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Short communication

## Optimization of the flight technique in ski jumping: The influence of wind

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## ABSTRACT

Ski jumping performance is strongly affected by wind. Flight technique optimization for maximizing jump length is a highly complex motor-control task that also depends on the wind. Pontryagin's minimum principle was used in this study to gain a better understanding on how wind influences flight technique optimization. Optimum time courses of the angle of attack  $\alpha$  of the skis and of the body-to-ski angle  $\beta$  were computed for seven realistic wind scenarios on the large hill and on the flying hill. The optimum values of  $\alpha$  were smaller at headwind, and larger at tailwind when compared to the optimum time course at calm wind. The optimum values of  $\beta$  were the smallest possible ones at the given flight technique constraints, except for the last part of the flight. Optimum adjustments of  $\alpha$  increased the jump lengths between 0 and 1.8 m on the large hill, and between 0 and 6.4 m on the flying hill. Maximum jump length increases were achieved at the highest headwind speed. Even larger jump length effects can be achieved by using smaller  $\beta$ -angles, which might be possible in headwind conditions, but this is associated with increased problems to keep the flight stable.

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## 1. Introduction

Ski jumping can be subdivided into four consecutive phases: in-run, take-off, flight, and landing. At a given hill, the jump length depends on the in-run velocity parallel to the ramp, the take-off velocity perpendicular to the ramp, and the forces that act on the athlete and his equipment during the flight: gravity and the aerodynamic forces drag and lift (Straumann, 1927; König, 1952; Denoth et al., 1987; Müller et al., 1995, 1996). The aerodynamic forces depend on the drag and lift areas, which are functions of the flight position angles: angle of attack  $\alpha$  of the skis, body-to-ski angle  $\beta$ , hip angle  $\gamma$ , and V-angle (angle between the skis to each other) (Müller et al., 1996). By controlling the flight position angles according to optimum aerodynamic forces, the athlete can maximize the jump length. This optimization task is constrained by the athlete's individual biomechanical and aerodynamical features and abilities and the pitching moment, which depends also on the flight position angles. The flight phase in ski jumping can be simulated and optimized using the equations of motion and accurate wind tunnel data of the drag and lift areas. Straumann developed the equations of motion already in 1927, and Remizov

(1984) applied optimal control theory (Pontryagin's minimum principle, Pontryagin et al., 1962) for the first time to compute the optimum time course of the angle of attack  $\alpha$  (with skis held in parallel). Müller et al. (1995, 1996) were the first to use time functions of drag and lift areas based on field studies and wind tunnel measurements for realistic computer simulations of V-style ski jumping. This approach was further developed and applied to study various scientifically and practically relevant topics in ski jumping (Schmölzer and Müller, 2002, 2005; Müller et al., 2006; Müller, 2009a,b). Jung et al. (2014) applied Pontryagin's minimum principle to optimize the time courses of the angle of attack  $\alpha$  of the skis and of the body-to-ski angle  $\beta$ , considering flight technique constraints. They found that substantially different time courses can result in similar jump lengths. This is in line with field study results during the 2002 Winter Olympic Games, which illustrated that the medalists used distinctively different flight techniques (Schmölzer and Müller, 2005).

The jump length effect of aerodynamic forces, and also the associated importance of flight technique optimization increase with increasing size of the hill (Schmölzer and Müller, 2002; Jung et al., 2014). Hills are characterized by the hill size (HS), which is the distance between the edge of the ramp and the L-point on the landing slope surface (Gasser, 2012). Competitions, including World Cup events, World Championships, and Olympic Games

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are held on three types of hills: normal hills (HS: 85–109 m), large hills (HS: 110–184 m), flying hills (HS ≥ 185 m) (Gasser, 2012).

Aerodynamic forces acting on the athlete and his equipment are influenced by wind because wind changes speed and direction of the airflow and alters the angle of attack  $\alpha$  of the skis. The resulting jump length effect depends on the wind speed and direction, the flight technique, the hill size, and the flight phase in which wind occurs. A detailed computer modelling analysis of wind effects in ski jumping can be found in Jung et al. (2018). Jung et al. (2015) considered wind as a possible factor influencing flight technique optimizations on all three types of hills for the first time. However, it had not been taken into account that the International Ski Federation (FIS) adapts the in-run length with respect to wind and thus the in-run speed to keep the jump length of elite athletes in the vicinity of the landing zone between the K-point and the L-point of the hill.

In this study, flight technique optimizations of the angle of attack  $\alpha$  of the skis and the body-to-ski angle  $\beta$  were performed for seven realistic wind scenarios on the large hill and flying hill.

## 2. Methods

### 2.1. Computer model of the flight phase

The in-run velocity parallel to the ramp  $\mathbf{v}_0$  and the take-off velocity perpendicular to the ramp  $\mathbf{v}_{p0}$  are the initial conditions for the flight path. During the flight, the flight path (in the  $x$ - $z$ -plane) is determined by the gravitational force  $\mathbf{F}_g = m\mathbf{g}$  ( $m = 69$  kg was used here for the athlete with equipment,  $g = 9.81$  m s<sup>-2</sup>), and by the aerodynamic drag force  $F_d$  and lift force  $F_l$ . The drag and lift force depend on the air density  $\rho$ , the drag area  $D$  and lift area  $L$ , respectively, and the magnitude  $w = |\mathbf{w}|$  of the airflow velocity  $\mathbf{w}$ :  $F_d = (\rho/2)Dw^2$ , and  $F_l = (\rho/2)Lw^2$ . Wind changes the direction and magnitude of the airflow velocity:  $\mathbf{w} = \mathbf{v}_w - \mathbf{v}$ ;  $\mathbf{v}_w$  is the wind velocity and  $\mathbf{v}$  is the flight velocity (tangential to the flight path).  $D = c_d A$  and  $L = c_l A$  are functions of the flight position angles (Müller et al., 1996);  $A$  is the cross sectional area of the athlete with his equipment, and  $c_d$  and  $c_l$  are the drag and the lift coefficients. The drag area  $D(t)$  and lift area  $L(t)$  time functions of the recently published reference jump L2017 (Jung et al., 2018) were used until  $t = 0.7$  s; from this time on, these functions were optimized for  $\alpha$  and  $\beta$ . It was taken into account that  $\alpha$  depends on the airflow velocity (Jung et al., 2018).

The motion of the ski jumper in the plane containing the flight path can be described by a coupled system of four differential equations (of motion):

$$\dot{\mathbf{s}} = \begin{bmatrix} \dot{v}_x \\ \dot{v}_z \\ \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\rho w}{2m} [D(v_{wx} - v_x) + L(v_{wz} - v_z)] \\ \frac{\rho w}{2m} [D(v_{wz} - v_z) - L(v_{wx} - v_x)] - g \\ v_x \\ v_z \end{bmatrix},$$

with  $\mathbf{s}(t)$  being the state matrix of the system and  $w = \sqrt{(v_{wx} - v_x)^2 + (v_{wz} - v_z)^2}$ . Initial conditions are  $\mathbf{s}(0) = (v_x(0), v_z(0), x(0), z(0))^T$  with  $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_{p0}$ . The in-run speed parallel to the ramp  $v_0$  was set in dependence on the wind scenario (see *Wind scenarios*). The take-off speed perpendicular to the ramp was set to  $v_{p0} = 2.8$  m s<sup>-1</sup>, which is a typical value for elite athletes (Virmvirta and Kivekäs, 2012). The large hill in Garmisch-Partenkirchen (HS 140 m; altitude: 730 m;  $\rho = 1.18$  kg·m<sup>-3</sup>) and the flying hill in Bad-Mitterndorf (HS 225 m; altitude: 840 m;  $\rho = 1.16$  kg·m<sup>-3</sup>) were used for this study. The air density  $\rho$  was set according to the barometric formula and a temperature of 0 °C. The equations of motion were integrated into the system of

equations that describes the optimization problem (Jung et al., 2014). For a given landing slope profile, the flight path coordinate  $x(t_f)$  at landing was used to compute the jump length  $l$  along the landing slope surface from the ramp edge to the landing point.

### 2.2. Optimization

Reaching maximum jump length is constrained optimization problem. The athlete has to find a flight technique that optimizes the aerodynamics forces drag and lift. However, the range of admissible flight positions is constrained. In this study, the flight technique was optimized with respect to the angle of attack  $\alpha$  of the skis  $\alpha$  and the body-to-ski angle  $\beta$  (controls). Constraints were set for the controls: the constraint for  $\alpha$  is a constant value ( $\alpha_{max}$ ), and the constraint for  $\beta$  is a function of time ( $\beta_{min}(t)$ ) (Jung et al., 2014). For this study, the constraints were selected based on typical flight positions found in the field:  $\alpha_{max} = 40^\circ$ , and  $\beta_{min}(t) = \beta_{refA}(t) - 5^\circ$  (No. 5 in Jung et al., 2014);  $\beta_{refA}(t)$  is the mean  $\beta$ -time course measured in elite athletes during four international competitions (*reference jump A*; Schmölzer and Müller, 2002). Penalty functions were added to the objective function of the Pontryagin's minimum principle (Pontryagin et al., 1962) to solve this constrained optimization problem as a series of unconstrained problems. Methodical details including the system of equations that describes the problem can be found in Jung et al. (2014).

### 2.3. Wind scenarios

Wind blew tangentially to the landing slope. It was mimicked here that the FIS determines the in-run length appropriately for a given range of wind speeds (so that the jumps of elite athletes remain in the vicinity of the landing zone between the K-point and the L-point). The wind speeds 0 m s<sup>-1</sup>,  $\pm 1$  m s<sup>-1</sup>,  $\pm 2$  m s<sup>-1</sup>, and  $\pm 3$  m s<sup>-1</sup> were the basis of seven wind scenarios (+ stands for headwind, - stands for tailwind). For these wind speeds, the in-run speed was set to reach a jump length of  $l = 132.5$  m on the large hill (HS 140 m), and  $l = 212.5$  m on the flying hill (HS 225 m). This is in the middle between the K-point and the L-point (and corresponds to typical jump lengths of elite athletes observed in international competitions). The wind speed we started out from in each of the seven scenarios was then increased by 1 m s<sup>-1</sup>, and also decreased by 1 m s<sup>-1</sup>, while the in-run speed was kept constant. Therefore, the study covers wind speeds from  $-4$  m s<sup>-1</sup> to 4 m s<sup>-1</sup>.

The in-run speeds were computed with a flight technique optimized for calm wind. In order to investigate the effect of wind on flight technique optimization, simulations using a flight technique optimized for calm wind (jump length  $l_{no}$ ) were compared with simulations using a flight technique optimized for the given wind speed (jump length  $l_o$ ). This was done for every wind speed in the seven wind scenarios.

## 3. Results

The jump length effects of flight technique optimizations for seven wind scenarios are given in Table 1 for the large hill (HS 140 m), and in Table 2 for the flying hill (HS 225 m). The maximum jump length effect on the large hill was 1.8 m at headwind, and 0.5 m at tailwind. The jump length effects were larger on the flying hill. Here, the maximum was 6.4 m at headwind, and 3.0 m at tailwind.

Fig. 1 exemplarily illustrates the optimum time courses of  $\alpha$  and  $\beta$  on the flying hill for calm wind (0 m s<sup>-1</sup>), and for light head- and tailwind ( $\pm 1$  m s<sup>-1</sup>). Compared to the optimum time course at calm

**Table 1**  
Jump length effect of flight technique optimization on the large hill (HS 140 m). Seven wind scenarios were investigated where wind with the speed  $v_w$  blew tangentially to the slope. In each wind scenario, a basis wind speed was varied by  $\pm 1 \text{ m s}^{-1}$ . The in-run speed  $v_0$  was set to reach  $l = 132.5 \text{ m}$  at the basis wind speed and was kept constant during wind speed variation.  $l_{no}$  denotes the jump length obtained by a flight technique optimized at calm wind, and  $l_o$  denotes the jump length obtained by a flight technique optimized for the given wind speed. The jump length effect is  $\Delta l = l_o - l_{no}$ .

No	$v_w[\text{m s}^{-1}]$	$v_0[\text{m s}^{-1}]$	$l_{no}[\text{m}]$	$l_o[\text{m}]$	$\Delta l[\text{m}]$
1	-4	26.84	125.5	126.0	0.5
	-3	26.84	132.5	132.6	0.1
	-2	26.84	139.3	139.3	0.0
2	-3	26.49	125.2	125.6	0.4
	-2	26.49	132.5	132.5	0.0
	-1	26.49	139.5	139.5	0.0
3	-2	26.13	124.9	125.2	0.3
	-1	26.13	132.5	132.5	0.0
	0	26.13	139.7	139.8	0.1
4	-1	25.77	124.6	124.8	0.2
	0	25.77	132.5	132.5	0.0
	1	25.77	139.9	140.2	0.3
5	0	25.41	124.2	124.3	0.1
	1	25.41	132.5	132.6	0.1
	2	25.41	140.0	140.7	0.7
6	1	25.05	123.9	124.0	0.1
	2	25.05	132.5	132.7	0.2
	3	25.05	140.1	141.2	1.1
7	2	24.70	123.7	123.7	0.0
	3	24.70	132.5	133.0	0.5
	4	24.70	140.2	142.0	1.8

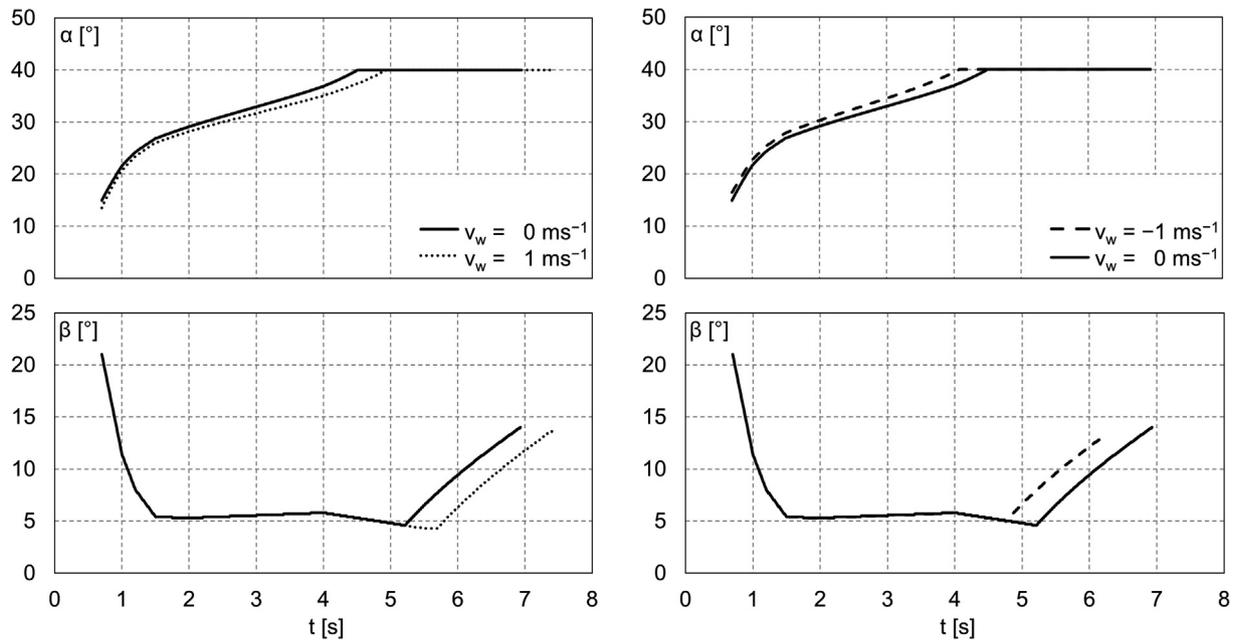
**Table 2**  
Jump length effect of flight technique optimization on the flying hill (HS 225 m). Seven wind scenarios were investigated where wind with the speed  $v_w$  blew tangentially to the slope. In each wind scenario, a basis wind speed was varied by  $\pm 1 \text{ m s}^{-1}$ . The in-run speed  $v_0$  was set to reach  $l = 212.5 \text{ m}$  at the basis wind speed and was kept constant during wind speed variation.  $l_{no}$  denotes the jump length obtained by a flight technique optimized at calm wind, and  $l_o$  denotes the jump length obtained by a flight technique optimized for the given wind speed. The jump length effect is  $\Delta l = l_o - l_{no}$ .

No	$v_w[\text{m s}^{-1}]$	$v_0[\text{m s}^{-1}]$	$l_{no}[\text{m}]$	$l_o[\text{m}]$	$\Delta l[\text{m}]$
1	-4	30.34	197.4	200.4	3.0
	-3	30.34	212.5	213.2	0.7
	-2	30.34	225.9	225.9	0.0
2	-3	29.89	196.6	199.3	2.7
	-2	29.89	212.5	213.1	0.6
	-1	29.89	226.1	226.6	0.5
3	-2	29.41	195.8	197.8	2.0
	-1	29.41	212.5	212.6	0.1
	0	29.41	226.2	226.9	0.7
4	-1	28.94	195.0	196.6	1.6
	0	28.94	212.5	212.5	0.0
	1	28.94	226.2	227.7	1.5
5	0	28.47	194.3	195.5	1.2
	1	28.47	212.5	212.7	0.2
	2	28.47	226.1	228.8	2.7
6	1	28.00	193.8	194.6	0.8
	2	28.00	212.5	213.3	0.8
	3	28.00	225.9	230.1	4.2
7	2	27.54	193.6	194.0	0.4
	3	27.54	212.5	214.3	1.8
	4	27.54	225.4	231.8	6.4

wind,  $\alpha$  was smaller at headwind, and larger at tailwind until the constraint  $\alpha_{\max}$  was reached. In the investigated wind scenarios on the large hill (flying hill), the differences between the  $\alpha$ -values optimized for calm wind and the  $\alpha$ -values optimized for given head- and tailwind were within  $0^\circ$  and  $3.8^\circ$  ( $4.9^\circ$ ). The optimum  $\beta$ -values were always the smallest possible ones with respect to the given constraint except for the last part of the flight. In the last part of the flight,  $\beta$  increases until landing, and this increase starts earlier in tailwind, and later in headwind when compared to calm wind.

#### 4. Discussion

The flight phase in ski jumping is a tightrope walk in which the athlete must find a technique that optimizes the aerodynamic forces. This problem is constrained by individual biomechanical and aerodynamical features and abilities. In addition, the athlete has to control the pitching moment to achieve these optimum positions and to keep the flight stable. Wind changes the aerodynamic forces and the pitching moment, which makes this difficult motor-control task even more difficult.



**Fig. 1.** Optimized time courses for  $\alpha$  and  $\beta$  on the flying hill (HS 225 m). Optimizations were performed for calm wind ( $0 \text{ m s}^{-1}$ ; solid line), headwind ( $1 \text{ m s}^{-1}$ ; dotted line), and tailwind ( $-1 \text{ m s}^{-1}$ ; dashed line). Wind blew tangentially to the slope. The in-run speed was set to  $v_0 = 28.94 \text{ m s}^{-1}$  to reach  $l = 212.5 \text{ m}$  at calm wind.

It was found that wind mainly influences the optimum value of the angle of attack  $\alpha$  of the skis, which is smaller in headwind conditions and larger in tailwind conditions (Fig. 1). The body-to-ski angle  $\beta$  should be, similar to calm wind, as low as possible with respect to the given constraint, except for the last part of the flight where it is advantageous to increase  $\beta$  again (Fig. 1; Jung et al., 2014). Since wind influences jump length and thus flight time, the increase of  $\beta$  starts at different times. The competition practice of the FIS was mimicked by adjusting the in-run speeds for each of the seven wind scenarios. This was not taken into account in the study published by Jung et al. (2015). At given wind speeds, jump length effects were smaller in the present (and more realistic) study. Optimum adjustments of  $\alpha$  increased the jump length on the large hill (flying hill) by up to 1.8 m (6.4 m) at headwind and by up to 0.5 m (3.0 m) at tailwind in the investigated wind scenarios (Tables 1 and 2). Loosing or winning a competition can be a factor of 0.5 m. However, the optimal adjustments of  $\alpha$  were only between 0 and  $3.8^\circ$  on the large hill and between 0 and  $4.9^\circ$  on the flying hill. This shows that jump length is very sensitive to small flight position changes, however, transferring such small adaptations into practice is at the limits of the athletes' sensory-motor abilities.

Headwind increases the speed of the airflow; this allows the athlete to lean forward in a more pronounced way without tumbling (smaller constraint for  $\beta$ ). Smaller  $\beta$ -angles increase jump length substantially (Jung et al., 2014). However, the pitching moment of flight positions at the very edge of stability can suddenly become unbalanced when gusts occur (Müller et al., 1996). This means that flight positions that may be advantageous in a given phase of the flight, may be very dangerous fractions of a second later. The athlete has to take this risk factor into consideration when he strives for maximum jump length.

#### Conflict of interest

None declared.

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