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Comment

Chimera states and hidden attractors
Comment on "Chimera states in neuronal networks: A review" by S. Majhi, B.K. Bera, D. Ghosh, M. Perc ☆

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When in 2002 Kuramoto and Battogtokh [1] for the very first time described a strange behavior of coupled oscillators, within which 'coherence and incoherence' co-existed, it took two years until this surprising pattern got confirmed [2] and found acceptance in the scientific environment. The so-called 'chimera states', unpredictable animals with 'body of a lion, head of a goat and a tail of a snake' (https://en.wikipedia.org/wiki/Chimera_(mythology)) formed a new branch of dynamical problems' area of science, making it possible to have different types of responses of oscillators (ordered and unordered), even though the scheme of coupling is identical for all of the units!

Co-existence patterns have been found in mechanical models [3,4], chemical oscillators [5] or neuron systems [6]. Neuronal networks, not only related with the biology in general, but particularly with human brains, deserve a special consideration in this subject. The review 'Chimera states in neuronal networks: A review' by Majhi et al. [7] tries to deal with this matter and its results are found to be very interesting.

Authors have generally discussed two types of neuronal models, i.e. the FitzHugh-Nagumo and the Hindmarsh-Rose ones. This choice is reasonable, since both types of systems are found to be commonly used in chimeras research. What should be noted, the explanation of coupling schemes that can occur for neuronal networks is included. This part may be especially valuable for those readers, who are not very familiar with the neurobiology models, since it explains nice and clearly the way in which two neurons communicate and share information. The biology magic is then transformed into the mathematical dynamical equations with different types of coupling (local, global and non-local; especially the latter one is typical for chimeras investigations) and examined in various scenarios, depending on the class of synapses occurring in the system.

Authors begin with the electrical synapses, discussing the cases of deterministic equations [8] for which typical, Kuramoto-like chimera states are present, stochastic equations [9] (where the intensity of noise has major influence

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on dynamics and possible appearance of desired states) and time-delayed cases (giving just a sketch of potential results). Then, the chemical synapses scenario is presented, with [10] and without [11] delay. The latter paper seems to be especially interesting since it includes the two-parameter plane, exhibiting the potential regions of chimeras' birth. This kind of results may be very useful, since we still do not know under which conditions this unpredictable states can arise. Moreover, a new type of traveling imperfect chimera state [12] is shown, which not only include the co-existence of coherence and incoherence, but also travels through the oscillators in time, changing the character of each single unit (the domain to which it belongs in the whole networks' profile). The variety of states possible for both electrical and chemical synapses [13,14], highly depending on the investigated model and its parameters proves that the subject is very complex and still requires many studies to be conducted.

Survey prepared by Majhi et al. is very clear and provides a thorough review on the most important results on chimera states for neuronal systems. Readers not very familiar with the concepts of chimeras may find here useful tools and methods of analysis to begin their own adventure with these amazing creatures (just to mention typical measures used in investigations on complex networks, e.g. mean phase velocity, or local order parameter). Taking the possibility and referring to the 'further research' points listed at the end of the survey, we would especially like to focus on point 7, i.e. *'Although there exists a few works concerning basin of attraction for the chimera states [119, 120], precise study on the influence of initial conditions in inducing chimera patterns in neuronal networks is missing'*, which is closely related with our own research.

From a dynamical systems perspective, chimera state corresponds to a certain attractor located somewhere in the phase space. Thus for its research, the problem of determining all co-existing local attractors in the phase space of a dynamic system describing a network of coupled oscillators is important. For numerical localization of attractor, one needs to choose an initial point in its basin of attraction and observe how the trajectory, starting from this initial point, after a transient process visualizes the attractor. Self-excited attractors, even co-existing in the case of multistability, can be revealed numerically by the integration of trajectories, started in small neighborhoods of unstable equilibria, while hidden attractors have the basins of attraction, which are not connected with equilibria and are "hidden somewhere" in the phase space. These types of attractors (along with the so-called 'rare attractors' [15]) can also have very small measures of the basins of attraction which are especially hard to trace. The classification of attractors as being hidden either self-excited [16,17] was introduced in connection with the discovery of the first hidden Chua attractor [18–20] and has captured much attention of scientists from around the world (see, e.g. [21–25]).

The concept of hidden attractors not only reflects the difficulties in studying fundamental problems, e.g. the second part of Hilbert's 16th problem on the number and mutual disposition of limit cycles in two-dimensional polynomial systems, and Aizerman's and Kalman's problems on absolute stability of automatic control systems, but it also allowed one to reveal new attractors in various applied models, such as phase-locked loops (PLL), electromechanical models with the Sommerfeld effect, drilling systems, aircraft control systems, and others (see, e.g. the survey [21]).

The problem with hidden (and rare) attractors is that we cannot use straightforward methods to trace such states, or even predict that they exist for the system's potential dynamics. Since the fixed points are useless and the searching through the whole phase space may be too much time (and energy) consuming, the problem becomes not trivial. And dangerous! One can simply imagine a model that determines the dynamics of a mechanical appliance, or the potential growth of a deadly disease, which may consist a hidden attractor, about which the researchers have no idea. If such a state is very undesired, leading to a catastrophe or a global epidemy, the idea about the possible conditions in which it can arise becomes extremely important.

Considering chimera states, the equilibria analysis for complex networks may be too complicated to conduct, and hence we look for such states also very uncertainly, usually using some random methods. By solving the problem with tracing of hidden attractors, we can get closer to the proper understanding of chimeras themselves. Especially if the chimera state is an undesired state, due to some reasons, we would be able to predict its appearance and prepare some protecting mechanism to prevent, or at least control it.

Concluding, even though very young, chimera states play a major role in the theory of nonlinear dynamics and its applications. They constitute a bridge between well-known coherent patterns (synchronization) and incoherent ones (chaotic dynamics, general irregularity), exhibiting that it is possible to have both types of behaviors within one system's profile. This co-existence is very surprising and when finally properly understood, can become a milestone for our environment and ourselves. As have been stated in the review by Majhi et al., co-existence dynamics in neural networks is closely related with the analysis of dreams, bump states or brain disorders (just to mention nowadays diseases of affluence, like Alzheimer's or Parkinson's ones). The development in this area of modern dynamics science cannot be neglected and requires hard and solid work to be conducted. Although the applications (and even experimental confirmation of chimera states in neural networks) is still beyond our reach, when it finally gets close enough, its results and consequences can be outstanding.

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