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Research Highlight

Scaling symmetry meets topology

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In 1970, Vitaly Efimov found an interesting phenomenon in a quantum three-body problem, which is now known as the Efimov effect [1]. Efimov found that when the two-body interaction potential is short-ranged and is tuned to the vicinity of an *s*-wave resonance, an infinite number of three-body bound states emerge and their eigenenergies E_n form a geometric sequence as $E_n = -|E_0|\exp(-2\pi n/s_0)$, where s_0 is a universal constant. This effect is directly related to a symmetry called the scaling symmetry. In short, the three-body problem with a resonant two-body interacting potential can be reduced to the following one-dimensional Schrödinger equation as

$$\left(-\frac{\hbar^2 d^2}{2md^2R}-\frac{(s_0^2+1/4)\hbar^2}{2mR^2}\right)\Psi = E\Psi. \quad (1)$$

It is very clear that Eq. (1) displays a scaling symmetry, that is to say, because both the kinetic and the interaction terms scale as $1/R^2$, by making a scale transformation $R \to \lambda R$ and $E \to E/\lambda^2$, the Schrödinger equation is invariant.

However, to solve the problem, one has to impose a boundary condition at the short distance, for otherwise the problem is ill-defined. In many situations, such a boundary condition will destroy the scaling symmetry completely. The most significant point of the Efimov effect is that the scaling symmetry is not completely destroyed, instead, a discrete scaling symmetry is retained. A discrete scaling symmetry means nothing but the scaling factor λ can not take an arbitrary value but can only take a set of discrete values of $\exp\{-\pi n/s_0\}$. Another significant point is that, although the short-range boundary condition is non-universal, the scaling factor is a universal one. Hence, one often also uses the term Efimov effect to refer to those phenomena with the discrete scaling symmetry and a universal scaling factor. In modern content of the renormalization group, this effect is also tied to the limit cycle behavior of the renormalization group flow equation [2].

The study of the Efimov effect has been an active topic in the few-body problems. The Efimov effect in the three-body system has been firstly observed in cold atom system [3], and later has also been observed in the helium trimer [4]. The Efimov effect has also been studied in few-body problems beyond three [5,6]. Dynamical phenomenon with such a universal discrete scaling symmetry has also been proposed and observed in ultracold atomic gases [7]. Very recently, it has also been pointed out that the Efimov effect can be linked to the fractal behavior in the time domain [8,9]. Nevertheless, these studies have so far been limited to atomic and nuclear systems. The recently published experiment by Wang et al. [12] brings the Efimov physics into condensed matter system by observing such a universal discrete scaling symmetry in the topological semi-metals. The topological semi-metal has received considerable attentions in the past decades. The interplay between this discrete scaling symmetry and topology introduces a new twist to the physics of topological material and will make its physics even richer.

As said earlier, Eq. (1) is scale invariant because both the kinetic and the interaction terms are proportional to $1/R^2$. In the semi-metals, the low-energy dispersion of electrons (or holes) is linear in momentum *k*. When we add a charged impurity to such systems, the Coulomb potential between electron and the impurity is $1/R$. Thus, in this case, both the kinetic and the interaction terms have the same scaling with $1/R$, and again the Hamiltonian is scale invariant. Similar to the Efimov effect discussed above, the short range boundary condition breaks it down to the discrete scale invariance and results in an infinite number of quasi-bound states forming a geometric sequence [10,11].

The experiment by Wang et al. [12] reported the observation of such discrete scale invariance in single-crystal three-dimensional Dirac semi-metal $ZrTe_5$. The $ZrTe_5$ crystallizes in a layered orthorhombic structure with the space group Cmcm. It contains layers of $ZrTe_5$ in the *ac* plane, coupled via the van der Waals interactions along the *b* axis. By applying magnetic field in the *b* axis and attaching leads in the *ac* plane, the authors measure the magnetoresistance of $ZrTe_5$ and the result is shown in Fig. 1a (for low temperature) and b (for high temperature). After subtracting the background, the resistance shows five oscillating cycles with respect to $\log(B)$, as shown in Fig. 1c. Fig. 1d shows that a peak can also be seen after the Fourier transformation. They also show that several different samples can reproduce this universal oscillations.

The theoretical understanding is given by considering the effect of the magnetic field with $l_B = \sqrt{\hbar c/eB}$. When l_B approaches the typical size of some quasi-bound state, the binding energy touches the Fermi surface, which results in a resonant scattering between the mobile carriers and the quasi-bound states. This influences the transport property and leads to the log-periodicity

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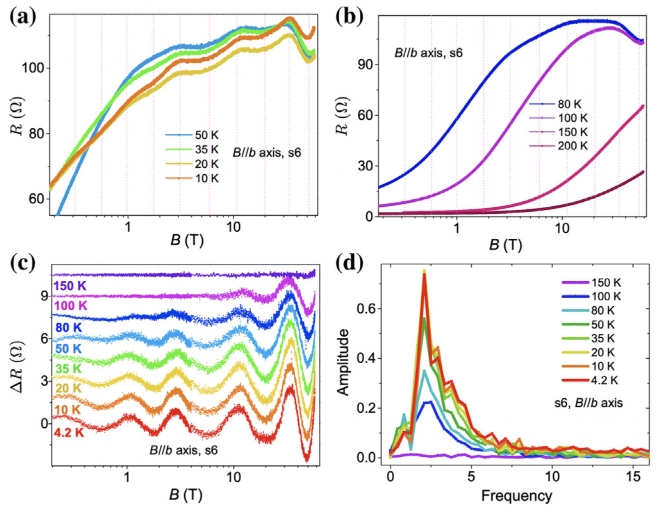

Fig. 1. Experimental observation of magnetoresistance oscillations in $ZrTe_5$. (a) Magnetoresistance at relatively low temperatures. (b) Magnetoresistance at relatively high temperatures. (c) Oscillations after subtracting a smooth background from the raw data. (d) Fourier transformation of magnetoresistance oscillations. Here s_6 labels a specific sample of single-crystal $ZrTe_5$. Reproduced from Ref. [12].

of magnetoresistance. The authors also use the T-matrix approximation to obtain the conductance formula as [13]

$$\sigma_{xx} = \frac{4e^2 l_B^2}{\hbar}\left(n_c C \sum_n \frac{\Gamma(B)}{\Gamma(B)^2 + (E_F - E_n(B))^2}\right), \quad (2)$$

where n_c is the density of impurities, C is some microscopic parameter, $E(B)$ are quasi-binding energy in the presence of the magnetic field and $\Gamma(B)$ is the broadening due to the coupling to mobile states. Because of the descrete scaling symmetry of the system, the resonant magnetic field B_n where $E_n(B_n) = E_F$ also forms an approximate geometric sequence. This result supports the presence of log-periodicity in magnetoresistance.

This interesting progress of realizing the discrete scale symmetry in the Dirac semi-metal open up many questions for future investigations. For example, similar experimental evidence for the discrete scale invariance has also been obtained from the local tunneling measurements near charged impurities in graphene [14], which is a two-dimensional Dirac semi-metal. Since the presence of this discrete scale invariance is insensitive to the dimensionality or the (pseudo-) spin, can such discrete scale invariance be realized in other systems or by other measurments? More interestingly, the system with filled bands is intrinsically a many-body system. However, in the considerations hitherto, the Coulomb interactions between electrons have not been included. The Coulomb interaction can lead to screening effect. The understanding of such effect is important, and some attempts to this problem can be found in Refs. [15,16].

Conflict of interest

The authors declare that they have no conflict of interest.

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