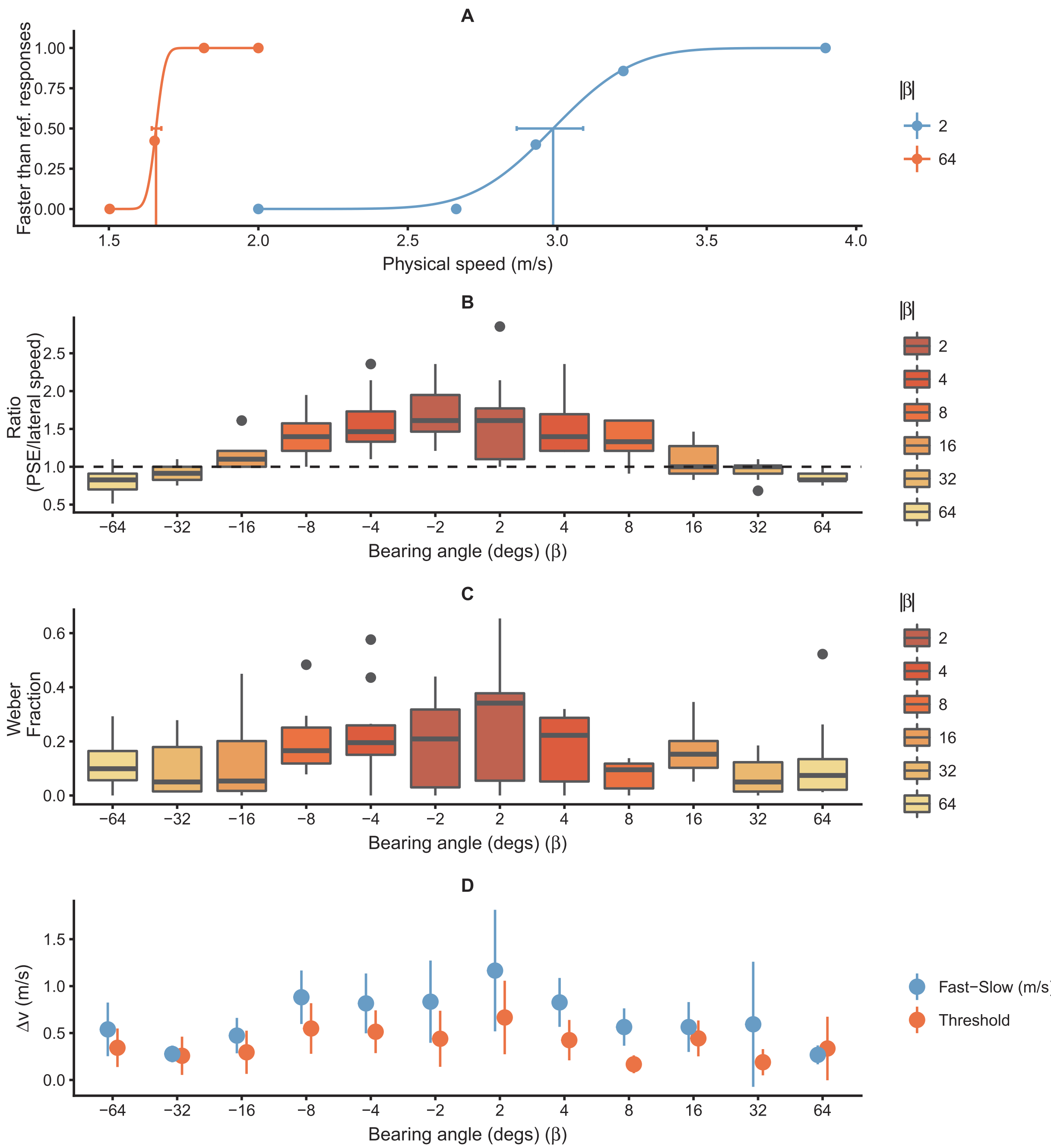

Fig. 3. (A) Psychometric functions for two different bearing angles for one participant (reference speed: 2 m/s). The y axis indicates the probability of judging the target's speed as faster than the reference speed. The horizontal error bar indicates the discrimination threshold. Speed in depth is underestimated with respect to the reference movement, as shown by the psychometric curve for $\beta = 2$. Discrimination thresholds are higher for motion in depth compared to lateral motion. (B) Average relative speed (PSE/Standard lateral). Values above the dashed line (Ratio > 1) denote underestimation of depth vs lateral speed. (C) Weber Fraction as a function of β. Weber fractions are higher for motion in depth, indicating that observers are less precise when judging differences for MID compared to lateral movement. (D) Representation of the differences between fast and slow trials (blue) and differential threshold (red) across β. Error bars indicate the 95% confidence interval. Mean differences for slow-fast trials are consistently higher than the discrimination threshold, therefore we assume speed was perceived as different. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

could be a simple mis-estimation of the initial distance on the target. Since viewing distance and target size are important variables to extract the rate of change in depth from the optic array, a constant underestimation of the initial distance would result in an overestimation of the trajectory of the target. We, therefore, checked if an underestimation of the initial distance could explain our results.

In order to do so, we specified a linear model of the perceived trajectory ($\hat{\beta}$) as a function of the initial distance (D_0), and derived the expected value of the parameter that multiplies D_0 assuming there is no underestimation.

Given the following equivalence:

$$tan(\beta) = \frac{V_x}{V_z} \tag{16}$$

Multiplying V_x and V_z by the total time of presentation t_0 we obtain:

$$tan(\beta) = \frac{V_x\ t_0}{D_0} \tag{17}$$

Table 1
Weber fractions for each β.

	Weber Fraction	CI
−64	0.13	0.06–0.2
−32	0.10	0.02–0.17
−16	0.14	0.02–0.26
−8	0.20	0.11–0.29
−4	0.26	0.14–0.38
−2	0.23	0.09–0.37
2	0.30	0.14–0.46
4	0.22	0.14–0.31
8	0.08	0.04–0.12
16	0.16	0.1–0.22
32	0.08	0.03–0.14
64	0.13	0.01–0.24

Table 2
Weber fractions for each reference speed.

	Weber Fraction	CI
2.00	0.18	0.14–0.23
2.50	0.16	0.12–0.21
3.00	0.16	0.12–0.2

Then, we can compute the value of (V_x t_0) in our experiment and use it as a constant (k) in the linear model:

$$tan(\beta) = \frac{k}{D_0} \tag{18}$$

Therefore, if D_0 is underestimated we expect a slope larger than 1 dependent on the term $\frac{k}{D_0}$ since D_0 is in the denominator. Based on Eq. (18), we applied a Linear Mixed Model to estimate this slope and check if it is larger than 1. Slopes of the ratio $\frac{k}{D_0}$ were introduced as random effects, and the intercept was dropped since it is not present in Eq. (18).

The results for the fixed effects showed that the slope do not differ from 1 (Estimate $= -6.479$, $t(9.267) = -0.389$, $p = 0.706$). Therefore, we did not find enough evidence to support the hypothesis that the initial viewing distance may have been underestimated.

6. Discussion

In this study we found a dependence between the speed and the perceived direction of an object, congruent with a Bayesian model of depth perception based on a slow prior as proposed by Stocker and Simoncelli (2006) and Welchman et al. (2008).

Our data show that the movement in depth is strongly underestimated with respect to lateral movement. A comparison between the Weber fractions for our trajectories (Fig. 3C) shows that reliability depends on the z component, that, in turn, translates into the bearing angle (β). Consistent with this, we have found that the variability in the estimation of the movement in depth is approximately two times greater. This is important, since, within a Bayesian framework depth estimates would be more affected than lateral estimates, presumably, causing the directional bias found in this and other studies.

In this study we were mainly interested in obtaining the PSE coupled with the estimated trajectory rather than obtaining a precise estimate of the discrimination threshold (ΔV). For this reason we chose the QUEST method. However, as Fig. 3A ($\beta = 64$) shows, for a few conditions, the amount of speeds sampled by this procedure might affect these estimates. Because of this reason we had to formulate our Bayesian model assuming a Weber Fraction based on previous literature for the lateral movement. In addition, our results show that the PSE for $\beta = \pm\ 64$ is significantly lower than the reference speed (t $(18.0) = -5.16$, p.001). This is against our first hypothesis, given that the rest of the angles show an underestimation of speed with respect to the reference. We hypothesized that, in this condition, for high speeds the target would have to move out of the projection frustrum before 1 s. (presentation time) possibly causing an effect on the perceived speed

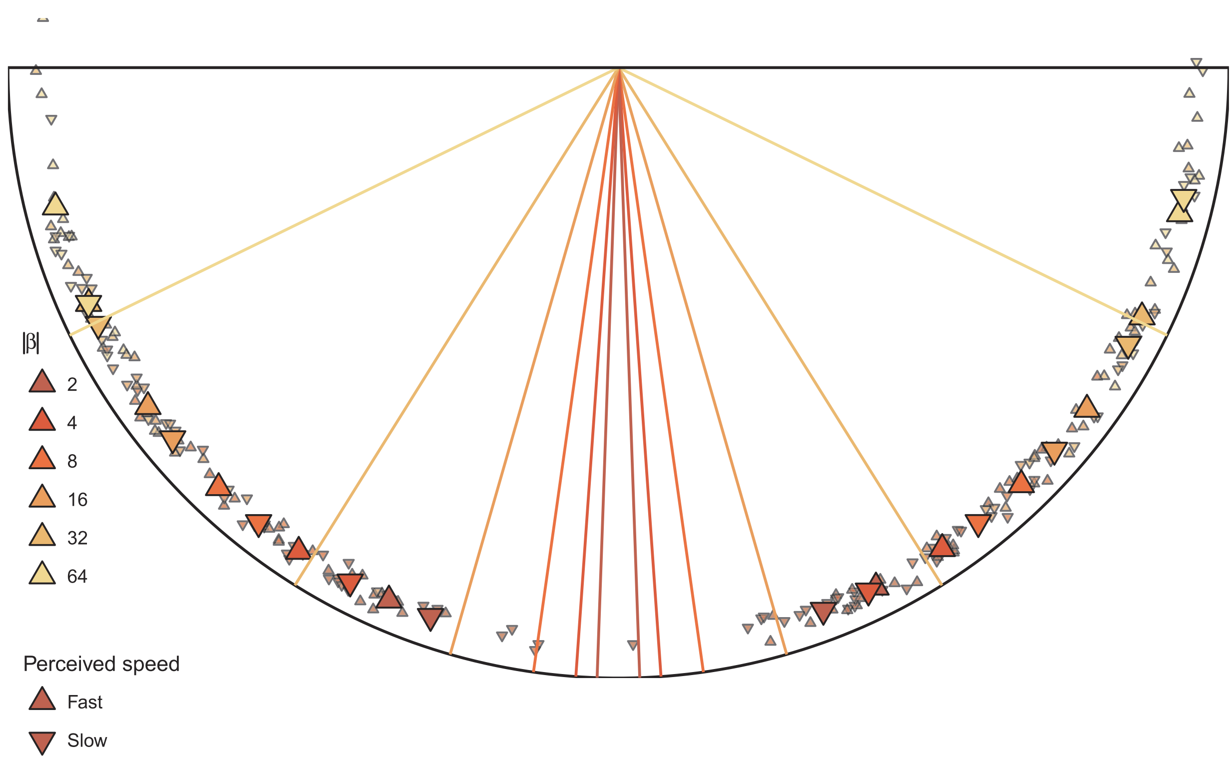

Fig. 4. Polar representation of the average perceived trajectory pooled across subjects for each β and slow/fast perceived speed (upwards/downwards triangles indicate fast or slow group of trials respectively). Smaller triangles indicate observer's mean perceived trajectory split by β, perceived speed and participant. Jittering was added to the smaller triangles in order to ease interpretation.

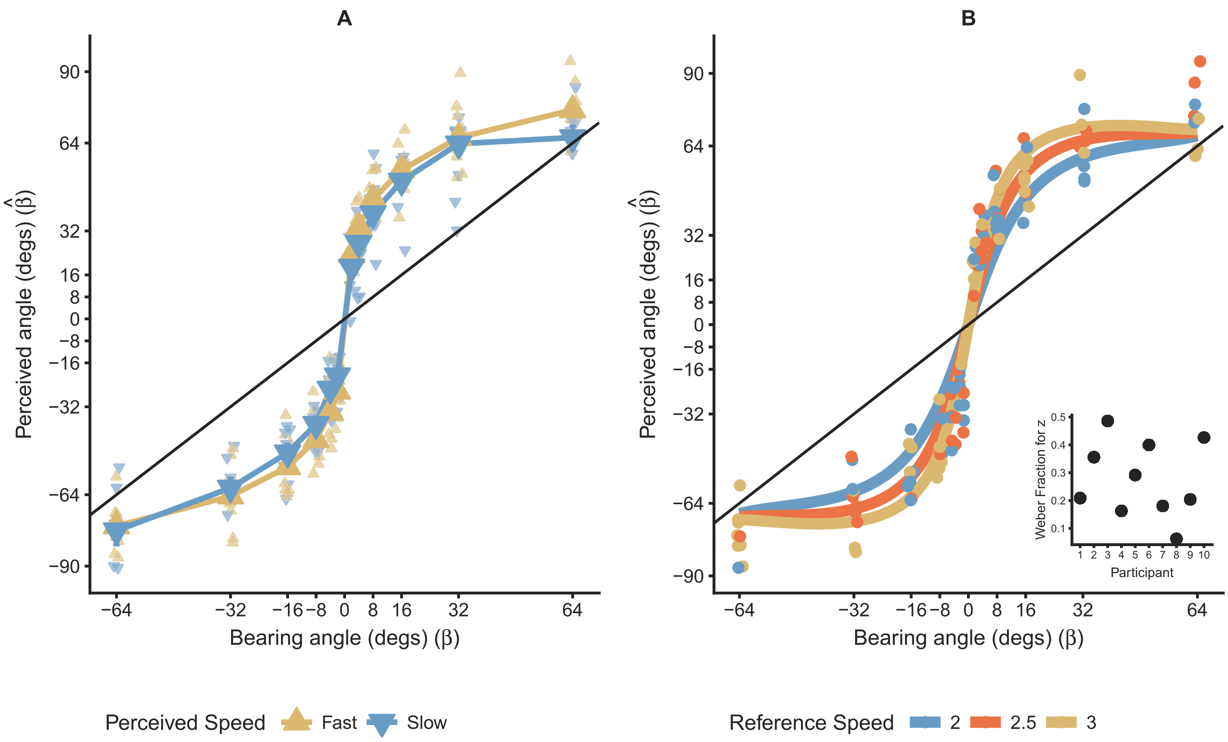

Fig. 5. (A) Average perceived trajectory between subjects as a function of the physical trajectory of the stimulus (β) split by fast/slow perceived speed (upwards/downwards triangles indicate fast or slow group of trials respectively). Error bar indicates the 95% confidence interval. Smaller triangles indicate observer's mean perceived trajectory split by β, perceived speed and participant. (B) The curves represent the fit of the model across β split by reference speed. The points indicate the individual mean reported trajectories for each participant split by reference speed and β ($WF_x = 0.1$; $WF_z = 0.28$; $\sigma_v = 0.33$). Horizontal jittering was added to the individual points in both figures to ease interpretation. Inset indicates the Weber fractions used to estimate the perceived trajectories in the model for each participant.

(Anstis & Stuart, 2018). Consequently, the participants could have judged the test target as faster than the reference.

Our results are in line with previous literature in which the speed in depth was strongly underestimated with respect to lateral movements (Brenner, Van Den Berg, & Van Damme, 1997; Brooks & Stone, 2006; Rushton & Duke, 2009; Welchman et al., 2008). For example, in Lages (2006), the participants were prompted to indicate both, the perceived trajectory angle and radial distance for stimuli moving in the $x - z$ space. Their results show a visual space describing an ellipsoidal shape as a result of V_z being underestimated with respect to V_x for the same physical speeds. Rokers et al. (2018) showed that a Bayesian model based on the slow motion prior can account for errors when judging the direction of an object. Their model is capable of predicting perceptual errors under different levels of contrast, eccentricity and distance to the stimulus. According to their results, the greater the distance to the stimulus, the greater the perceived lateral bias. The extent of trajectory bias in our study is higher than shown in the previous literature (Harris & Dean, 2003; Welchman et al., 2008). However, just as the Rokers et al. (2018) model would predict, given that our initial distance is the highest (5, 1, and 0.5 m. respectively), the trajectory bias we would found would be the strongest.

Recently, Wei and Stocker (2017) have proposed a simple mathematical relationship that could be applied to this study. They hypothesize that perceptual bias is proportional to the rate of change of the discrimination threshold along the stimulus space, a notion that is supported by our results. At high angular speeds, lateral motion perception seems to follow Weber's law (Stocker and Simoncelli 2006). Also, as we found in this study, Weber's law seems to apply to motion in depth. Therefore, the attraction towards the slow prior would be stronger for higher angular speeds (see Fig. 1). This would account for an increased directional bias for higher speeds as shown by our results.

Even though our results seem clear, simulations of 3D movement in stereoscopic setups are known to introduce problems such as the conflict between vergence and accommodation. Conflict between these two cues could be responsible for consistent depth underestimation shifting perceived position closer to the screen due to accommodation for simulated depth movements (Regan et al., 1986), that is, increasing the bias in the perceived bearing angle. However, this conflict has been studied with contradictory outcomes for perceived distance, which according to our results would be a by-product of speed estimation: Watt, Akeley, Ernst, and Banks (2005) found that when accommodation was changed by manipulating the distance between display and observer, disparity scaling was corrected. Willemsen, Gooch, Thompson, and Creem-Regehr (2008), however, found that vergence-accommodation conflict does not affect the perceived distance.

Future studies may explore the effect of a target moving with a non-null vertical component in order to constrain a full 3D motion model. It is known that sensitivities for stimulus oriented vertically or horizontally are nearly equal (Manning, Thomas, & Braddick, 2018; Portfors-Yeomans & Regan, 1996) and reports in the $x - y$ plane show little directional bias (Welchman et al., 2008). We thus expect our results to generalize to the $y - z$ plane. On the other hand, Poljac et al. (2006) found that bias in the plane $y - z$ is lower than $x - z$, suggesting that vertical estimation may be less reliable than lateral estimation. Interestingly, Poljac et al. (2006) found that direction was estimated more precisely when the target crossed the eye height. This suggests that additional visual cues may be used to estimate the direction of moving targets in this case. Furthermore, the participants performed the experiment in a free gaze situation. Since pursuit is known to influence the accuracy of speed estimates (Schütz, Braun, Kerzel, & Gegenfurtner, 2008), an interesting future direction would be to investigate the effect of smooth pursuit or fixation on the speed precision estimation for motion in depth and the resulting lateral bias.

Given that we have found a dependence between the estimation of speed and direction, there might be some common neural processes integrating both features. Lages and Heron (2010) previously proposed a parallel processing of 2D velocity estimates and disparity to extract estimates of 3D motion. 2D motion information would be encoded in V1 under a preference for slow speeds and selective directions (Perrone, 2006; Series, Georges, Lorenceau, & Frégnac, 2002). This could be interpreted as a prior for slow speeds (Vintch & Gardner, 2014). Then, this information is further processed by MT (Braddick et al., 2001; Burge & Johannes, 2015; Sanada & DeAngelis, 2014). Concerning disparity, an early computation of this signal is carried out by V1 (Nienborg, Bridge, Parker, & Cumming, 2005). Further indirect projections to MT relay through V2 and V3 (Ponce, Lomber, & Born, 2008) as an intermediate processing of disparity (Thomas, Cumming, & Parker, 2002). As a result, speed, direction and disparity processing are performed by MT. Therefore, it is the most probable candidate for the integration of these signals in order to obtain the structure of 3D motion perception (Rokers, Cormack, & Huk, 2009).

7. Conclusions

We show that the direction of an object on a near-collision course with the observer is overestimated as a function of the perceived speed. Objects are consistently judged as passing the observer further away than they actually do. Our methodology allowed us to couple the perceived speed for different bearing angles (β) with directional biases in depth perception. Our results indicate that a Bayesian model of speed discrimination in depth following Weber's law can successfully simulate both types of perceptual biases denoting a coherence between speed and direction estimation. Thus, future research could manipulate the reliability of motion signals to further investigate the relation between motion and direction estimates.

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