



Review

The multidimensional brain

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Abstract

Brain activity takes place in three spatial-plus time dimensions. This rather obvious claim has been recently questioned by papers that, taking into account the big data outburst and novel available computational tools, are starting to unveil a more intricate state of affairs. Indeed, various brain activities and their correlated mental functions can be assessed in terms of trajectories embedded in phase spaces of dimensions higher than the canonical ones. In this review, I show how further dimensions may not just represent a convenient methodological tool that allows a better mathematical treatment of otherwise elusive cortical activities, but may also reflect genuine functional or anatomical relationships among real nervous functions. I then describe how to extract hidden multidimensional information from real or artificial neurodata series, and make clear how our mind dilutes, rather than concentrates as currently believed, inputs coming from the environment. Finally, I argue that the principle “the higher the dimension, the greater the information” may explain the occurrence of mental activities and elucidate the mechanisms of human diseases associated with dimensionality reduction.

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0. Introduction

Physical or biological activities can be depicted in terms of moving particles embedded in three-dimensional space. In this simple, rather intuitive abstraction, the x , y and z axes stand for the canonical three spatial dimensions, while time passing is portrayed in terms of particles' trajectories [116] (Fig. 1A). For example, the trajectories displayed by the BOLD activation of cortical areas after a visual stimulus can be either drawn as interactive static plots or visualized as animations. However, the current increased availability of big data and innovative computational tools have led to a growing interest for multidimensional approaches [17]. To understand what “multidimensionality” truly means, we need to think in more abstract terms than usual: while the visualization of low-dimensional data is relatively straightforward in a three-dimensional plot, it is not always obvious how to visualize high-dimensional datasets in a similarly intuitive way [33]. One possibility consists in adding further dimensions to the canonical ones, in order to attain other axes apart from x , y and z . If we introduce a further axis, the particles are allowed to

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move in more than three dimensions (plus time) (Fig. 1B). This methodological step is highly profitable, because it allows the study of systems dynamics in higher dimensions, achieving more information about the particles' trajectories.

The trend towards multidimensional strategies is taking place in far-flung disciplines. Inspired by the periodic table, Reymond [82] grouped chemical compounds in a multidimensional space, in which neighbors have related properties; further, Cardesa et al. [9] uncovered energy cascades in five dimensions (three-dimensional space, time and scale) endowed in the turbulent flows of simple water-like fluids. This paves the way for similar analyses in geophysical and industrial flows, conducting and quantum fluids, plasmas. Not surprisingly, the assessment of the most complex object of all, e.g., the human brain, is also amenable to a multidimensional representation. As brain activities are portrayed as locations at different points in time, either linear/nonlinear nervous dynamics or neural functions/mental observables can be described in terms of scalars/vectors measured at different locations of multidimensional phase spaces. The key question here is: can further dimensions be added to the representation of brain activity? The choices at neuroscientist's disposal are manifold: they are allowed to insert all the axes they require (Fig. 1C), and to locate whatever parameter they want on the further axes, from spike frequencies to synchronized oscillations, from nonlinear fields to attractors, from color perceptual spaces to tactile qualities, from other spatial dimensions to the neural clusters' number (see Figs. 1B–F). Of course, to leave the realm of the pure abstraction and acquire scientific relevance, the arbitrarily chosen dimensions must provide testable hypotheses and experimental validation. In the following, I will describe recently-developed techniques that allow the use of extradimensions in neuroscience. I will then illustrate the multidimensional approach for a wide range of brain activities and mental functions. Furthermore, I will discuss how recent data point towards the real presence of hidden, further dimensions where nervous activity might take place. I will conclude that the brain operates in dimensions higher than its surrounding environment.

1. The unreasonable multidimensional power of the novel computational tools

Electroencephalography (EEG), functional MRI (fMRI) and magnetoencephalography (MEG) have been proven very useful in unveiling neural network dynamics taking place in the classical three spatial dimensions-plus time. However, the data extracted from these standard neurotechniques can be successfully assessed also in the language of innovative, sophisticated multidimensional tools [63]. In this Section, I describe new mathematical weapons that should allow to analyze in multidimensional contexts the huge data sets now commonly generated by standard experimental protocols [14]. Innovations make it feasible to scale efficiently to high dimensions: e.g., Shah and Koltun [91] introduced a clustering algorithm that untangles heavily mixed clusters and achieves high accuracy across multiple domains. It was successfully tested on large datasets of faces, hand-written digits, objects, newswire articles, protein expression levels and sensor readings from the Space Shuttle [91]. Another novel tool assesses the general multidimensional space that encompasses linear combinations of neuronal patterns: Rigotti and Fusi [83] proposed a method exploiting fMRI Repetition Suppression, in order to measure the hidden dimensionality in response to specific task conditions. This technique is able to detect microscopic overlaps even in brain areas that do not display discernible average differential BOLD signal, identifying the cortical circuits that integrate multiple independent information pathways. The importance of higher dimensionality lies in its relationships with the functionality of neural circuits: indeed, the dimensionality (measured, in this case, in terms of response vectors) is higher in areas involved in multi-stream integration and lower in regions where inputs from independent sources do not interact [83].

Another approach takes into account a feature that is generally hidden in standard series: the “fractal dimension” of data. The fact that the neural activity encompasses self-similarity at different temporal and spatial scales has been proven very useful for the assessment of biomedical analysis waveforms, in particular the complexity endowed in EEG time series [79,31,23]. In my framework, self-similarity might stand for a further (non-integer or fractional) dimension that allows the extraction of larger amounts of information from EEG and functional fMRI data sets [115]. Another important, recently developed issue is the “energy landscape analysis”, a data-driven approach to neuroimaging [113], that has been used, in particular, for the assessment of fMRI series recorded from healthy individuals [22] and of energy landscapes shaping the dynamics of subcortical networks during resting state [39]. The latter authors evaluated alterations in energy landscapes after perturbation of subcortical networks' multidimensional parameters. Even slight network perturbations of different parameters (such as maximal number of attractors, unequal temporal occupations

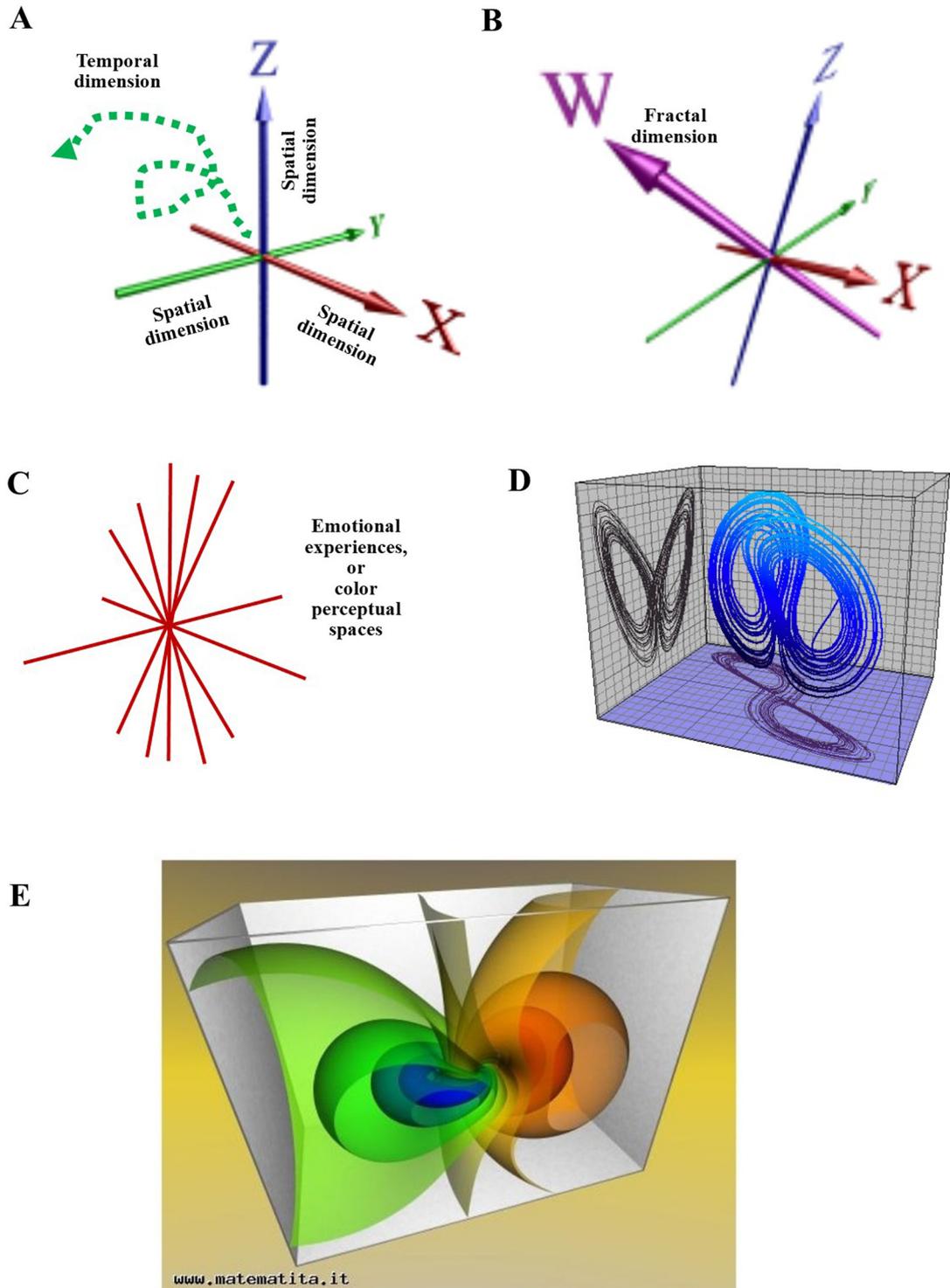


Fig. 1. Different ways to cope with further nervous dimensions. Fig. 1A depicts the canonical picture of three spatial dimension plus time. Time is described by a particle trajectory. Fig. 1B illustrates the arbitrary insertion of a further dimension, in this case a fractal one. Fig. 1C: a sheaf of functional multidimensions might stand for different mental or biological parameters (in this case, either emotions, or colors' perception). Fig. 1D: Multidimensional attractors produced by the complex, nonlinear dynamics of the brain at the edge of chaos. Fig. 1E. A Clifford torus, displaying four-dimensional, donut-like nervous spatial trajectories. Credits: Figs. 1A and 1B are modified from <http://eusebeia.dyndns.org/4d/vis/01-intro>; Fig. 1D is modified from <https://www.mat.ucsb.edu/g.legrady/academic/courses/03w200a/multidimensional/index.html>.

and readiness for system's reconfiguration) caused significant shifts in the energy landscape. Therefore, diverse nervous functions emerging due to the reconfiguration of the default brain network at rest can be easily assessed through the add of such multidimensional parameters.

The connectome turns multidimensional. Network science, an interdisciplinary spin-off of graph theory, envisages the brain in terms of the “connectome”, i.e., a system of nodes and edges that describes the nervous system as a series of various interacting regions. Conventionally, connectome has been studied in terms of static or time-averaged links; however, recent improvements are starting to take into account also a dynamical component, i.e., the time-varying, non-permanent relationships among edges and nodes [52]. Complex network (graph-theory) analysis applied to MEG/EEG signals allows the assessment of both canonical and non-canonical dimensions, the latter standing for a series of non-spatial parameters, such as, e.g., spike frequencies, connectivity, activity magnitude and complex network properties [42,27]. Developments in source reconstruction algorithms are leading to novel neuronal metrics (standing, in my framework, for further dimensions) that can provide insights into the multidimensional dynamics of various neural representations [42]. The organization of complex networks can be assessed through higher-order connectivity patterns: such approaches allowed Benson et al. [5] to reveal unexpected hubs and geographical elements endowed in the very structure of the nervous connectome. On the other hand, Cai and Pestilli [8] described multidimensional arrays able to assess diffusion-weighted magnetic resonance data. They evaluated 1490 connectomes, thirteen tractography methods and three data sets and elucidated the relationships among nodes, edges, white matter fascicles and diffusion data. This method provides anatomical manipulation in multidimensions on white matter tracts, allowing either statistical inference, or the study of the white matter's geometrical organization. Novel statistical machine-learning methods, such as “multivariate pattern analysis”, generate stimulus response models in a data-driven fashion that allows fMRI studies of dynamical functional network connectivity in human cognition [56]. Vergara et al. [108] developed an “information theoretical framework” that quantifies the information flowing among subsets of neural networks' functional domains. Clusters of cross domain mutual information point towards scarce information sharing between cognitive and sensory areas. These findings show that, when information theory is applied to states estimated from pairs of multi-network functional domains, additional, multidimensional features of connectivity can be easily unveiled.

The novel multidimensional tools might also help researchers to improve already existing neurotechniques. To make an example, the utility of the successful “independent vector analysis” is circumscribed, because of the extremely noisy nature and large variability of vector distributions. In order to overcome these limitations, Kuang et al. [46] proposed an adaptive fixed-point algorithm that analyzes multiple-subject complex-valued data, allowing an easier detection of the information in fMRI series. Their model, tested both on simulations and real fMRI data, achieves two goals: a) decomposing multi-subject complex-valued information, and b) addressing the troublesome problem of subjective variability of brain function [46]. The nervous connectome can be currently assessed not just in terms of anatomical or dynamical nodes derived from standard imaging neurotechniques, but also of molecular, cellular and neurochemical ones. Here we provide two recent examples. Sousa et al. [95] performed transcriptome sequencing of 16 regions of adult human, chimpanzee, and macaque brains. Integration with human single-cell transcriptomic data revealed global, regional and “cell-type-specific” species-expression differences in genes which denote distinct functional categories. Noori et al. [65] resorted to advanced data mining and supervised machine learning to develop a multiscale, multilayer “neurochemical connectome” of the rat brain. Their tool is freely available for systems analysis of motor, sensory, emotional and cognitive information processing. The last statement emphasizes the growing role of open sources in neuroscience. Advances in neuroimaging of healthy adults, including the ones driven by the Human Connectome Project, allow large free-available data sets to be investigated through recently developed computational tools [21], thus offering exciting prospects for accelerated progress in characterizing brain networks' anomalies [107].

The reverse is feasible too: back from higher to lower connectome dimensions. Despite their huge potentialities, high-dimensional reconstructions of the brain are currently plagued by a severe technical limitation: their computational cost. These restrictions can be tackled through a recently-introduced approach, termed “dimensionality reduction”. Diverse dimension reduction techniques are available, from the popular “principal component analysis”, to more advanced “factor analysis”, including ICA, MAVE [64]. Ye et al. [117] assessed specific patterns encoded in the brain connectivity graphs of healthy subjects, in order to explain how high-dimensional, complex geometries can be mathematically embedded into lower dimensions. Their “tractography-derived” approach tested both linear

and nonlinear dimensionality reduction techniques and demonstrated that the most influential regions of the structural connectome, such as rich-club members, are more centrally mapped [117]. In turn, Heusser et al. [33] introduced “HypeTools”, a Python toolbox for the visualization and manipulation of large, high-dimensional datasets. Dimensionality reduction was used in order to embed high-dimensional sets in a lower-dimensional space that allows simpler options for data manipulation. Just as the location of an object moving through a canonical space can be visualized in terms of a three-dimensional trajectory, HyperTools uses dimensionality reduction algorithms to create two- and three-dimensional paths for time series of high-dimensional observations [33]. Hence, multidimensional reconstructions extracted from real neurodata provide a huge amount of information; their main defect, i.e., computationally untreatable complexity, will be soon tackled by powerful, next-to-come advances that will achieve the proper dimensionality reduction.

In sum, this Section emphasizes how the recent free availability of big data sets which encompass thousands of features prompted the development of novel computational tools, able to cope with the underlying high-dimensional nature. This huge amount of information can be tackled in terms of paths taking place in phase spaces of dimensions higher than the usual three plus time: indeed, every increase in dimensions leads to supplementary available information, in the same way as a sphere encompasses more information than a circle.

2. Topology and symmetries come into play

Once recognized the limitations of the classical techniques in accommodating brain activity, neuroscientists are trying to move beyond the classical approaches [77]. One of the most promising novel tools is topology, a branch of mathematics which allows the description of systems equipped with any number of dimensions. When topology meets the physics of the brain, we achieve a general model of neuronal activity that can be described in terms of multidimensional manifolds/computational geometry and has the potential to be operationalized. Indeed, topology works like a telescope and microscope at the same time: allowing to zoom into networks to find hidden neuronal structures and at the same time look at the larger cortical areas. In touch with old scholars [114,50] current scientists suggest that the world and its dynamics can be described in terms of mappings and projections taking place on topological structures [61]. Systems’ properties of physical and biological spaces can be translated into abstract mathematical spaces, and *vice-versa*, leading to the development of higher dimensional constructs. Consistent with this trend, the last decade has seen an explosion of topological approaches in neuroscience, especially to understand neural codes [14] and to extract higher dimensional models of percept, concept, emotion, memory from experimental data [77]. For example, Baas [3] mapped sets of spike trains to a topological manifold, achieving a novel space with a structure richer enough to encompass non-trivial spatial stimuli. Recovering the manifold in which rats are moving, the author detected and separated the influence of the various stimuli that shape the neural (cognitive) space of spike trains. To sum up, topology allows to build phase spaces with more dimensions and therefore with more information to analyze: indeed, a particle moving in, say, five dimensions, describes more paths than a particle moving in the canonical three dimensions plus time.

The geometry inside: nested multilevel correlations. It has recently been argued that the real networks are not just random combinations of single features, rather are organized according to “hidden” geometric correlations between their layers [43]. These correlations allow the detection of multidimensional communities, e.g., sets of nodes that are simultaneously similar in multiple layers. Such kind of hidden multidimensionality enables accurate trans-layer link estimates, so that connections in one layer can be predicted by observing the hidden geometric space of another [43]. As suggested above, the recent availability of huge datasets leads to the necessity to develop methods able to extract as many hidden patterns as possible from experimentally detected traces. The challenge lies in transforming the detected patterns into scientific knowledge, using topological tools which are strictly intertwined with the complex network analysis and the language of graph theory. When connectivity exhibits integrated network topology, structural and functional features become more closely linked to each other, because direct structural connections mediate a larger proportion of neural communication within functional modules [26]. Complementing the conventionally-applied algebraic topology methods, Maletić and Zhao [55] proposed new “information entropy measures” for the characterization of multidimensional structures extracted from complex datasets. They assessed topological relationships, multilevel entropy measures and simplicial complexes endowed in taxi driver’s cognitive maps, in order to track transitions to high-dimensional structures [55]. In turn, Merelli et al. [60] proposed a methodology, based on topological data analysis, that captures the changes of internal organization in complex systems: this means that the topological invariants

in multivariate time series can be used for the computation of a novel entropy measure, the so-called “weighted persistent entropy”. This measure allows to track the nonlinear changes which are correlated with phase transitions and network reorganization.

[28] focused on “simplicial complexes”, i.e., a construct from algebraic topology to assess higher-order networks, by capturing both emergent community structures and the dependence of the degree distribution on the number of the underlying dimensions [6]. It is noteworthy that higher-order networks have a natural geometric interpretation which describes the interactions between two or more nodes. Indeed, multidimensional networks might encompass simplicial complexes made not only by classic nodes and links, but also by triangles and tetrahedra glued along their faces. Simplicial complexes’ network geometry attracts increasing attention, due to the wide range of possible applications, from data mining to Internet routing protocols [6]. The technique is promising for the assessment of neurodata extracted both from electrophysiological recordings in animal models and hemodynamic fluctuations in humans. The authors posit that simplicial complexes have the potential to eclipse graph theory in unraveling the fundamental mysteries of cognition [28].

Two is better than one. Last, but not least, another important concept from topology has recently come into play in neuroscience: the Borsuk–Ulam theorem (BUT), which states that two antipodal points on a sphere map to a single point on a circumference [7,57]. In neuroscientific terms, this means that a cortical region embedded in lower dimensions gives rise to two detectable regions with matching description in higher dimensions [105]. The dimensions described by BUT do not stand just for spatial ones (as in the case of a circle and a sphere), but also for abstract dimensions (such as biological complexity, fractional quantities, fractal dimensions) [72,103]. In BUT terms, every novel axis of the above-mentioned phase space stands for a different dimension. Indeed, the BUT perspective allows a feature (e.g., a shape, a trajectory, a vector or an energy) located in the three-dimensional environment to be translated to an abstract space of different dimensions, and vice versa. Achieving maps from one system to another, researchers are enabled to enrich and elucidate the information extracted from real neurodata [102].

Some of the BUT predictions have recently been confirmed, suggesting that this topological framework can be fruitfully used in neuroscience. The BUT predicts the presence of at least two brain regions equipped with the same features. Recent papers support this claim: for example, Karaman et al. [40] identified the existence of antipodal behavioral patterns of neural activities. Also, Hoffmann and Triesch [32] demonstrated that brain connectivity comes in pairs. They described, in a network in which connection probabilities are symmetric in pairs, the occurrence of abundant bidirectionally connected neuron pairs, which far exceeds what would be expected in a random network. The BUT framework envisages that a) matching cortical areas with different anatomical location might encompass the same mental faculty, and b) the activity of such a mental faculty halves when other ones occur in the brain. In touch with this prevision, Waskom and Wagner [112] stated that: “examining targeted prefrontal regions, we found that pairs of voxels with similar context preferences exhibited spontaneous correlations that were approximately twice as large as those between pairs with opposite context preferences.”

The hidden relationships between symmetry breaking and multidimensionality. Many complex dynamical approaches to brain function have been proposed throughout the last years [16,4,120,100,18]. I would like to remark that one key point of these frameworks is that nonequilibrium phase transitions may lead to symmetry breaking [4,16,71]. In touch with my claim, Jirsa et al. [35], focusing on the left-right cortical symmetry, provided a bimodal description of the brain activity connected to behavioral dynamics. Moreover, it has been demonstrated that, when an acoustic stimulus frequency is systematically changed in an experimental setting, a spontaneous transition in coordination occurs at a critical frequency, in both motor behavior and brain activity [36]. The study of the temporal organization of large-scale brain networks in whole-brain resting fMRI data established that spontaneous transitions between networks of interacting brain areas are predictable [111]: more specifically, network activity is organized in a hierarchy of two distinct metastates (i.e., higher-order cognition and sensorimotor systems), such that transitions are more probable within, than between, metastates.

Dimensional reduction and symmetry break display close relationships. It has been even proposed that symmetry stands for features with matching description lying in higher dimensions, while the corresponding symmetry break for a single point lying one dimension lower [103]. Recent approaches highlighted the relationships between symmetry and energy requirements, because the former is strictly correlated with thermodynamic activity [86]. By introducing novel topological tools that analyze enthalpy, free-energy and entropy in fMRI neurodata, Tozzi and Peters [104] provided a testable approach to proceed from topology to thermodynamic of nervous activity/symmetry breaking. Hence,

during symmetry breaks and transitions from one functional state to another, changes in thermodynamic parameters and dimensions take place in the nervous system. This means that abstract topological approaches may represent real, energetic physical counterparts accessible to current neurotechniques.

A recent paper describes physical “gauge theories” in the context of the brain activity [90]. The authors assessed nervous activity through the physical and mathematical procedures of gauge theory, examining the possibility that brain function is driven by one or more continuous forces, called gauge fields, that originate inside or outside the CNS. In particular, their framework highlights the concept of symmetries, which represent the cornerstone of gauge theories. The approaches based on symmetry breaking allow a methodological assessment of brain dynamics, described and operationalized in terms of energetic gradient-descent trajectories taking place on curved paths of the nervous phase space. In touch with these symmetry issues, Robinson et al. [84] analyzed the activity eigenmodes of the bi-hemispheric brain and its spatial structure. They found that interhemispheric connections between homologous regions via the corpus callosum lead to splits which depend on symmetric or anti-symmetric activity between opposite areas. The symmetric properties of the lowest eigenmodes are able to constrain the interhemispheric connectivity strengths, so that, consistent with observations, spontaneous brain activity turns symmetric between hemispheres. In order to achieve and maintain the required brain function, neural activity needs to be tightly coordinated within the brain networks’ manifolds. In this vein, Pillai and Jirsa [76] described structured flows on manifolds able to provide a formal description of behavior in terms of low-dimensional processes that emerge from the symmetric and invariant properties of network’s dynamics. They found that symmetry breaking in network connectivity gives rise to timescales hierarchies, resulting in the emergence of attractive functional subspaces, so that behavior ultimately emerges when appropriate conditions imposed by energetic constraints upon the couplings are satisfied. In short, a topological, abstract framework, consisting of brain symmetric functions lying on multidimensional manifolds, describes the general thermodynamic and information principles [59,60] that predict how behavior and task rules are represented in the real brain.

In sum, this Section suggests that the best available strategy to tackle the elusive issue of nervous multidimensionality is to make use of the proper topological weapons. The current state of the art suggests that the very general and “deductive” topological approaches afford accurate previsions that can be tested in the proper scientific experimental setting. This is because abstract topology displays a real energetic counterpart that can be measured in the nervous system.

3. A journey into mental hidden dimensions

In the previous Section, I described how additional nervous dimensions, suggested by simulations or extrapolated from real neuro-data sets, stand for methodological devices that allow mathematical treatment of the brain activity. Here I will focus on multidimensional (anatomical or functional) structures that may implement it in the central nervous system. Indeed, recent papers point indirectly towards the human brain as truly encompassing hidden dimensions. This claim is doomed to be highly controversial, because the mind of a human observer, perhaps due to evolutionary constraints, is not capable to perceive, or even imagine, spaces and structures made up of four or more dimensions.

The hidden dimensions of perceptual spaces. A “perceptual space” of a sensory domain is a mental workspace of points (or vectors, or tensors) that capture similarities and differences in judgments and enables further processing such as classification and naming [118,109]. Perceptual spaces are not traceable to single-neuron properties, because they span across all the sensory cues: from colors to environmental sounds, from faces recognition to auditory textures, from tactile quality to odor. The best studied, i.e., the color’s perceptual space, is correlated with the three cone absorption spectra and is believed to lie in more than three dimensions-plus time [109]. Combinatorial strategies are needed in order to represent each separate neuronal population in high-dimensional spaces. The authors focused on a ten-dimensional domain of visual textures, assessed through psychophysical measurements in human observers. The multidimensional approach to color vision suggests two combinatorial strategies of perceptual space’s representation: 1) the first is based on projections onto coordinate-like axes equipped with Euclidean geometry, while 2) the second is based on activity patterns. Such patterns are scattered throughout the whole multidimensional space, which is equipped with peculiar global curvatures [109]. Multiple types of local image statistics capture different dimensions, such as luminance distributions, spatial contrast, oriented segments and corners, allowing the analysis of textures, surfaces and forms [110]. Such multifaceted visual processing is difficult to assess, because of the very high dimensionality and the interaction of different kinds of statistics. In order to tackle the problem, Victor et al. [110] focused on binary images

on a square lattice, using a reduced set of stimuli able to tap several kinds of local statistics. Assessing 16 subjects in a ten-parameter (ten-dimensional, in my terminology) space, the authors found that local image statistics shapes a perceptual space which is highly stereotyped across observers. The same approach and experimental treatment might hold for other cues. For example, during complex sounds processing, auditory cortex neurons integrate across multiple dimensions of sound (such as center frequency, intensity and bandwidth), and not just across the few ones detected by traditional studies [94]. The latter authors built an evolutionary algorithm and a statistical method for nonlinear regression that allow a quantitative exploration of cortical responses to five-dimensional auditory stimuli in mice. In summary, sensory cortical neurons seem to be driven by interactions across different inputs dimensions, in ways that are not detected by low-dimensional characterizations [19].

Stimuli modify the dimensionality of cortical activity. Recent works assessing connectome arrangements provide indirect proofs of human brain multidimensionality. Mazzucato et al. [58] evaluated the activity of ensembles of simultaneously recorded neurons and demonstrated that stimuli change the dimensionality of cortical activity: the more the network's ensemble size grows, the more neurons are recruited, the more the dimensions. In particular, the default mode network displays higher dimensionality, compared with other nervous functional networks. In touch with these claims, Zamora-López et al. [119] compared the connectomes of *C. elegans*, cats, macaques and humans with surrogate networks in which either modules or hubs were destroyed, showing that functional complexity always decreases in the perturbed networks. A comparison between simulated and empirically attained resting-state functional connectivity suggests that the human brain at rest has the largest complexity allowed by its anatomical connectome. Roland et al. [85] examined the space-time dynamics of excitation/inhibition in dendrites and axons, over four visual areas of ferrets exposed to scenes with stationary and moving objects. The visual stimuli broke the tight balance, such that networks exhibited longer episodes of net excitation, followed by a compensatory mechanism of net inhibition. Within 80 milliseconds, the excitation-inhibition's space-time dynamics produced a flow on a low three-dimensional manifold that reduced the complexity of neuron interactions over the whole network. In contrast to pure temporal dynamics, low dimensional flows evolved to distinguish the simple visual scenes [85].

The discovery of stimulus-induced synchronization in the visual cortex suggests that the correlations among low-level stimulus features are encoded by the temporal relationship between neuronal discharges. Therefore, temporal coherence might represent a signature of perceptual grouping, providing a link between temporal coordination and cognitive functions [93]. Performing chronic multisite recordings from the visual cortex of anesthetized cats and monkeys trained to solve cognitive tasks, these authors assigned different coding functions to low-dimensional dynamics features (such as synchronized oscillations and phase shifts) and high-dimensional non-linear ones. They proved that the low-dimensional dynamics, characterized by synchronized oscillations and large-scale correlations, are nothing more than the results of computations performed in the high-dimensional state-space of recurrently coupled networks. In brief, the multiplicity of the neural architecture in different sensory systems provides the opportunity to compare their dissimilar solutions to two common requirements: the need for dimensionality reduction and the effectiveness of the higher-order statistical structure inherent in natural sensory inputs [118]. In the reminder, I will discuss an alternative way to elucidate the relationships between the different dimensionalities of the brain and the environment.

Emotions and goals in extra dimensions. A few experimental examples illustrate how multidimensional approaches can be used to assess various mental activities. Emotional states (also termed emotional feelings) are traditionally viewed as innately programmed in subcortical areas [48]. Are such emotional experiences embedded in a complex, high-dimensional categorical mental space? Claims about the geometric organization of the semantic space of emotion have sparked intense debate, in particular among authors who support modified versions of the "higher-order theory of consciousness" [47]. This theory investigates the relationships between conscious states and higher-order representations, predicting that the latter portray oneself as being in particular mental states [47]. It is tempting to trace the described higher-order mental activities to dimensional considerations. In order to quantify the boundaries between emotion categories, Cowen et al. [11] analyzed a series of self-reported emotional states elicited by short videos. The array of reported emotional experiences was assessed in terms of discrete and dimensional geometries. The videos elicited 27 distinct varieties of reported emotional experience uncovering gradients of emotions (e.g., from anxiety to fear to horror to disgust) corresponding to smooth variation in affective dimensions, such as valence and dominance. As stated above, the temporal sequence of a brain activity can be represented as a trajectory (termed in this case a "thought chart") in a phase space. Assuming the topological space containing all the possible brain states to be a very high-dimensional manifold, Xing et al. [116] proposed an unsupervised manifold learning framework to

reconstruct and visualize such huge structure and to provide a novel data-driven framework for brain states' classifications. They examined the temporal EEG connectomes of 20 healthy volunteers, both at rest and during an emotion regulation task. A computerized procedure of nonlinear dimensionality reduction was then performed, in order to reconstruct and embed the learned manifold in a lower-dimensional Euclidean space. The thought charts of resting and emotion regulation task turned out to describe two distinct trajectories on the high-dimensional topological space. Such an approach has potential applications in neurofeedback, via real-time thought chart visualization, as it allows new trajectories to be inserted on-line through out-of-sample embedding Xing et al. [116].

Patterns in sensory cortex are organized across circuits equipped with an intrinsic functional architecture, arranged along relevant stimulus dimensions [112]. Waskom and Wagner examined whether similar organizational principles might also underlie distributed representations of more abstract information, such as rules or goals. Because the human prefrontal cortex supports goal-directed behavior by representing task context information through higher-order processes, they used multivariate decoding and spontaneous correlations' analysis to show that context representations are distributed across subnetworks within the prefrontal cortex. They found that pairs of voxels with similar context preferences exhibited spontaneous correlations, twice as large as those between pairs with opposite context preferences. Such subnetwork organization was stable across both task-engaged and resting states, suggesting that the abstract multidimensional representations supporting goal-directed cognition are constrained by an intrinsic functional architecture. In turn, Agus et al. [1] investigated psychology undergraduates' performances on problems presented in both verbal-numerical and graphical-pictorial formats. The best predictor of good performance turned out to be the confidence in solution correctness under the condition of time pressure administration. This finding suggests that the occurrence of a graphical facilitation could be the result of a multifactorial interaction among multiple contextual and individual dimensions (such as, e.g., visuo-spatial and numerical prerequisites, statistical anxiety, attitudes towards statistics and the confidence in response correctness), rather than being strictly related to the problem presentation format [1]. This lends support to the notion that brain function's multidimensionality may also refer to the subjective features of the studied population.

Consciousness: the multidimensional device of the brain? Recent claims suggest that brain modifications correlated with consciousness may reflect the action of projective transformations. In particular, Rudrauf et al. [87] introduced a mathematical model of embodied consciousness, in which the spatial field of consciousness is described in terms of projective geometry under the control of a process of active inference. Such spatial field, according to the authors, would be able to combine multisensory evidence with prior beliefs in memory. The single individual's standpoints and perspectives are selected according to his preferences, so that the choice of projective frames governs how expectations are transformed by consciousness [87]. An increasing interest has been raised by the integrated information theory of consciousness [98,44]. However, this framework is plagued by practical and theoretical problems. Indeed, measuring integrated information from neuronal signals requires the problematic assessment of all the elements at the same time; furthermore, the interpretation of the required spatial partition becomes rather vague in continuous time-series variables extracted from nonlinear dynamics. In order to tackle these issues, Tajima and Kanai [97] correlated the topological dimensionality of shared continuous attractor dynamics with integrated information. The effects of unobserved nodes on the attractor dynamics can be assessed using "delay-embedding techniques", able to quantify the dimensionality of an embedded attractor even in case of partial observations [97]. In touch with the above-mentioned gauge theories of the brain, topological dimensionality points towards a critical property of integrated information, i.e., its invariance to general coordinate transformations. This means that topological approaches allow the extension of the current notions of integrated information theory of consciousness to continuous dynamical systems too [97].

Whereas a pairwise causal influence between two neural elements can be assessed with existing methods, the quantification of multiple influences poses two major mathematical problems: overestimation and non-causal confounding influences. To solve these problems, Oizumi et al. [70] derived a measure of integrated information where causal influences among elements are statistically disconnected. Integrated information is here geometrically interpreted as the divergence between the actual probability distribution of a system and an approximated one. This framework allows a methodological unification of a wide range of information theoretic measures, such as mutual information, transfer entropy, stochastic interaction and integrated information. Such a novel concept of "geometrical integrated information" is very promising, as it quantifies the strength of multiple causal influences among elements, by simply projecting the probability distribution of complex systems onto constrained multidimensional manifolds.

In this section, I showed how experimentally detectable mental functions can be assessed in terms of changes in nervous dimensions. I concluded that mental processes such as sensations, perceptions, emotions and consciousness are associated with aspects of neural dynamics occurring in hidden nervous dimensions.

4. Multidimensional lessons from virtual and real neurodata

Multidimensional lessons from artificial neuronal networks. Within the brain networks, tens of millions of multidimensional geometrical structures have been recently uncovered. Reimann et al. [81] assessed a virtual network of artificial cortical neurons, using simplicial complexes' techniques that assess the routes of network information flows. In response to virtual stimuli, synaptically connected neurons are bounded into multidimensional geometrical structures, termed functional cliques. Functional cliques evolve in a stereotypical sequence towards the transitory appearance of more and more complex structures, up to eleven dimensions. The more neurons in a clique, the higher the dimension of the resulting geometric object. Higher-dimensional cliques temporarily assemble to enclose high-dimensional holes (referred to as cavities by the authors). The brain would therefore react to stimuli by building a tower of temporary multidimensional blocks, starting with mono-dimensional rods, then bi-dimensional planks, three-dimensional cubes and more complex multidimensional geometries. Experiments were then performed on real brain tissue, suggesting that, during development, the brain constantly rewires in order to build as many high-dimensional structures as possible [81].

Multidimensional lessons from human neurodata. Because the nervous symmetric states display dimensions higher than the asymmetric ones, the space of interest might not just refer to abstract dynamical neural manifolds, but also to detectable physical cortical locations. In this vein, Stemmler et al. [96] demonstrated that animals can navigate by reading out a simple population of grid cell activity across multiple spatial scales. The population vectors' combination at different microscopic dimensions allows to predict neural and behavioral correlates of multiscale grid cell readout, encompassing both entorhinal grid cells and hippocampal place cells. Despite the spatial activity of a single grid cell cannot constitute a metric, a hierarchically organized ensemble provides a distance measure [96]. The human brain exhibits the ability, peculiar among the living beings, to connect past, present and future events in a coherent picture, as if *Homo sapiens* was allowed to watch past-present-future glued together in a sort of mental kaleidoscope [102,24]. This observation led to the hypothesis that the brain activity might take place on a multidimensional torus. Indeed, recent findings suggest that the trajectories followed by human thoughts during mind-wandering and/or memory retrieval might take place in the functional space of a donut-like, four-dimensional (plus time) hypersphere, undetectable in the usual three dimensions (plus time) (Figure E). A retrospective evaluation of fMRI data, looking for the topological hallmarks of a fourth dimension, appears to be consistent with the existence of a functional hypersphere [102]. Such hypersphere is made observable through “hallmarks” on the brain surface, i.e., the simultaneous activation of areas opposite each other. The three-dimensional “shadows” of a four-dimensional torus are a direct consequence of the BUT, which predicts that “antipodal points” will be simultaneously activated in a multidimensional brain.

The last, but not the least, novel topological techniques of “computational proximity” provided evidence that a four-dimensional nervous activity does exist: in other words, a ceaseless, functional four-dimensional cap surrounds the brain [73]. The authors introduced a method for the measurement of hidden temporal patterns of entropy/information in fMRI data, based on image tessellation into small polygons encompassing dissimilar entropic content. During spontaneous brain activity, this method highlighted entropic patterns that closely resemble the trajectories foreseen by the presence of a brain hypersphere.

Does the brain activity lie in dimensions higher than the environment? Topological models predict that: 1) increases in anatomical/functional dimensions and symmetries might occur in the path from lower to higher activities of the brain; therefore, 2) information in the primary sensory areas is lower than in the higher associative ones. Contrary to common belief, this framework points towards a brain that progressively increases the complexity of the perceived object: an imagined (or perceived) dog encompasses more information than the real dog, because the former is filled with novel emotional and cognitive meanings (for example, when watching a dog, one might think: “how nice is it!” “it is a canid”, and so on). In order to demonstrate the counter-intuitive hypothesis that the brain activity might occur in dimensions higher than the three-dimensional (plus time) environment, Peters et al. [74] retrospectively assessed fMRI series and found that, during visual stimulation, the Rényi entropy (i.e., information) is higher in associative cortices than in the visual primary ones. Therefore, rather than distillate the message from the external world, our

brain scatters the incoming input in higher functional dimensions. A recent paper by Dasgupta et al. [15] substantiates the idea that the insects' nervous system too displays such increases in dimensions, compared with the environment. The authors assessed fruit flies during performance of a similarity search-for example task. The fly's olfactory circuit assigns similar neural activity patterns to similar odors, so that behaviors learned from one odor are used when a similar one is experienced. Surprisingly, flies solve this problem with a variant of a computer science algorithm termed "locality-sensitive hashing", that spreads the incoming message across many group of neurons, instead of concentrating it on a few ones [15].

In touch with these claims, Axelrod et al. [2] argued that we generate internal experience through the simultaneous activation of different cognitive systems, interrelated in higher dimensions. In order to test the hypothesis that our mental experience is mediated by a combination of activities of multiple cognitive processes, the authors evaluated four fMRI experiments with the same participants. They found that, during a self-generated mental activity (in their case, recalling personal memories), three cognitive processes (retrieving memory of events, building a vivid scene in our mind, moving back in time) are pooled over in order to operate simultaneously. Therefore, the subjective impression that each of our internal experiences is a unitary, indivisible entity is accomplished by the brain through a combination of different components, that might lie in higher dimensions. This also means that the concept of the "grandmother neuron" is likely condemned to disappear.

In this Section, I provided the "strongest" available claims, i.e., the recently published papers that more explicitly point towards the possible existence of real brain extra-dimensions. Once again, I stress that even these extreme approaches are built upon topological weapons. This is not purely coincidental, because topology seems to stand for a higher-order approach able to catch the otherwise elusive presence of hidden neural dimensions of the anatomical/functional structure of the brain.

5. Brains in higher dimensions: what next?

Despite the limitations due to the still insufficient computational power of the currently available algorithms, the study of hidden nervous dimensions is a promising approach. It is therefore legitimate to ask about the possible future developments of such powerful tools. In this review, I limited my description to the hidden dimensions of the brain; however, recent studies start to highlight other underestimated actors displaying the computational powers previously believed to be exclusive of the brain. Here are a few examples. In previous Sections, we noticed that the best candidate for the role of multidimensional neural phase space is the nervous connectome. However, the standard description of the connectome confined to cortical/subcortical structures [106] might not suffice. Indeed, recent paper point towards the connectome's nodes and hubs not just confined to the "higher" structures of the brain, but also extended to the central and peripheral nervous systems [49,99,69,89,54]. This means that multidimensionality might not be restricted to the brain, rather it might encompass the whole nervous system. Despite the integration of somatosensory information is generally assumed to be a function of the central nervous system, Du et al. [20] described fully functional GABAergic communication within rodent peripheral sensory ganglia. Sensory neurons express major proteins necessary for GABA synthesis and release GABA in response to depolarization: this means that sensory ganglia can modulate transmission of pain-related signals from the peripheral sensory nerves to the CNS [20]. In addition, it has been demonstrated that peripheral receptors play a role not just in information processing and perceptual synthesis, but may also have cognitive effects (for a review, see [101]). For example, Pruszynski and Johansson [80] reported that first-order neurons in the tactile system provide signal edge orientation, via both intensity and temporal structure of their responses. These peripheral neurons perform feature extraction computations that were typically attributed just to cortical neurons. These findings point towards nervous peripheral structures, such as somatosensory ganglia and receptors, representing a hitherto underappreciated site of sensory integration. Therefore, it is possible that, in next years, the concept of connectome will be also extended to the peripheral receptors, which reflect the abstract understanding of the immediate environment [34]. This enlarged framework of nervous activity offers a potential target for multidimensional approaches to neural function that are not circumscribed to the brain [20].

The last frontier: looking for the biochemical nervous correlates of further dimensions. Once assumed that multidimensional brain dynamics may occur, could its neurophysiological mechanisms be hypothesized? The incorporation of cellular and transcriptional data on individual nodes with network models of the connectome enabled the assessment

the microscopic mechanisms underpinning the macroscopic organization of the whole-brain networks [13]. Further insights could be provided through the assessment of the pharmacodynamics of active chemical substances [88]. For instance, high doses of lysergic acid diethylamide (LSD) cause the subjective report of additional spatial or temporal dimensions' perception. Individuals under the effect of LSD describe static objects in three dimensions that seem to move relative to additional spatial dimensions [41]. Carhart-Harris et al. [10] described how LSD provokes increased correlation among different brain regions, compared with placebo. The reported increases in connectivity between visual cortex and other brain areas might help to explain the subjective account of spatial and temporal dimensions' increase. LSD leads to a prolonged cross-activation of 5-HT_{2A} receptor heteromers and to increase in glutamate release in specific cortical layers [62,78]: hence, it might be speculated that the functional occurrence of brain additional dimensions could be correlated with the biomolecular mechanisms underlying the mental effects associated with the use of active mental substances.

Is it feasible to build a real (or artificial) neural network to detect (or simulate) multidimensional brain activities? A recent experimental technique could contribute to solve this crucial question, throwing an unexpected bridge between biophysical neural function and . . . the devices of theoretical physics. A magnetic field with the proper angulation is able to modify the trajectory and the voltage difference of a given electric current. This phenomenon is termed “Hall effect” [30]. A similar process, called “quantum Hall effect” [66], occurs when an electric charge is sandwiched between two surfaces cooled down to near absolute-zero temperature. When such two-dimensional apparatus is exposed to a strong magnetic field, the electric amount that it can conduct becomes “quantized”. This phenomenon is straightforwardly elucidated, if we just consider it as occurring in four, instead of the canonical three, spatial dimensions [121,45,122]. In operational terms, Lohse et al. [53] built a two-dimensional waveguide for light flow and superimposed complex oscillatory patterns that act as a manifestation of higher-dimensional coordinates. They used a two-dimensional lattice consisting of superlattices along the x and y axes. Each superlattice is achieved by superimposing two standing waves of different wavelength (Fig. 2A). A third wave is introduced along the x direction: this corresponds to tilting the long lattice along the x-axis' one-dimensional path (Fig. 2B). Lohse et al. [53] and Zilberberg et al. [122] provided the wavelengths, the angles and the proper equations required in order to detect the four-dimensional quantum Hall effect. Two different responses are attained: 1) a linear one (two-dimensional) along the axis x, and 2) a nonlinear one (four-dimensional) along the axis y (Fig. 2C). In sum, the authors provided a relatively simple approach able to describe peculiar quantum dynamics in terms of pure oscillations. Here I ask: could such “four-dimensional-building apparatus” be transferred, with the due amendments, to the neuroscientific field, in order to assess and quantify the presence of further spatial dimensions in the brain? I speculate that the typical conformation of the waves located in the lattice might be achieved by using real nervous oscillations of different frequency. The required waves might stand for the nervous electric oscillations detected *in vivo* through well-known neurotechniques such as EEG and fMRI. Therefore, the superimposition of neural waves of different wavelengths might produce the superlattice illustrated in Fig. 2A. The superimposition of further brain waves (Fig. 2B) might give rise to both (two-dimensional) linear and (four-dimensional) nonlinear nervous dynamics. My theoretical claim would allow to transfer the framework of the four-dimensional quantum Hall effects provided by Lohse et al. [53] to the realm of neuroscience, in order to: a) describe the real multidimensional brain dynamics and b) demonstrate the feasibility of a synthetic nervous network equipped with four spatial dimensions (plus time), instead of the classical three (plus time).

Dimensional changes in human diseases. To better understand, diagnose, and treat psychiatric primary and secondary disorders, it is crucial to achieve deeper insights into brain circuits in health and disease, both in humans and animal models [107], in order to elucidate abnormal brain connectivity and network dysfunction [25,13]. For example, Odish et al. [68] demonstrated the presence of longitudinal modifications of the structural connectome in (pre) Huntington's disease, compared with healthy controls: these dynamics correlated with changes in clinical and cognitive measures. Other brain disorders, including schizophrenia and Alzheimer's disease are associated with specific lesions, more likely to be located in the connectomic hubs [12]. Furthermore, diffusion tensor imaging-based studies, together with multi-shell tractography and probabilistic fiber tracking, have identified also in Parkinson's disease global and local connectome disruptions (reduced global clustering and efficiency, as well as increased global path length) and extensive patterns of reduced connectivity [38,92]. Gleichgerrcht et al. [29] provided evidence that epilepsy too is likely to be correlated with abnormal function within specific brain networks. Last, but not least, the Human Connectome Project, gathering data from healthy young adults aged 21–35, is achieving a high-quality dataset for comparability with other populations. Their ongoing studies include: Amish Connectome Mental Illness,

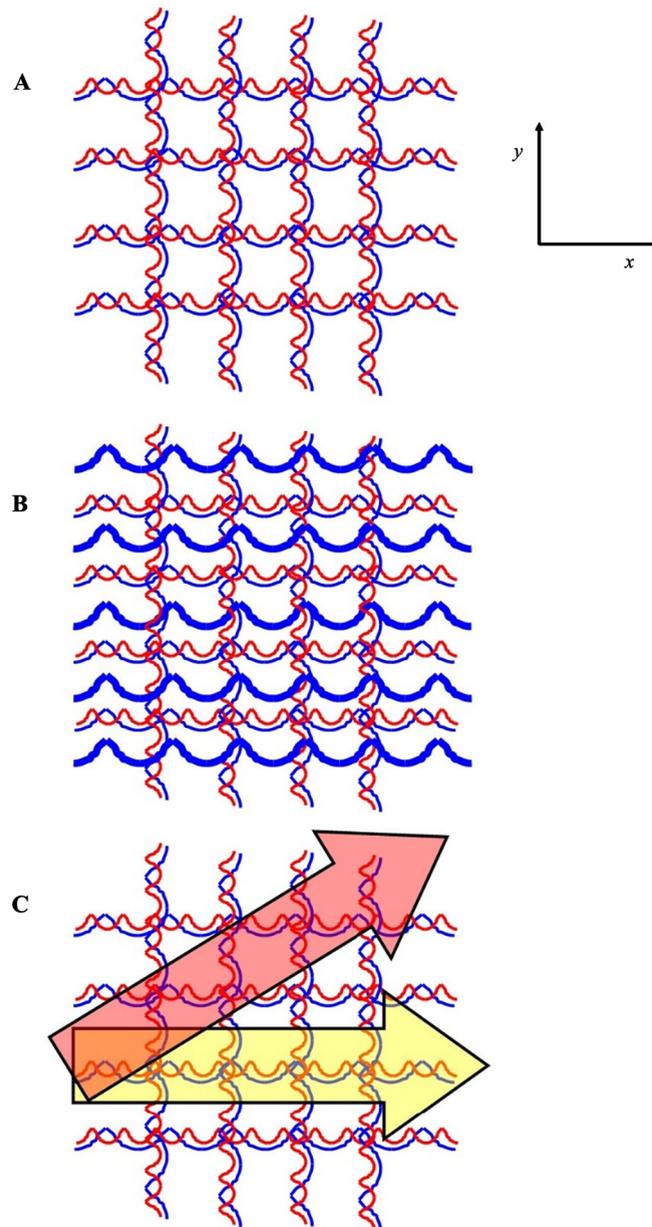


Fig. 2. Four-dimensional physical activities taking place on a two-dimensional superlattice. Fig. 2A depicts a topological lattice equipped with two waves of different wavelength (red and blue thin lines). Fig. 2B: another wave (blue thick line) with the proper wavelength and angulation is superimposed to the lattice along the direction x . In the experimental assessment of Lohse et al. [53], the proper angulation of the third wave is achieved by tilting the lattice. Fig. 2C: the superimposition of the three waves gives rise to two different motions: a two-dimensional, linear one along the axis x (yellow arrow), and a four-dimensional, nonlinear one, along the axis y (red arrow). Modified from Lohse et al. [53].

Anxiety and Depression in Teenagers, Human Connectomes for Low Vision, Blindness, Sight Restoration (<https://www.humanconnectome.org/disease-studies>).

One could readily anticipate that multidimensional approaches to network theory will allow a novel level of assessment of human diseases, currently undetectable by standard techniques. The problems to tackle are huge, because the large number of different network organizations and fiber tracts' connections make changes detectable enough to distinguish groups not sufficient to diagnose the disease of an individual patient [37]. Nevertheless, just to give an example of the huge potentialities of the novel approaches, these challenges could be addressed by a multidimensional

description of structural, functional, effective and dynamic connectomes. Simulated lesion studies of both targeted and random removal allowed the visual assessment of generated abnormalities in brain connectivity and the inspection of topological alterations in neural connections [117,12]. Kang et al.'s [39] suggest that energy landscape analysis could be an important tool to investigate the brain networks' alterations correlated with brain diseases. Neurodevelopmental brain disorders might arise from imbalances in excitatory and inhibitory (E/I) brain circuitry that cannot be captured by the classical low-dimensional model. O'Donnell et al. [67] combined computational simulations with analysis of in vivo two-photon Ca²⁺ imaging data, recorded from the somatosensory cortex of a mice model of Fragile-X Syndrome [67]. Their results suggest that the basic E/I imbalance model should incorporate higher dimensional approaches, which could better capture the multifaceted computational functions of neural circuits.

Recent approaches demonstrate the feasibility of precision medicine paradigms in the assessment of complex, multifactorial diseases [51]. For example, patients affected by the Type 2 diabetes (T2D) display a wide range of phenotypes and susceptibilities to diabetes-related complications, including neurologic impairment. Li et al. [51] developed a precision medicine approach that characterizes the complexity of these patient populations. The use of high-dimensional electronic medical records, genotype data from more than 11,000 individuals and topology-based patient-patient networks allowed the authors to identify three distinct subgroups of T2D, characterized by different complications and outcomes. A genetic association analysis of the emergent T2D subtypes was also performed, in order to identify subtype-specific genetic markers. This technique highlighted thousands of single-nucleotide polymorphisms which map to unique genes, specific to the three T2D subtypes [51]. The use of novel meta-analytic approaches in neuroscience has been described by Crossley et al. [13], who applied network analysis methods to previously published studies and/or publicly accessible databases. This strategy of combining connectivity with other brain features, termed "meta-connectomics", shows how network analysis of task-based neuroimaging studies can be used in order to infer functional co-activation from primary data of regional activations. This approach, relating cognition to functional network topology, shows that the brain is composed of cognitively specialized functional subnetworks or modules, linked by a rich club of less specialized regions. Furthermore, meta-connectomics makes it possible to correlate meta-analytic maps of disorder-related MRI abnormalities with the complex topology of the healthy connectome.

In conclusion, the examples provided in this Section suggest once again that brain multidimensionality is a novel exciting field of research, potentially able to provide integrative insights into nervous organization. It is easy to foresee that the hidden dimensions of the brain, either abstract or real, will play a prominent role in the future assessment of both neural function at different coarse-grained levels, and psychiatric disorders.

6. Conclusions: it's not true, but I still believe it

In this paper, I emphasized how numerous clues in the recent literature point towards the possibility to assess brain activities in terms of multidimensional phase spaces. Powerful computational methods allow the extraction of a large amount of otherwise inaccessible information from the massive neurodata sets already available in literature and in the web. I stressed that such hidden information can be more easily extracted, when embedded in multidimensional phase spaces. I showed how such novel approaches in multi-dimensions could be useful in the assessment of countless brain activities, from perceptions to consciousness, from emotions to probabilistic reasoning, and so on.

However, a last, crucial issue needs to be tackled: are nervous multidimensions a convenient technical device, or do they reflect genuine brain features? In the experimental assessment of nervous activity, two ways are viable in order to cope with multidimensions. In many of the papers quoted in this review, extra-dimensions stand for a technical device used just for methodological purposes, which makes it possible to deal with the overwhelming complexity of the nervous activity. There is, however, another conceivable use of additional dimensions. Some of the above-mentioned authors provided further axes in order to describe true brain extra-dimensions: this means that these added dimensions are not mere abstractions. Yet, the borders between the two approaches are not so neat at all, because the standard math that usually assesses networks and brain function is unable to detect the required high-dimensional structures and spaces. This may also explain why it is so hard to understand the brain. It is difficult to understand whether these papers consider neural activity somehow analogous to a multidimensional structure, or we are asked to believe that the brain actually is, or contains, one. In spite of the abundance of both proofs and available methods for the empirical assessment of imperceptible hidden dimensions, this type of multidimensional realism of the brain may not be readily accepted, because of our limited three-dimensional perception. But then, does it really matter? As

suggested by Petlevski [75], the topological models of multidimensional brain offer a very large conceptual module encompassing all the branches of neuroscience, including human diseases.

To sum up, are we in front of a multidimensional brain, or a three-dimensional brain that operates in multidimensions? The answer may just not matter, as long as the novel models are able to elucidate nervous processing and provide testable hypotheses. Apart from epistemological debates on the realism or anti-realism of entities and theories, what matters now is that the study of changes in brain dimensions is a promising novel tool. Approaching brain multidimensions, a large amount of deductive rationalism is required, because this framework is about mappings and projections among different activity levels. We must remind that, while neural networks are known to exhibit rich, lower-order connectivity patterns at the level of individual nodes and edges, higher-order organization is still largely unexplored. I believe that a complete description of nervous phenomena can only be achieved by looking at their higher levels, where differences are more easily detectable and assessable.

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