



## News &amp; Views

## A high-stability quasi-spherical resonator in SPRIGT for microwave frequency measurements at low temperatures

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To meet the demands of high stability and sustainable temperature measurements over time, TIPC-CAS in China and LNE-Cnam in France have jointly developed single-pressure refractive-index gas thermometry (SPRIGT), which is expected to be an alternative primary thermometry because of its 0.25 mK accuracy and high measurement speeds [1].

In SPRIGT, the refractive index of a working gas in a resonator is determined by measuring the ratio of microwave resonance frequencies under vacuum and with a gas bounded in the conductive resonant cavity at a single pressure and temperature. Then the temperature of the gas can be determined by comparing its refractive index or resonance frequencies with that at a reference temperature (e.g., fixed point of neon,  $T_{\text{Ne}}$ ). The simplified relation between temperature and resonance frequencies at a fixed pressure is as below [1–4]:

$$\frac{T}{T_{\text{Ne}}} \approx \frac{f_0^2(T_{\text{Ne}}) - 1}{f_1^2(T_{\text{Ne}})} \frac{f_1^2(T)}{f_0^2(T) - 1}, \quad (1)$$

where  $f_0$  and  $f_1$  are resonant frequencies measured in vacuum and at a fixed pressure, respectively.

High-accuracy 2 ppb (1 ppb  $\equiv 10^{-9}$ ) measurements of the resonant frequency are critical for realizing SPRIGT at low temperatures [1]. Therefore, a high-quality factor copper quasi-spherical resonator (QSR) was built from two hemispheres whose inner surfaces were machined by precision diamond turning. The QSR dimensions were the same as those of the resonator used at LNE-Cnam [5]; nominal semi-major axes of the tri-axial ellipsoid 49.50, 49.75 and 50.00 mm, and a nominal shell thickness of 10.0 mm. The QSR was closed at room temperature, its shape being controlled via the relative half-width of well-resolved microwave

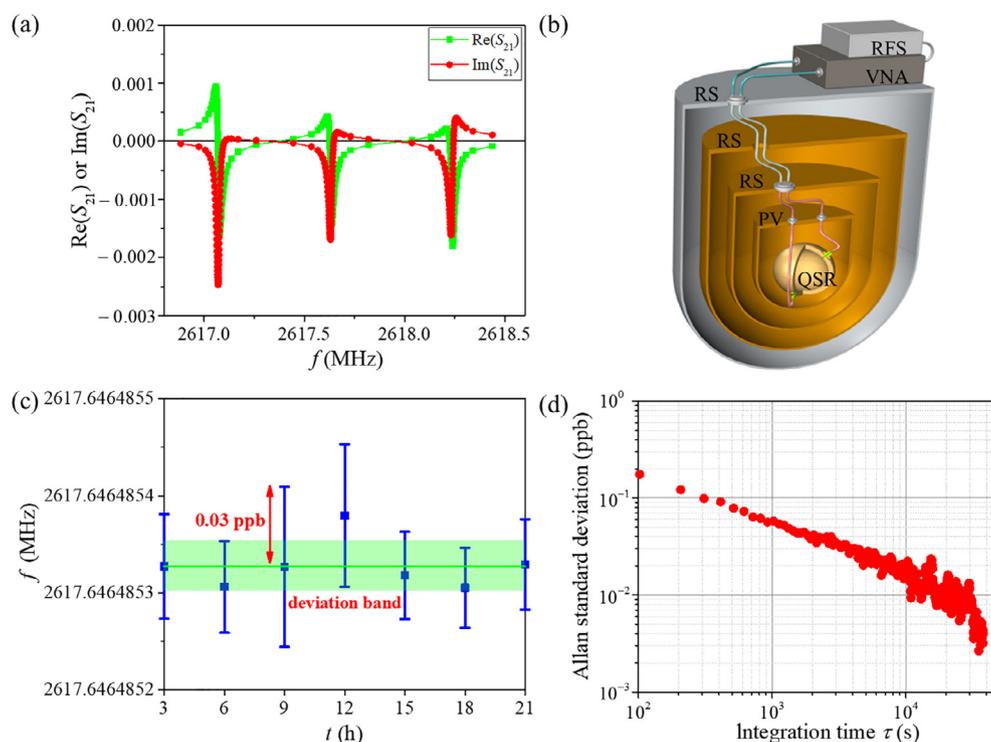
resonance. Microwaves are coupled in and out of the resonator via two loop antennae, one on each hemisphere. A two-port vector network analyzer (KEYSIGHT N5241A PNA-X) and a 10 MHz reference signal, provided by a Rubidium Frequency Standard (Stanford Research Systems FS725), were used to measure the complex scattering parameters  $S_{21}$ . Scans were performed for several different modes  $n$  (e.g. TM11, TE11 etc.). For each of these, the resonant frequencies  $f_n$  and half-widths  $g_n$  were determined from non-linear least-squares fits of  $S_{21}$  as a function of frequency [2]. Fig. 1a shows a plot of the complex parameters  $S_{21}$  for a 1.5 MHz wide scan over the TM11 mode at 8.0 K.

To perform SPRIGT at TIPC-CAS [6–8], a cryogen-free cryostat has been developed that can maintain temperatures over the range 5–25 K, with a variation of only 20  $\mu$ K, whatever the timescale. Fig. 1b shows a schematic of the cryostat containing the resonator protected by three layers of thermal shielding.

The relative standard frequency uncertainty  $u(f)/f$  for a single 100 s scan over the TM11 mode was 0.20 ppb, 10 times lower than the predicted uncertainty of 2 ppb [1]. The relative standard frequency uncertainties for single 100 s scans over the TM11, TE11, TM12, and TE13 modes were respectively 0.13, 0.10, 0.12, and 0.69 ppb at 5.0 K. These are up to 20 times lower than the predicted uncertainties [1]. The larger uncertainty for the TE13 mode arises most likely because its 10.4 GHz resonant frequency lies beyond the 8.0 GHz maximum bandwidth of the four feedthroughs making its signals weaker and therefore noisier. Higher bandwidth feedthroughs and microwave signal amplifiers should solve this problem in future work [9]. To characterize frequency stability, scans were performed over the TM11 triplet for a duration of 21 h with the resonator at 8 K (well above 4.2 K to avoid pre-condensation of helium-4). Fig. 1c shows the results grouped in periods of 3 h. The relative standard uncertainty for each group was 0.02 ppb and the global average 0.01 ppb. In this condition, the adopted distribution is a t-Student, with 6 degrees of freedom. A plot of the generalized variance has been presented to demonstrate how frequency stability varies as a function of integration time, shown in Fig. 1d. In addition, this frequency instability lies close

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**Fig. 1.** The structure schematic diagram of the resonator used in SPRIGT and its performance at low temperatures. (a) Frequency scan over the three components of the TM11 mode performed at 8.0 K showing the real and imaginary components of the parameter  $S_{21}$ . (b) Cryostat containing the quasi-spherical microwave resonator (QSR). Key: RFS: rubidium frequency standard; VNA: vector network analyzer; RS: outer and middle radiation shields; PV: pressure vessel and innermost radiation shield. (c) Resonant frequency stability of the TM11 mode at 8.0 K (the green line shows the global average frequency, and the light green area shows the standard deviation band). (d) Resonant frequency Allan standard deviation of the TM11 mode as a function of integration time at 8.0 K.

to the manufacturer's specified limit for the FS725. With a helium-4 filled resonator, such an uncertainty would correspond to a temperature uncertainty of 2  $\mu$ K. Note that such performance possible because of the low linear thermal expansion coefficient of copper at low temperatures, where a 0.05 mK temperature deviation would change the frequency by 0.007 ppb [10].

These measurements have demonstrated the frequency stability obtainable from the microwave source and resonator designed for SPRIGT. The performance already exceeds expectations. Moreover, improvements are expected with the implementation of a GPS frequency reference and higher bandwidth feedthroughs.

### Conflict of interest

The authors declare that they have no conflict of interest.

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