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Functional forms of the negative binomial models in safety performance functions for rural two-lane intersections

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ABSTRACT

Safety Performance Functions (SPFs) play a prominent role in estimating intersection crashes, and identifying the sites with the highest potential for safety improvement. To maximize the crash prediction accuracy, this paper describes the application of different functional forms of the Negative Binomial (NB) models (*i.e.* NB-1, NB-2 and NB-P) in estimating safety performance functions by crash type for three types of rural two-lane intersections, including three-leg stop-controlled (3ST) intersections, four-leg stop-controlled (4ST) intersections and four-leg signalized (4SG) intersections. Crash types were aggregated into same-direction, opposite-direction, intersecting-direction and single-vehicle crashes. Major and minor road Annual Average Daily Traffic (AADT) were used as predictors in the SPF estimation. In addition, major and minor road AADT were also used as predictors in the estimation of the over-dispersion parameter of the NB models to account for the crash data heterogeneity. In the end, all NB models were compared based on both the model estimation goodness-of-fit and the prediction performance.

The model goodness-of-fit indicates that the NB-P model outperforms the NB-1 and NB-2 models for most crash types and intersection types, by providing a flexible variance structure to the NB approaches. The parameterization of the over-dispersion factor verifies that the over-dispersion parameter of the NB models highly depends on how the variance structure is defined in the model, and the over-dispersion parameter is shown to vary among different intersections for each crash type and can be estimated using both the major and minor road AADT at rural two-lane intersections. The NB-P model is found to more effectively capture the variation of over-dispersion among intersections in NB models, which benefits the accommodation of data heterogeneity in intersection SPF development. The prediction performance comparison illustrates that the NB-P model slightly improves the crash prediction accuracy compared with the other two models, especially for the 3ST and 4SG intersections. In conclusion, the NB-P model with parameterized over-dispersion factor is recommended to provide more unbiased parameter estimates when estimating SPFs by crash type for rural two-lane intersections.

1. Introduction and motivation

In the United States, reducing traffic crashes at intersections has continuously been a high priority of the transportation agencies in the past few decades, due to the fact that intersection and intersection-related crashes contribute to about 50% of total crashes per year, and lead to one of the largest economic and societal losses (National Highway Traffic Safety Administration (NHTSA, 2015). In order to improve traffic safety at intersections, there has been increasing interest in estimating crash prediction models and identifying locations with the highest potential for safety improvement.

The Highway Safety Manual (HSM) (2010) provides the Safety Performance Functions (SPFs) for intersection crash predictions of several highway facilities including rural two-lane highways, rural multi-lane highways, urban and suburban arterials and freeway ramp terminals. Of which, traffic volumes for both major and minor roads are used as predictors to estimate the crash counts. The SPFs in HSM were estimated using data collected from a limited number of States, including Washington, Minnesota, Texas and Ohio. Because crash relationships in these states are not necessarily representatives of those in the entire country, the HSM recommends a calibration procedure to adjust the predicted crash counts for individual jurisdictions in using the prediction from the intersection SPFs. To achieve a better crash prediction, instead of calibrating the HSM SPFs, a variety of states have collected sufficient intersection data, and estimated their own intersection SPFs, including Colorado, Florida, Georgia, Illinois, Kansas,

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North Carolina, Oregon, Utah, Virginia and Michigan (Gates et al., 2018).

In the current HSM, the intersection SPFs are estimated in total, which might not be feasible as the crash patterns may vary by crash type and severity (Wang et al., 2017a,b; Wang et al., 2018; Zhao et al., 2018). To account for crash counts variations among crash types, a variety of studies estimated the intersection SPFs by crash type (Gates et al., 2018; Wang et al., 2017a,b; Wang et al., 2018; Hauer et al., 1988; Poch and Mannering, 1996; Hauer, 2000; Ivan et al., 2000; Qin, 2002; Abdel-Aty et al., 2005; Geedipally and Lord, 2010; Geedipally et al., 2010; Dixon et al., 2015; Wu et al., 2018; Harwood et al., 2000; Lyon et al., 2005; Liu et al., 2018). Similarly, in order to account for crash variations among crash severity levels, studies were conducted to estimate crashes by severity (Dixon et al., 2015; Harwood et al., 2000; Wu et al., 2013; Lyon et al., 2005; Abdel-Aty and Radwan, 2000; Lord and Persaud, 2000; Ulfarsson and Shankar, 2002; Lyon et al., 2003; Lord et al., 2008; Tarko et al., 2008; Oh et al., 2004; Russo et al., 2016; Xie et al., 2011; Wang et al., 2017a, b; Liu and Sharma, 2018). In terms of the methodologies, the Poisson regression model has been initially used in the SPF estimation since the crash frequencies are non-negative integers (Lord and Mannering, 2010). However, the Poisson model has its implicit restriction on the distribution of data – the variance of the data is constrained to be equal to the mean. This constraint might be questionable as the variance of crash data is normally greater than the mean, which is known as the over-dispersion (Washington et al., 2011). To address the over-dispersion issue, researchers generally employ the Negative Binomial (NB) or Poisson-Gamma model in SPF estimation because the NB model allows the variance of crash counts to be larger than the mean. Two well-known non-nested NB model forms, *i.e.* NB-1 and NB-2 (Cameron and Trivedi, 1986) have been increasingly applied in estimating motor vehicle crashes. The difference between the two NB models is that the NB-1 model defines a linear relationship to the variance and mean, while the NB-2 model defines the relationship between the variance and mean as a quadric form. Details about the two NB models are shown in the next section.

In recent studies, Chang and Xiang (2003) used both the NB-1 and NB-2 models to investigate the relationship between congestion levels and accidents for freeways. The results show that the relationship between freeway crashes and AADT, median width and the number of through lanes is consistent between the two NB models. Mehta and Lou (2013) applied both the NB-1 and NB-2 models to estimate the SPFs for rural two-lane two-way highways and four-lane divided highways in the State of Alabama, respectively. They verified that the NB-2 model outperforms the NB-1 model in terms of the crash prediction accuracy. Details about the estimation procedures of NB-1 and NB-2 models can be found in Hilbe (2007) and Lord and Park (2015). Nevertheless, both NB-1 and NB-2 models restrict the variance structure in the SPF estimation (Park, 2010), *i.e.* the relationship between variance and mean is either linear or quadric, which may lead to biased model estimation results. In order to address the restriction of variance structure in NB models, Greene (2008) extended the two traditional NB models and introduced an encompassing model formula entitled as NB-P model which nests both the NB-1 and NB-2 models, where both NB-1 and NB-2 regressions are special cases of the NB-P model when P = 1 and P = 2 respectively. The advantage of using NB-P regression is that it parametrically nests both NB-1 and NB-2 regressions, and hence, allowing statistical tests of the two functional forms against a more general alternative (Greene, 2008; Ismail and Zamani, 2013). Greene (2008) developed the NB-P model and applied it with the NB-1 and NB-2 models in a German health care study. The results illustrated that the NB-P model outperforms both the NB-1 and NB-2 models based on the likelihood ratio tests. Similarly, Ismail and Zamani (2013) compared the NB-1, NB-2 and NB-P models in estimating the Malaysian private car own damage claim counts, and verified that the NB-P model was the best approach among three methods based on the model goodness-of-fit. Considering that no applications of the NB-P model have yet been applied in highway safety literature, it is worthwhile to investigate the performance of this type of model in crash prediction comparing with the traditional NB-1 and NB-2 models.

Despite the consistent examinations of variance structures in the NB models, research has also been conducted to investigate the nature of over-dispersion in crash prediction models for decades. Traditional NB models have assumed a fixed over-dispersion parameter in NB regression models, which might not always be true due to the unobserved heterogeneity in the crash data (Mannering et al., 2016; Miaou and Lord, 2003). Not accounting for the over-dispersion variations might result in biased estimation results, especially in the Empirical Bayes (EB) application (Miaou and Lord, 2003; Hauer, 2001). The SPFs estimated for the highway segments in the HSM define the over-dispersion parameter as a function of segment length. The study conducted by Miao and Lord (2003) found that the parametrization of over-dispersion factor significantly improves the model goodness-of-fit of estimating crashes for urban four-leg signalized intersections compared with the fixed over-dispersion model. The study conducted by Mitra and Washington (2007) indicated that the over-dispersion of intersection SPFs depends greatly on how the mean-variance structure is modeled, and the over-dispersion is a function of covariates if the crash prediction model suffers from the omitted variables. Geedipally and Lord (2008) estimated the SPFs for three-leg rural unsignalized intersections and found models with a varying dispersion parameter usually produce smaller confidence intervals, and hence more precise estimates, than models with a fixed dispersion parameter. Studies have also been investigated to examine the time- and spatial-varying over-dispersion parameters in the SPF estimation (Lord and Park, 2008).

Overall, this paper is an attempt to explore the appropriate functional forms of NB models in the planning-level intersection SPF estimation, where the traffic volumes are considered as predictor variables. To this end, four regression models, *i.e.* Poisson, NB-1, NB-2 and NB-P models are respectively estimated to predict crashes at rural two-lane intersections by crash type, using the data collected from the Highway Safety Information System (HSIS). The over-dispersion parameter of NB models is parameterized by traffic volumes to account for the potential unobserved heterogeneity in the crash data. The rest of paper is organized as follows. Section two presents the framework and estimation approaches for the four regression models. The third section describes the data in detail and the fourth section presents the model estimation results. The model prediction comparisons are displayed in section five, and concluding remarks are provided in the final section.

2. Statistical methodologies

2.1. Poisson model

The Poisson regression model is initially used by researchers to estimate intersection SPFs, with the formula as (Washington et al., 2011):

$$Prob[y_i|\mu_i] = \frac{exp(-\mu_i)\mu_i^{y_i}}{y_i!} \tag{1}$$

where $Prob[y_i\mu_i]$ is the probability of y crashes occurring at intersection i and μ_i is the expected number of crashes at intersection i. Given a vector of explanatory variables X_i and a vector of estimable coefficients β, the μ_i can be estimated by the equation:

$$\mu_i = exp(\boldsymbol{\beta X_i}) \tag{2}$$

The variance of the Poisson regression model is expressed as:

$$Var(y_i) = E(y_i) = \mu_i \tag{3}$$

2.2. NB-2 model

As depicted in Eq. (3), one of the limitations of Poisson regression model is that the variance of the data is constrained to be equal to the mean. This constraint might be problematic as the variance of crash data is usually greater than the mean, which is known as the over-dispersion (Washington et al., 2011). To account for the over-dispersion issue, the Negative Binomial (NB)/Poisson-Gamma model is applied, which is derived by rewriting Eq. (2) as:

$$\mu_i = exp(\boldsymbol{\beta X_i} + \varepsilon_i) \tag{4}$$

where $exp(\varepsilon_i)$ is an error term. If this error term is assumed to follow a gamma distribution with mean 1 and variance $1/\sigma = k$, where k denotes the over-dispersion parameter in NB/Poisson-Gamma related regression models, the traditional NB model formula is derived, which is denoted as the NB-2 regression model in crash prediction area. Then the distribution of the NB-2 model can be derived as (Washington et al., 2011):

$$Prob[y_i|\mu_i] = \frac{\Gamma[(\sigma) + y_i]}{\Gamma(\sigma)y_i!}\left[\frac{\sigma}{(\sigma) + \mu_i}\right]^{\sigma}\left[\frac{\mu_i}{(\sigma) + \mu_i}\right]^{y_i} \tag{5}$$

where Γ is a gamma function; the mean and variance of the NB-2 model can be written as:

$$E(y_i) = \mu_i \tag{6}$$

$$Var(y_i) = \mu_i + k\mu_i^2 \tag{7}$$

then the marginal distribution of NB-2 model can be rewritten as:

$$Prob[y_i|\mu_i] = \frac{\Gamma\left[\left(\frac{1}{k}\right) + y_i\right]}{\Gamma\left(\frac{1}{k}\right)y_i!}\left[\frac{\frac{1}{k}}{\left(\frac{1}{k}\right) + \mu_i}\right]^{\frac{1}{k}}\left[\frac{\mu_i}{\left(\frac{1}{k}\right) + \mu_i}\right]^{y_i} \tag{8}$$

2.3. NB-1 model

Another widely used formula of NB model is obtained by replacing $\frac{1}{k}$ with $\frac{1}{k}\mu_i$ in Eq. (8) (Greene, 2008), which is denoted as the NB-1 regression model. The marginal distribution of NB-1 model can be derived as:

$$Prob[y_i|\mu_i] = \frac{\Gamma\left[\left(\frac{1}{k}\mu_i\right) + y_i\right]}{\Gamma\left(\frac{1}{k}\mu_i\right)y_i!}\left[\frac{\frac{1}{k}\mu_i}{\left(\frac{1}{k}\mu_i\right) + \mu_i}\right]^{\frac{1}{k}\mu_i}\left[\frac{\mu_i}{\left(\frac{1}{k}\mu_i\right) + \mu_i}\right]^{y_i} \tag{9}$$

and the mean and variance of the NB-1 model can be written as:

$$E(y_i) = \mu_i \tag{10}$$

$$Var(y_i) = \mu_i + k\mu_i \tag{11}$$

2.4. NB-P model

As shown in Eqs. (7) and (11), both the NB-1 and NB-2 regression models constrained the variance structure of the NB models. The only difference between the two NB models is that the NB-1 model defines a linear relationship to the variance and mean, while the NB-2 model defines their relationship as a quadric form. In order to eliminate the variance structure constraint, Greene (2008) introduced a NB-P regression model which nests both NB-1 and NB-2 regressions and allows the parameterization of the variance function of the NB model. Similar to the NB-1 model, let's replace $\frac{1}{k}$ by $\frac{1}{k}{\mu_i}^{2-P}$ in Eq. (8). The marginal distribution of the NB-P model is then obtained as:

$$Prob[y_i|\mu_i] = \frac{\Gamma\left[\left(\frac{1}{k}{\mu_i}^{2-P}\right) + y_i\right]}{\Gamma\left(\frac{1}{k}{\mu_i}^{2-P}\right)y_i!}\left[\frac{\frac{1}{k}{\mu_i}^{2-P}}{\left(\frac{1}{k}{\mu_i}^{2-P}\right) + \mu_i}\right]^{\frac{1}{k}{\mu_i}^{2-P}}\left[\frac{\mu_i}{\left(\frac{1}{k}{\mu_i}^{2-P}\right) + \mu_i}\right]^{y_i} \tag{12}$$

and the mean and variance of the NB-P model can be written as (Park, 2010):

$$E(y_i) = \mu_i \tag{13}$$

$$Var(y_i) = \mu_i + k{\mu_i}^P \tag{14}$$

where the P is the functional parameter to be estimated. It is noticed from Eq. (14) that the NB-1 and NB-2 models are two special cases of NB-P model. Specifically, the NB-P model reduces to the NB-1 model when $P = 1$, and reduces to the NB-2 model when $P = 2$. Therefore, the NB-P regression model parametrically nests both the NB-1 and NB-2 models and the likelihood ratio test can be implemented to test the appropriateness of NB-P against the NB-1 and NB-2 models (Greene, 2008; Ismail and Zamani, 2013). The log likelihood functions of the Poisson, NB-1, NB-2 and NB-P models can be derived from the Eqs. (1), (8), (9) and (12), respectively, and the model coefficients can be estimated by the maximum likelihood estimation (MLE) approach. Similar studies about the MLE model estimation procedures can be referred to Lord and Park (2015); Greene (2008) and Ismail and Zamani (2013).

2.5. Intersection SPF formula

Given the major road AADT - $AADT_{major,i}$, minor road AADT - $AADT_{minor,i}$ and the estimable coefficients $\beta_{AADT_{major}}$ and $\beta_{AADT_{minor}}$, the expected total crash counts for intersection i ($i.e.$ μ_i) were estimated in this study by the equation:

$$\mu_i = N \times {AADT_{major,i}}^{\beta_{AADT_{major}}} \times {AADT_{minor,i}}^{\beta_{AADT_{minor}}} \times exp(\beta_0) \tag{15}$$

Where N is the number of years and β_0 is the intercept. The over-dispersion parameter of the NB models might vary among different crashes due to the unobserved heterogeneity in crash data (Mannering et al., 2016). In order to account for the over-dispersion variations, the over-dispersion parameters in NB-1, NB-2 and NB-P models were estimated using the major and minor road AADT, with the formula as (Geedipally and Lord, 2008):

$$k_i = exp(\gamma_{AADT_{major}} \times AADT_{major,i} + \gamma_{AADT_{minor}} \times AADT_{minor,i} + \gamma_0) \tag{16}$$

In summary, the coefficients to be estimated for the Poisson model are $\beta_{AADT_{major}}$, $\beta_{AADT_{minor}}$ and β_0; the coefficients to be estimated for the NB-1 and NB-2 models are $\beta_{AADT_{major}}$, $\beta_{AADT_{minor}}$, β_0, $\gamma_{AADT_{major}}$, $\gamma_{AADT_{minor}}$ and γ_0; the coefficients to be estimated for the NB-P model are similar to the NB-1 and NB-2 models, with an additional functional parameter P. The SAS software is used to estimate all model coefficients in this study.

3. Data preparation and analyses

In this study, SPFs for intersections on rural two-lane highways were estimated. Seven-year crash data (2003–2009) (which are the latest datasets that are available) at rural two-lane intersections were collected from the State of Minnesota from the Highway Safety Information System (HSIS, 2019), and a 250-feet radius is used to distinguish between the segment crashes and intersection crashes. Of which, 755 are three-leg stop-controlled (3ST) intersections, 1064 are four-leg stop-controlled (4ST) intersections and 165 are four-leg signalized (4SG) intersections.

To account for SPF variations among different crash types, in addition to the SPF estimation of total (TOT) crashes, intersection SPFs were also estimated by crash type. In order to obtain sufficient observations in each crash type level, crash types were aggregated into four categories, based on the original travel direction of involved vehicles (Qin, 2002; Qin et al., 2004). They are **1)** same-direction crashes (SDC) which include turning-same direction crashes, sideswipe-same direction crashes and rear-end crashes; **2)** intersecting-direction crashes (IDC) which include turning-intersecting crashes and angle crashes; **3)** opposite-direction crashes (ODC) which include turning-opposite direction crashes, sideswipe-opposite direction crashes and head-on

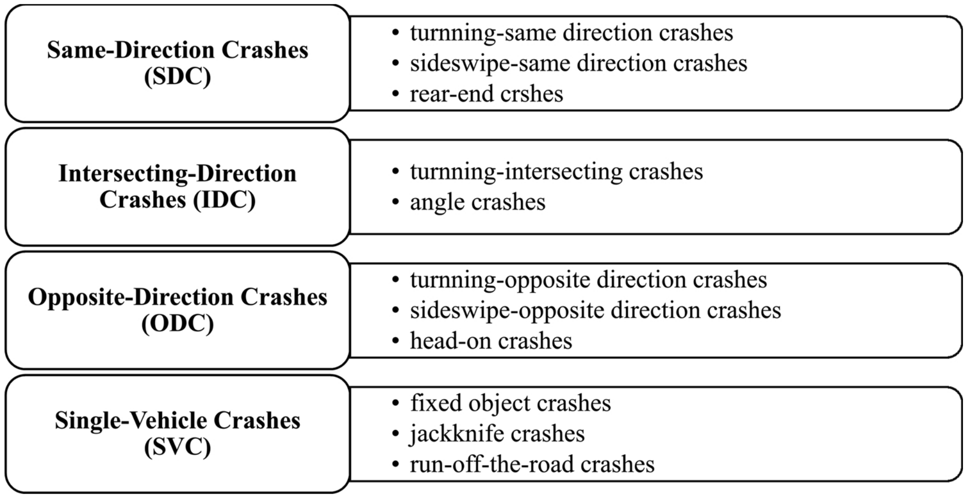

Fig. 1. Classification of Crash Types.

Table 1
Summary Statistics of Intersection Data.

Intersection Types	Intersection Elements	Min.	Max.	Mean	Std. Dev.
3ST Intersections (755 Intersections)	**TOT**	0.00	27.00	2.66	3.70
	SDC	0.00	19.00	0.61	1.43
	ODC	0.00	7.00	0.37	0.84
	IDC	0.00	11.00	0.35	0.93
	SVC	0.00	15.00	1.34	1.85
	AADT-Major (*10^3)	0.31	21.33	3.59	3.43
	AADT-Minor (*10^3)	0.01	6.42	0.59	0.74
4ST Intersections (1064 Intersections)	**TOT**	0.00	35.00	3.18	4.08
	SDC	0.00	16.00	0.76	1.46
	ODC	0.00	5.00	0.40	0.77
	IDC	0.00	28.00	1.08	2.05
	SVC	0.00	11.00	0.94	1.30
	AADT-Major (*10^3)	0.11	19.67	3.04	2.28
	AADT-Minor (*10^3)	0.01	6.88	0.68	0.80
4SG Intersections (165 Intersections)	**TOT**	0.00	42.00	9.21	8.29
	SDC	0.00	32.00	4.73	4.96
	ODC	0.00	13.00	1.58	2.12
	IDC	0.00	18.00	2.08	2.60
	SVC	0.00	8.00	0.82	1.23
	AADT-Major (*10^3)	2.00	26.20	10.01	4.19
	AADT-Minor (*10^3)	0.35	12.40	4.53	2.48

crashes; and **4)** single-vehicle crashes (SVC) which include fixed object crashes, jackknife crashes and run-off-the-road crashes. The classification of crash types is summarized in Fig. 1. AADT for both major and minor roads were used as predictors in the intersection SPFs. The explanations and summary statistics of crash data are presented in Table 1.

4. SPF estimation results

Table 2 through Table 4 show the SPF estimation results of the Poisson, NB-1, NB-2 and NB-P models by crash type, and the results for total crashes for 3ST, 4ST and 4SG intersections, respectively. Coefficients labeled as βs are the parameters estimated for the SPF structures, and coefficients labeled as γs are the parameters estimated for the variance structures in the NB models. Coefficients in boldface are statistically significant at the 5% level, coefficients with "*" are statistically significant at the 10% level, and the log likelihood (LLs) with "a" represents the model with the highest log likelihood value among the four regression models. Table 2 shows the SPFs estimated for the 3ST intersections. Generally, the coefficients of AADT are shown to be very different across different crash types, which verifies the necessity of estimating SPFs by crash type for rural two-lane 3ST intersections. The coefficients for both major road AADT and minor road AADT are very similar, and are all statistically significant across the four different regression models for each crash type.

In terms of the model comparisons, the Poisson model can be compared with each of the NB models respectively using the likelihood ratio test (Greene, 2008), since the Poisson model is a special case of all NB models where the over-dispersion parameter (*i.e.* k in Eqs. (7) and (11) and (14)) is equal to 0. It is confirmed from the results that all NB models outperform the Poisson model based on the likelihood ratio test, which indicates that the over-dispersion exists in the crash data and should be considered in SPF estimation (The procedure of performing likelihood ratio test is described in the following sentence). The likelihood ratio test can also be used in comparing either the NB-1 or the NB-2 with the NB-P model, since the NB-P model parametrically nests both the NB-1 and NB-2 models (Greene, 2008). In this case, the likelihood ratio can be calculated as $2 \times [LL_{NB-P} - LL_{NB-1\,or\,NB-2}]$ and compared to the $\chi^2{}_{df_{alt}-df_{null}} = \chi^2{}_1$ ($df_{alt} - df_{null} = 1$ since NB-P model has one more estimated parameter than both the NB-1 and NB-2 models; $\chi^2{}_1 = 3.84$ when the 5% significance level is used; $\chi^2{}_1 = 2.71$ when the 10% significance level is used). However, the likelihood ratio test cannot be used to conduct statistical test between the NB-1 and NB-2 models, as these two models are not nested. Both the AIC and BIC values can be used to compare the NB-1 and NB-2 models. In addition, the Vuong, (1989) can also be applied to compare the NB-1 model with the NB-2 model. The Vuong Test can be calculated as $V = \frac{\sqrt{n}\,\bar{m}}{S_m}$, where $m_i = LL_{i,NB-2} - LL_{i,NB-1}$, $\bar{m}$ is the mean of m_i, S_m is the standard deviation of m_i, and n is the sample size. Positive value favors the NB-2 model while negative value favors the NB-1 model. The tests of model results in Table 2 illustrate that the NB-P model favors both NB-1 and NB-2 models for SDC, ODC and IDC crashes. The NB-P and NB-1 models perform very closely, and both perform better than the NB-2 model for the TOT and SVC crashes. The estimated variance functional parameters *Ps* are statistically significant in all NB-P models. In specific, the estimated parameter *Ps* in the NB-P models for the TOT, ODC, IDC and SVC are all significantly different from both 1 and 2, while the estimated parameter for the SDC is very close to 2. These findings verify the assumption that the restriction of variance structure in traditional NB models may lead to biased model estimation. In terms of the over-dispersion parameter estimates, with regard to the NB-2

Table 2
Model Estimation Results of 3ST Intersections (N = 604).

	TOT	SDC	ODC	IDC	SVC
Poisson					
$\beta_{AADTmajor}$	**0.68 (< 0.01)**	**1.27 (< 0.01)**	**0.58 (< 0.01)**	**0.79 (< 0.01)**	**0.43 (< 0.01)**
$\beta_{AADTminor}$	**0.26 (< 0.01)**	**0.37 (< 0.01)**	**0.36 (< 0.01)**	**0.64 (< 0.01)**	**0.12 (< 0.01)**
β_0	**−8.06 (< 0.01)**	**−15.24 (< 0.01)**	**−9.86 (< 0.01)**	**−13.56 (< 0.01)**	**−5.79 (< 0.01)**
LL	−1359.28	−527.80	−445.15	−386.81	−1007.85
AIC	2724.60	1061.60	896.30	779.60	2021.70
BIC	2737.80	1074.80	909.50	792.80	2034.90
NB-2					
$\beta_{AADTmajor}$	**0.68 (< 0.01)**	**1.25 (< 0.01)**	**0.60 (< 0.01)**	**0.62 (< 0.01)**	**0.42 (< 0.01)**
$\beta_{AADTminor}$	**0.28 (< 0.01)**	**0.46 (< 0.01)**	**0.35 (< 0.01)**	**0.51 (< 0.01)**	**0.13 (< 0.01)**
β_0	**−8.11 (< 0.01)**	**−15.72 (< 0.01)**	**−9.93 (< 0.01)**	**−11.27 (< 0.01)**	**−5.78 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	**−1.00 (0.01)**	**−0.20 (< 0.01)**	−0.60 (0.37)	**−0.20 (< 0.01)**	−0.40 (0.21)
$\gamma_{AADTminor\ (*10)^{-4}}$	**−4.44 (< 0.01)**	**−5.10 (< 0.01)**	−0.18 (0.59)	**−8.60 (< 0.01)**	−4.20* (0.07)
γ_0	**0.42 (< 0.01)**	**0.38 (< 0.01)**	**0.41 (0.03)**	**1.05 (< 0.01)**	**0.23 (< 0.01)**
LL	−1171.15	−509.90	−432.15	−376.70	−924.05
AIC	2354.30	1031.8	876.30	765.50	1860.10
BIC	2380.71	1058.2	902.70	791.80	1886.51
NB-1					
$\beta_{AADTmajor}$	**0.68 (< 0.01)**	**1.18 (< 0.01)**	**0.58 (< 0.01)**	**0.75 (< 0.01)**	**0.42 (< 0.01)**
$\beta_{AADTminor}$	**0.26 (< 0.01)**	**0.25 (< 0.01)**	**0.35 (< 0.01)**	**0.58 (< 0.01)**	**0.13 (< 0.01)**
β_0	**−8.09 (< 0.01)**	**−13.47 (< 0.01)**	**−9.76 (< 0.01)**	**−12.70 (< 0.01)**	**−5.80 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	0.13 (0.29)	**1.33 (< 0.01)**	−204.40 (0.79)	−0.03 (0.92)	−0.66 (0.74)
$\gamma_{AADTminor\ (*10)^{-4}}$	−0.20 (0.49)	**0.29 (< 0.01)**	−300.30 (0.46)	−0.15 (0.91)	−0.30 (0.84)
γ_0	**0.53 (< 0.01)**	**−1.24 (< 0.01)**	**−29.10 (0.03)**	**2.14 (0.04)**	**−1.12 (0.03)**
LL	−1161.66	−510.58	−431.54[a]	−374.61	−921.53
AIC	2335.30	1033.20	875.10	761.20	1855.10
BIC	2361.70	1059.60	901.50	787.60	1881.50
NB-P					
$\beta_{AADTmajor}$	**0.65 (< 0.01)**	**1.27 (< 0.01)**	**0.62 (< 0.01)**	**0.73 (< 0.01)**	**0.39 (< 0.01)**
$\beta_{AADTminor}$	**0.26 (< 0.01)**	**0.41 (< 0.01)**	**0.38 (< 0.01)**	**0.58 (< 0.01)**	**0.15 (< 0.01)**
β_0	**−7.78 (< 0.01)**	**−15.55 (< 0.01)**	**−10.25 (< 0.01)**	**−12.64 (< 0.01)**	**−5.67 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	**−0.40 (< 0.01)**	−0.99 (0.85)	**0.25 (< 0.01)**	**0.38 (0.03)**	**−0.10 (< 0.01)**
$\gamma_{AADTminor\ (*10)^{-4}}$	**−2.00 (< 0.01)**	**−0.70 (0.01)**	**1.88 (< 0.01)**	**−2.80 (0.04)**	**−3.20 (< 0.01)**
γ_0	**0.48 (< 0.01)**	**−0.88 (0.04)**	**−1.32 (< 0.01)**	**−0.38 (< 0.01)**	**0.16 (< 0.01)**
P	**1.45 (< 0.01)**	**1.97 (< 0.01)**	1.79* (0.07)	**1.45 (< 0.01)**	**1.36 (0.03)**
LL	−1161.15[a]	−486.65[a]	−435.2	−373.05[a]	−921.30[a]
AIC	2336.30	987.3	884.4	760.1	1856.60
BIC	2367.11	1018.1	915.3	790.9	1887.41

Notes: Bold coefficients are statistically significant at the 5% level of significance; coefficients with "*" are statistically significant at 10% level of significance; LL with "*a*" represents the model with the highest log likelihood value among the four regression models.

model, the traffic volumes for both the major and minor roads are statistically significant at the 5% level of significance for TOT, SDC and IDC crashes, while only the minor road AADT is significant at the 10% level of significance for SVC crashes. The traffic volumes are found to be only significant for SDC crashes in the NB-1 model, while the traffic volumes are statistically significant for all crash types in the NB-P model. These findings verify that the over-dispersion parameter varies among different intersections with regard to each crash type, and the variation of the over-dispersion parameter in NB models highly depends on how the variance structure is defined in the model. The over-dispersion can be estimated by both the major road and minor road AADT.

Table 3 shows the SPF estimation results for the 4ST intersections. Similar to the 3ST intersections, both the major and minor road AADT are statistically significant for all crash types in each of the four models. The NB models outperform the Poisson model for all crash types, based on the model goodness-of-fit. The NB-P model slightly outperforms the NB-2 model at the 10% significance level for the SDC and IDC crashes, but are significantly better than the NB-1 model. The performances of NB-P and NB-2 models are consistent for the TOT, ODC and SVC crashes, but the NB-P model considerably outperforms the NB-1 model for these three crash types. The estimated functional parameters for the variance structure in all NB-P models are statistically significant at the 5% level of significance. The estimated functional parameter for the variance structure in the NB-P model is shown to be close to 2 for the TOT crashes, and 1 for the SDC crashes. However, the functional parameter is shown to be dramatically different from 1 and 2 for the ODC, IDC and SVC crashes. With regard to the estimation of over-dispersion parameter in NB models, the over-dispersion only significantly varies among intersections in terms of the SDC crashes in the NB-2 model, and among intersections for the IDC and SVC crashes in the NB-P model with the 5% significance level. The over-dispersion is shown to fairly vary by AADT for all crash types except for the ODC crashes in the NB-1 model. These findings verify the conclusion made in the analysis of 3ST intersections that the over-dispersion parameter of NB models highly depends on how the variance structure is defined in the model.

Table 4 presents the SPF estimation results for the 4SG intersections. Both the major and minor road AADT are statistically significant for the TOT, SDC and ODC crashes in all models. Only the major road AADT is shown to be significant at the 5% level of significance for IDC and SVC crashes in each of these four models. The NB models are preferable than the Poisson regression model for all crash types based on the likelihood ratio test. The NB-P model outperforms both the NB-1 and NB-2 models at the 10% significance level for the TOT, SDC and ODC crashes. There is no obvious evidence showing that the performances of NB-1, NB-2 and NB-P models are statistically different for the IDC and SVC crashes. The variance functional parameters estimated in the NB-P model are all statistically significant at the 5% level of significance, with the values falling between 1.38 and 1.75. In terms of the estimation of over-dispersion parameter, the results show that the over-dispersion parameter only significantly varies among intersections for the TOT and SDC crashes in both NB-1 and NB-2 models. However, both the major road and minor road AADT are significant for the TOT, SDC and SVC crashes, and only the major road AADT is significant for the IDC crashes in the

Table 3
Model Estimation Results of 4ST Intersections (N = 851).

	TOT	SDC	ODC	IDC	SVC
Poisson					
$\beta_{AADTmajor}$	**0.57 (< 0.01)**	**0.97 (< 0.01)**	**0.70 (< 0.01)**	**0.35 (< 0.01)**	**0.44 (< 0.01)**
$\beta_{AADTminor}$	**0.47 (< 0.01)**	**0.47 (< 0.01)**	**0.41 (< 0.01)**	**0.81 (< 0.01)**	**0.14 (< 0.01)**
β_0	**−8.33 (< 0.01)**	**−13.07 (< 0.01)**	**−11.09 (< 0.01)**	**−9.96 (< 0.01)**	**−6.39 (< 0.01)**
LL	−1946.39	−910.75	−640.13	−1129.89	−1121.82
AIC	3898.80	1827.50	1286.30	2265.80	2249.60
BIC	3913.00	1841.70	1300.50	2280.00	2263.90
NB-2					
$\beta_{AADTmajor}$	**0.57 (< 0.01)**	**0.92 (< 0.01)**	**0.70 (< 0.01)**	**0.41 (< 0.01)**	**0.41 (< 0.01)**
$\beta_{AADTminor}$	**0.44 (< 0.01)**	**0.48 (< 0.01)**	**0.42 (< 0.01)**	**0.75 (< 0.01)**	**0.15 (< 0.01)**
β_0	**−8.16 (< 0.01)**	**−12.68 (< 0.01)**	**−11.14 (< 0.01)**	**−9.98 (< 0.01)**	**−6.27 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	−0.20 (0.46)	**1.17 (< 0.01)**	−0.30 (0.81)	−0.90 (0.22)	1.02* (0.07)
$\gamma_{AADTminor\ (*10)^{-4}}$	−0.20 (0.12)	**−2.70 (< 0.01)**	−10.60 (0.29)	1.16 (0.79)	−3.00 (0.41)
γ_0	**−0.77 (0.02)**	**−1.13 (< 0.01)**	0.35* (0.07)	−0.30* (0.08)	**−0.83* (0.01)**
LL	−1737.77[a]	−877.61	−634.00	−1040.86	−1083.94
AIC	3487.50	1767.20	1280.00	2093.70	2179.90
BIC	3516.00	1795.70	1308.50	2122.20	2208.40
NB-1					
$\beta_{AADTmajor}$	**0.53 (< 0.01)**	**0.88 (< 0.01)**	**0.53 (< 0.01)**	**0.35 (< 0.01)**	**0.39 (< 0.01)**
$\beta_{AADTminor}$	**0.44 (< 0.01)**	**0.47 (< 0.01)**	**0.48 (< 0.01)**	**0.70 (< 0.01)**	**0.15 (< 0.01)**
β_0	**−7.75 (< 0.01)**	**−12.25 (< 0.01)**	**−10.15 (< 0.01)**	**−9.19 (< 0.01)**	**−6.08 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	**1.17 (< 0.01)**	**2.71 (< 0.01)**	−126.5 (0.83)	**0.25 (< 0.01)**	**1.73 (< 0.01)**
$\gamma_{AADTminor\ (*10)^{-4}}$	**3.17 (< 0.01)**	1.95 (0.33)	56.33 (0.56)	**5.52 (< 0.01)**	−0.11 (0.58)
γ_0	**−0.44 (< 0.01)**	**−2.35 (< 0.01)**	**−0.40 (< 0.01)**	**−1.00 (< 0.01)**	**−1.30 (< 0.01)**
LL	−1748.46	−897.79	−636.85	−1055.52	−1088.50
AIC	3508.90	1801.60	1285.70	2123.10	2189.00
BIC	3537.40	1830.10	1314.20	2151.50	2217.50
NB-P					
$\beta_{AADTmajor}$	**0.57 (< 0.01)**	**0.92 (< 0.01)**	**0.68 (< 0.01)**	**0.44 (< 0.01)**	**0.37 (< 0.01)**
$\beta_{AADTminor}$	**0.44 (< 0.01)**	**0.48 (< 0.01)**	**0.42 (< 0.01)**	**0.74 (< 0.01)**	**0.17 (< 0.01)**
β_0	**−8.17 (< 0.01)**	**−12.66 (< 0.01)**	**−11.00 (< 0.01)**	**−10.17 (< 0.01)**	**−6.05 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	0.80 (0.13)	−1.80 (0.31)	−3.40 (0.12)	**−0.70 (< 0.01)**	**1.31 (< 0.01)**
$\gamma_{AADTminor\ (*10)^{-4}}$	2.24* (0.08)	−14.10* (0.06)	−35.40 (0.36)	**1.86 (< 0.01)**	**−2.20 (< 0.01)**
γ_0	**−0.53 (0.01)**	0.99 (0.42)	**5.22 (0.02)**	**−0.40 (< 0.01)**	**−1.03 (< 0.01)**
P	**1.92 (< 0.01)**	**1.10 (0.02)**	**1.75 (< 0.01)**	**1.49 (< 0.01)**	**1.59 (< 0.01)**
LL	−1737.81	−876.23[a]	−633.69[a]	−1039.26[a]	−1083.87[a]
AIC	3489.60	1766.50	1281.40	2092.50	2181.70
BIC	3522.80	1799.70	1314.60	2125.70	2215.00

NB-P model. It seems that the flexibility of variance structure in the NB-P model can more effectively capture the variation of over-dispersion among intersections in NB models, which ultimately helps account for the data heterogeneity in intersection SPF development.

5. SPF validation and prediction comparison

Other than the model goodness-of-fit, the models were also validated and compared based on the prediction ability. We randomly selected 80% of the data to estimate the SPFs, and used the remaining 20% data to evaluate and compare the SPF prediction performance based on four criteria, *i.e.* percentage of data points outside of the two standard deviation limits in the cumulative residual analysis (% CURE Deviation), calibration factor of the estimated SPFs (Calibration Factor), R^2 and mean absolute deviation (MAD). The % CURE Deviation analysis is used to verify if the model assumption is valid and the model prediction is robust. It measures the residuals as the difference between predicted and observed crashes, sorts the sites based on the ascending order of AADT values, calculates the cumulative residuals for the sorted sites, and plots the cumulative residuals against the AADT. If the estimated SPF is valid and robust, the CURE plot should locate between the two threshold limits, *i.e.* from the negative two standard deviation to the positive two standard deviation calculated for each data point. The % CURE Deviation statistic measures the percentage of data points outside of the aforementioned two standard deviation limits. Lower value represents a better estimated SPF. The calibration factor is used to calculate the ratio between the total predicted and total observed crashes for the validation data sets, with a value close to 1 indicating a better SPF. The value of R^2 is between 0 and 1, and the SPF is preferable if the R^2 is close to 1. The MAD measures the average absolute difference between the predicted and observed crashes, and a lower MAD value represents a better SPF (Wang et al., 2017a,b). A detailed discussion of these evaluation criteria is available in Lyon et al. (2016).

Table 5 shows the model validation and comparison results. Bold statistics represent the model with the best performance. The crash prediction performance at rural-two lane intersections by crash type is similar across the three NB regression models. With regard to both 3ST and 4SG intersections, the NB-P model slightly outperforms the other two models for the TOT, SDC, ODC and SVC crashes, and the NB-1 model performs the best for the IDC crashes. In terms of the 4ST intersections, the model prediction performance of the three NB models varies among different crash types. Specifically, the NB-1 model is verified to be the best model for predicting the TOT crashes, the NB-2 model is shown to perform the best for predicting the SDC and ODC crashes, while the NB-P model outperforms the NB-1 and NB-2 models for predicting the IDC and SVC crashes. Note that the R^2 values for the 4SG models are very low. This is not unexpected and it can be verified from the model estimation results that the estimated coefficients for the 4SG models are rarely significant, due to the low sample size of 4SG intersections.

Overall, based on the model goodness-of-fit and prediction performance, the NB-P model improves the crash prediction accuracy by allowing a flexible variance structure for most types of crashes at rural two-lane intersections, especially for the 3ST and 4SG intersections. Furthermore, the parameterization of the over-dispersion factor illustrates that the over-dispersion parameter of the NB models highly depends on how the variance structure is defined in the model, and the NB-P model can more effectively capture the variation of over-

Table 4
Model Estimation Results of 4SG Intersections (N = 132).

	TOT	SDC	ODC	IDC	SVC
Poisson					
$\beta_{AADTmajor}$	**1.07 (< 0.01)**	**1.28 (< 0.01)**	**0.86 (< 0.01)**	**0.76 (< 0.01)**	**1.01 (< 0.01)**
$\beta_{AADTminor}$	**0.27 (< 0.01)**	**0.34 (< 0.01)**	**0.36 (< 0.01)**	0.13 (0.18)	0.15 (0.33)
β_0	**−11.74 (< 0.01)**	**−14.93 (< 0.01)**	**−12.33 (< 0.01)**	**−9.18 (< 0.01)**	**−12.64 (< 0.01)**
LL	−556.01	−390.72	−226.67	−268.41	−153.69
AIC	1118.00	787.40	459.30	542.80	313.40
BIC	1126.60	796.10	468.00	551.40	322.00
NB-2					
$\beta_{AADTmajor}$	**0.94 (< 0.01)**	**1.07 (< 0.01)**	**0.86 (< 0.01)**	**0.72 (0.01)**	**1.05 (< 0.01)**
$\beta_{AADTminor}$	**0.38 (0.01)**	**0.40 (< 0.01)**	**0.39 (< 0.01)**	0.16 (0.37)	0.21 (0.25)
β_0	**−11.54 (< 0.01)**	**−13.51 (< 0.01)**	**−12.57 (< 0.01)**	**−9.08 (< 0.01)**	**−13.53 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	**−0.11 (0.03)**	−1.00* (0.09)	−1.60 (0.65)	−0.90 (0.19)	−1.20 (0.61)
$\gamma_{AADTminor\ (*10)^{-4}}$	**2.05 (0.02)**	**2.14 (0.04)**	3.39 (0.46)	1.07 (0.43)	1.01 (0.32)
γ_0	**−0.51 (0.04)**	**−0.70 (0.05)**	**−0.72 (< 0.01)**	0.04* (0.08)	**0.11 (< 0.01)**
LL	−404.15	−323.92	−212.90	−242.45	−150.80
AIC	820.30	659.80	437.80	496.90	313.60
BIC	837.60	677.10	455.10	514.20	330.90
NB-1					
$\beta_{AADTmajor}$	**0.93 (< 0.01)**	**1.10 (< 0.01)**	**0.45 (< 0.01)**	**0.80 (0.01)**	**1.00 (< 0.01)**
$\beta_{AADTminor}$	**0.38 (< 0.01)**	**0.53 (< 0.01)**	**0.28 (< 0.01)**	0.11 (0.54)	0.14 (0.44)
β_0	**−11.11 (< 0.01)**	**−14.35 (< 0.01)**	**−7.94 (< 0.01)**	**−9.41 (< 0.01)**	**−12.53 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	**−0.30 (< 0.01)**	**0.03 (< 0.01)**	−1.00 (0.86)	−0.40 (0.58)	0.34 (0.82)
$\gamma_{AADTminor\ (*10)^{-4}}$	**3.06 (< 0.01)**	**3.02 (< 0.01)**	4.35 (0.75)	1.78 (0.23)	1.56 (0.98)
γ_0	**0.34 (< 0.01)**	**−0.66 (0.02)**	**−1.37 (< 0.01)**	**−0.12 (0.03)**	**−2.05 (0.05)**
LL	−405.44	−326.94	−212.10	−242.39	−150.41[a]
AIC	822.90	665.90	436.20	496.80	321.80
BIC	840.10	683.10	453.50	514.00	330.10
NB-P					
$\beta_{AADTmajor}$	**1.10 (< 0.01)**	**1.16 (< 0.01)**	**0.86 (< 0.01)**	**0.73 (< 0.01)**	**0.95 (< 0.01)**
$\beta_{AADTminor}$	**0.21 (< 0.01)**	**0.43 (< 0.01)**	**0.38 (< 0.01)**	0.18* (0.07)	0.16 (0.71)
β_0	**−11.48 (< 0.01)**	**−14.55 (< 0.01)**	**−12.46 (< 0.01)**	**−9.31 (< 0.01)**	**−12.14 (< 0.01)**
$\gamma_{AADTmajor\ (*10)^{-4}}$	**−0.80 (< 0.01)**	**−0.80 (< 0.01)**	−4.5 (0.69)	**−2.60 (< 0.01)**	**6.46 (< 0.01)**
$\gamma_{AADTminor\ (*10)^{-4}}$	2.40* (0.09)	**2.32 (< 0.01)**	0.02 (0.77)	−0.20 (0.84)	**4.14 (< 0.01)**
γ_0	**−0.21 (< 0.01)**	**−0.69 (< 0.01)**	**2.00 (< 0.01)**	**0.63 (< 0.01)**	**−11.60 (< 0.01)**
P	**1.59 (< 0.01)**	**1.73 (< 0.01)**	**1.38 (< 0.01)**	**1.52 (< 0.01)**	**1.75 (< 0.01)**
LL	−402.62[a]	−322.45[a]	−210.26[a]	−241.66[a]	−150.75
AIC	819.20	658.90	434.50	497.30	315.50
BIC	839.40	679.0	454.70	517.50	335.60

dispersion among intersections in NB models which benefits the accommodation of data heterogeneity in intersection SPF development. Inside of each intersection type, the over-dispersion parameter is proved to vary among different intersections corresponding to each crash type and can be estimated using both the major and minor road AADT at rural two-lane intersections.

6. Discussion and conclusions

This study demonstrates the application of different NB models for

Table 5
Model Performance Validation and Comparison.

Crash Type	Criteria	3ST Intersections (N = 152)			4ST Intersections (N = 213)			4SG Intersections (N = 34)		
		NB-2	**NB-1**	**NB-P**	**NB-2**	**NB-1**	**NB-P**	**NB-2**	**NB-1**	**NB-P**
TOT	% CURE Deviation	1%	1%	**1%**	0%	**0%**	0%	24%	24%	**18%**
	Calibration Factor	0.94	1.05	**0.99**	1.10	**1.01**	1.11	1.10	0.78	**0.98**
	R^2	0.36	0.35	**0.35**	0.67	**0.67**	0.67	0.00	0.00	**0.00**
	MAD	2.25	2.27	**2.25**	1.81	**1.82**	1.81	5.88	5.87	**5.74**
SDC	% CURE Deviation	19%	44%	**16%**	**3%**	3%	3%	38%	29%	**26%**
	Calibration Factor	0.95	0.76	**0.96**	**0.86**	0.84	0.85	0.52	0.84	**0.89**
	R^2	0.51	0.49	**0.53**	**0.72**	0.71	0.72	0.00	0.01	**0.05**
	MAD	0.64	0.68	**0.64**	**0.55**	0.55	0.55	2.99	2.94	**2.90**
ODC	% CURE Deviation	13%	17%	**11%**	**5%**	17%	6%	0%	12%	**0%**
	Calibration Factor	1.02	1.02	**0.97**	**1.13**	1.14	1.16	1.08	1.17	**1.05**
	R^2	0.29	0.28	**0.30**	**0.71**	0.66	0.71	0.17	0.13	**0.17**
	MAD	0.54	0.54	**0.54**	**0.45**	0.46	0.45	1.75	1.75	**1.75**
IDC	% CURE Deviation	13%	**9%**	9%	12%	18%	**11%**	9%	**6%**	9%
	Calibration Factor	1.17	**1.00**	1.12	1.06	1.10	**1.05**	1.17	**1.17**	1.13
	R^2	0.26	**0.27**	0.27	0.41	0.42	**0.40**	0.00	**0.00**	0.00
	MAD	0.50	**0.50**	0.49	0.90	0.91	**0.90**	2.23	**2.20**	2.24
SVC	% CURE Deviation	1%	1%	**1%**	26%	29%	**26%**	3%	3%	**0%**
	Calibration Factor	1.02	1.02	**1.00**	1.29	1.25	**1.25**	1.37	1.44	**1.31**
	R^2	0.20	0.19	**0.20**	0.42	0.41	**0.41**	0.04	0.08	**0.07**
	MAD	1.32	1.32	**1.32**	0.91	0.91	**0.89**	1.12	1.11	**1.11**

Notes: Bold statistics represent the model with the best performance.

estimating SPFs by crash type at rural two-lane intersections. Intersection data from the State of Minnesota were collected. Crash types were aggregated into same-direction, opposite-direction, intersecting-direction and single-vehicle crashes. Major and minor road AADT were used as predictors in the SPF estimation for the Poisson, NB-1, NB-2 and NB-P models. Major and minor road AADT were also used as predictors in the estimation of the over-dispersion parameter of all NB models to account for the crash data heterogeneity.

The SPF estimation results show that the traffic volume is statistically significant in most of the intersection crash prediction models, and the estimated coefficients of AADT are significantly different across different crash types, which verifies the necessity of developing SPFs by crash type for rural two-lane intersections. The model goodness-of-fit indicates that the NB-P model outperforms the NB-1 and NB-2 for most crash types and intersection types, which verifies that the NB-P model can lead to more accurate coefficient estimates by providing a flexible variance structure to the NB models. Instead of being considered as a constant value, the over-dispersion parameter of the NB models was estimated by the AADT values. The parameterization of the over-dispersion factor verifies that the over-dispersion parameter of the NB models highly depends on how the variance structure is defined in the model, and the over-dispersion parameter is shown to vary across different intersections with regard to each crash type and can be estimated using both the major and minor road AADT at rural two-lane intersections. The NB-P model can more effectively capture the variation of over-dispersion among intersections in NB models which benefits the accommodation of data heterogeneity in intersection SPF development. Finally, the estimated SPFs by different NB models were validated and compared using a hold-out dataset. The comparison results illustrate that the NB-P model slightly improves the crash prediction accuracy compared with the other two models, especially for the 3ST and 4SG intersections. In summary, we conclude that the NB-P model with parameterized over-dispersion factor can lead to more accurate SPF coefficient estimates and crash predictions, and should be considered in estimating SPFs by crash type for rural two-lane intersections.

It is expected that this research can offer additional insight about the selection of functional forms of the negative binomial models in estimating safety performance functions for rural two-lane intersections. This study focuses on the estimation of planning-level SPFs where only the traffic volumes were considered in the models. Future work can focus on collecting other roadway geometric factors to estimate the fully specified SPFs and verify the performance of the NB-P model. One significant challenge of this study is that the crash data used in the SPF estimation is outdated, due to the current data availability. Future work can focus on collecting the most recent data to estimate and validate the model performance. Future work can also target on estimating the NB-P model for other intersection types, such as rural multilane intersections, urban and suburban arterials intersections and freeway ramp terminals.

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References

Abdel-Aty, M., Radwan, A., 2000. Modeling traffic accident occurrence and involvement. Accid. Anal. Prev. 32 (5), 633–642.

Abdel-Aty, M., Kellerand, J., Brady, P., 2005. Analysis of types of crashes at signalized intersections by using complete crash data and tree-based regression. Transp.Res. Rec. 1908: J. Transp. Res. Board 37–45.

Cameron, A., Trivedi, P., 1986. Econometric models based on count data. Comparisons and applications of some estimators and tests. J. Appl. Econ. 1 (1), 29–53.

Chang, G., Xiang, H., 2003. The Relationship between Congestion Levels and Accidents. State Highway Administration. Research Report. MD-03-SP 208B46. Maryland State Highway Administration.

Dixon, K., Monsere, C., Avelar, R., Barnett, J., Escobar, P., Kothuri, S., Wang, Y., 2015. Improved Safety Performance Functions for Signalized Intersections. Oregon Department of Transportation.

Gates, T., Savolainen, P., Avelar, R., Geedipally, S., Lord, D., Ingle, A., Stapleton, S., 2018. Safety Performance functions for Rural Road Segments and Rural Intersections in Michigan. Michigan Department of Transportation Final Report.

Geedipally, S., Lord, D., 2008. Effects of varying dispersion parameter of Poisson–Gamma models on estimation of confidence intervals of crash prediction models. Transp. Res. Rec. 2061: J. Transp. Res. Board 46–54.

Geedipally, S., Lord, D., 2010. Investing the effect of modeling single-vehicle and multi-vehicle crashes separately on confidence intervals of Poisson-Gamma models. Accid. Anal. Prev. 42 (4), 1273–1282.

Geedipally, S., Patil, S., Lord, D., 2010. Examination of methods to estimate crash counts by collision type. Transp. Res. Rec. 2165: J. Transp. Res. Board 12–20.

Greene, W., 2008. Functional forms for the negative binomial model for count data. Econ. Lett. 99 (3), 585–590.

Harwood, D., Council, F., Hauer, E., Hughes, W., Vogt, A., 2000. Prediction of the Expected Safety Performance of Rural Two-Lane Highways. Federal Highway Administration, U.S. Department of Transportation Report Number: FHWA-RD-99-207.

Hauer, E., 2000. Observational Before-After Studies in Road Safety: Estimating the Effects of Highway and Traffic Engineering Measures on Road Safety. Elsevier Science Ltd.

Hauer, E., 2001. Overdispersion in modelling accidents on road sections and in empirical bayes estimation. Accid. Anal. Prev. 33 (6), 799–808.

Hauer, E., Ng, J., Lovell, J., 1988. Estimation of safety at signalized intersections. Transp. Res. Rec. 1185: J. Transp. Res. Board 48–61.

Highway Safety Manual (HSM), 2010. American Association of State Highway and Transportation Officials. Washington D.C. 1st edition. .

Hilbe, J., 2007. Negative Binomial Regression. Cambridge University Press, Cambridge, U.K.

HSIS, 2019. Highway Safety Information System (HSIS). Federal Highway Administration (FHWA). http://www.hsisinfo.org/datarequest.cfm.

Ismail, N., Zamani, H., 2013. Estimation of Claim Count Data using Negative Binomial, Generalized Poisson, Zero-Inflated Negative Binomial and Zero-Inflated Generalized Poisson Regression Models. Casualty Actuarial Society E-Forum. . https://www.casact.org/pubs/forum/13spforum/Ismail%20Zamani.pdf.

Ivan, J., Wang, C., Bernard, N., 2000. Exploring two-lane highway crash rates using land use and hourly exposure. Accid. Anal. Prev. 32 (6), 787–795.

Liu, C., Sharma, A., 2018. Using the multivariate spatio-temporal bayesian model to analyze traffic crashes by severity. Anal. Methods Accid. Res. 17, 14–31.

Liu, C., Zhao, M., Li, W., Sharma, A., 2018. Multivariate random parameters zero-inflated negative binomial regression for analyzing urban midblock crashes. Anal. Methods Accid. Res. 17, 32–46.

Lord, D., Mannering, F., 2010. The statistical analysis of crash-frequency data: a review and assessment of methodological alternatives. Transp. Res. Part A Policy Pract. 44 (5), 291–305.

Lord, D., Park, P., 2008. Investigating the effects of the fixed and varying dispersion parameters of poisson-gamma models on empirical bayes estimates. Accid. Anal. Prev. 40 (4), 1441–1457.

Lord, D., Park, B., 2015. Appendix D: Negative Binomial Regression Models and Estimation Methods. https://www.icpsr.umich.edu/CrimeStat/files/CrimeStatAppendix.D.pdf.

Lord, D., Persaud, B., 2000. Accident prediction models with and without trend: application of the generalized estimating equations procedure. Transp. Res. Rec. 1717: J. Transp. Res. Board 102–108.

Lord, D., Geedipally, S., Persaud, B., Washington, S., Schalkwyk, I., Ivan, J., Lyon, C., Jonsson, T., 2008. Methodology to Predict the Safety Performance Function of Rural Multilane Highways. NCHRP 126. Transportation Research Board of the National Academics., Washington D.C.

Lyon, C., Oh, J., Persaud, B., Washington, S., Bared, J., 2003. Empirical investing of interactive highway safety design model accident prediction algorithm: rural intersections. Transp. Res. Rec. 1840: J. Transp. Res. Board 78–86.

Lyon, C., Haq, A., Persaud, B., Kodama, S., 2005. Safety performance functions for signalized intersections in large urban areas: development and application to evaluation of left-turn priority treatment. Transp. Res. Rec. 1908: J. Transp. Res. Board 165–171.

Lyon, C., Persaud, B., Gross, F., 2016. The Calibrator – an SPF Calibration and Assessment Tool User Guide. Report Number: FHWA-SA-17-016. .

Mannering, F., Shankar, V., Bhat, C., 2016. Unobserved heterogeneity and the statistical analysis of highway accident data. Anal. Methods Accid. Res. 11, 1–16.

Mehta, G., Lou, Y., 2013. Calibration and development of safety performance functions for Alabama two-Lane two-way rural roads and four-lane divided highways. Transp. Res. Rec. 2398: J. Transp. Res. Board 75–82.

Miaou, S., Lord, D., 2003. Modeling traffic crash-flow relationships for intersections: dispersion parameter, functional form, and Bayes versus empirical bayes methods. Transp. Res. Rec. 1840: J. Transp. Res. Board 31–40.

Mitra, S., Washington, S., 2007. On the nature of Over-Dispersion in motor vehicle crash prediction models. Accid. Anal. Prev. 39 (3), 459–468.

National Highway Traffic Safety Administration (NHTSA), 2015. Traffic safety factors 2015. A Compilation of Motor Vehicle Crash Data From the Fatality Analysis Reporting System and the General Estimates System. . https://crashstats.nhtsa.dot.gov/Api/Public/Publication/812384.

Oh, J., Washington, S., Choi, K., 2004. Development of accident prediction models for rural highway intersections. Transp. Res. Rec. 1897: J. Transp. Res. Board 18–27.

Park, J., 2010. Application of the Finite Mixture Models for Vehicle Crash Data Analysis. PhD Dissertation. Texas A&M University, College Station, TX.

Poch, M., Mannering, F., 1996. Negative binomial analysis of intersection-accident. Frequencies. J. Transp. Eng. 122 (2), 105–113.

Qin, X., 2002. Selecting Exposure Measures for Predicting Crash Rates on Two-lane Rural

Highways. Doctoral Dissertations. Paper AAI3066254. http://digitalcommons.uconn.edu/dissertations/AAI3066254.
Qin, X., Ivan, J., Ravishanker, N., 2004. Selecting exposure measures in crash rate prediction for two-lane highway segments. Accid. Anal. Prev. 36 (2), 183–191.
Russo, F., Busiello, M., Dell, A., 2016. Safety performance functions for crash severity on undivided rural roads. Accid. Anal. Prev. 93, 75–91.
Tarko, A., Inerowicz, M., Ramos, J., Li, W., 2008. Tool with road-level crash prediction for transportation safety planning. Transp. Res. Record 2083: J. Transp. Res. Board 16–25.
Ulfarsson, G., Shankar, V., 2002. Accident count model based on multiyear cross-sectional roadway data with serial correlation. Transp. Res. Rec. 1840: J. Transp. Res. Board 193–197.
Vuong, Q., 1989. Likelihood ratio test for model selection and non-nested hypotheses. Econometrica 57, 307–344.
Wang, K., Ivan, J., Nalini, R., Jackson, E., 2017a. Multivariate poisson lognormal modeling of crashes by type and severity on rural two lane highways. Accid. Anal. Prev. 99 (Part A), 6–19.
Wang, K., Ivan, J., Burnicki, A., Mamun, S., 2017b. Predicting local road crashes using socioeconomic and land cover data. J. Transp. Saf. Secur. 9 (3), 301–318.
Wang, K., Zhao, S., Jackson, E., 2018. Multivariate poisson lognormal modeling of weather-related crashes on freeways. Transp. Res. Rec.: J. Transp. Res. Board. 1–15.
Washington, S., Karlaftis, M., Mannering, F., 2011. Statistical and Econometric Methods for Transportation Data Analysis, 2nd ed. Chapman and Hall/CRC, Boca Raton, FL.
Wu, Z., Sharma, A., Mannering, F.L., Wang, S., 2013. Safety impacts of signal-warning flashers and speed control at high-speed signalized intersections. Accid. Anal. Prev. 54, 90–98.
Wu, Y., Abdel-Aty, M., Cai, Q., Lee, J., Park, J., 2018. Developing an algorithm to assess the rear-end collision risk under fog conditions using real-time data. Transp. Res. Part C Emerg. Technol. 87, 11–25.
Xie, F., Gladhill, K., Dixon, K., Monsere, C., 2011. Calibration of highway safety manual predictive models for Oregon state highways. Transp. Res. Rec. 2241: J. Transp. Rese. Board 19–28.
Zhao, M., Liu, C., Li, W., Sharma, A., 2018. Multivariate Poisson-lognormal model for analysis of crashes on urban signalized intersections approach. J. Transp. Saf. Secur. 10 (3), 251–265.